Calculation of the Hubble Constant, the Minimum Mass and the Proton Charge Radius Using the Dirac’s Hypothesis on the Ratio of the Electrostatic Force to the Gravitational Force

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Abstract: Currently, several physical constants are determined by observation. This is the case for the Hubble constant and the proton charge radius whose observed values involve large uncertainties. This publication suggests that these values could be calculated more precisely using algebraic equations involving other physical constants. To do so, some assumptions must be put forward, namely, the Dirac’s hypothesis on the observed ratio of the electrostatic force to the gravitational force. The approach used also allows calculating the value of a minimum mass. The calculated value of the Hubble constant is: \( H \approx 72.013 \, \text{km s}^{-1} \text{Mpc}^{-1} \), and that of the minimum mass: \( M_{\text{min}} \approx 1.7206 \times 10^{-68} \, \text{kg} \). Recent observations suggest that the proton charge radius could also be calculated using an additional but related assumption: \( r_p \approx 0.8264 \, \text{fm} \).

Keywords: Cosmology, Hubble Constant, Minimum Mass, Graviton, Proton Charge Radius, Dimensionless Constants

Introduction

The local value of the Hubble constant is determined by observation. The observed values are more or less precise and oscilate between 70 and 74 km s\(^{-1}\) Mpc\(^{-1}\), depending on the techniques used (Riess et al., 2022; Freedman, 2021; Khetan et al., 2021; Liao et al., 2019; Pesce et al., 2020; Yang et al., 2020).

As for the proton charge radius, large uncertainties are also observed (Gao and Vanderhaeghen, 2022).

To calculate these values more precisely, we must assume some inherent relationships with others physical constants. The proposed relationships in this study rely on the Dirac’s hypothesis on the observed ratio of the electrostatic force to the gravitational force.

Materials and Methods

We know that some physical quantities are limited by minimum values. For example, the quantum of action \( h \) is the minimum value of the action. The elementary charge \( e \) can also be regarded as the minimum charge. Quarks are associated with a fractional charge, but they are confined inside hadrons, which do have an integer charge (Perl et al., 2004).

Hypothesis About Minimum Values

The working hypothesis is that other physical quantities would also be limited by minimum values (subscript \( \text{min} \)).

The Hubble constant \( H \) (of dimension T\(^{-1}\)) would correspond to the minimum frequency:

\[
f_{\text{min}} = \frac{\hbar}{\Delta E} \tag{1}
\]

Since \( E = h f \), the minimum energy would be:

\[
E_{\text{min}} = h f_{\text{min}} = hH \tag{2}
\]

Since \( M = E/c^2 \), the minimum mass would be:

\[
M_{\text{min}} = E_{\text{min}} / c^2 = hH / c^2 \tag{3}
\]

To calculate \( H \), another, but more precise value must be involved. Since May 2019, the value of the elementary charge is fixed at \( e = 1.602176 \, 634 \times 10^{-19} \, \text{C} \).
Fig. 1: Chart of the assumed relationships between physical values; Physical values are represented by circles and relationships between them by arrows, each with a boxed formula. Each arrow indicates the direction of the relationship expressed by the formula linking two quantities. This direction is arbitrary and can be reversed by changing the formula. For instance, the relationship \((H = 2ac/\beta r_p)\) can be written \((r_p = 2ac/\beta H)\). The dotted lines depict the additional assumption (Eq. 16) about the calculated value of \(r_p\).

The Ratio of the Electrostatic Force to the Gravitational Force

The ratio, say \(\beta\), of the electrostatic force to the gravitational force between an electron and a proton is:

\[
\beta = \frac{e^2}{4\pi \varepsilon_0 G m_e m_p} \approx 2.26866 \times 10^{36}
\]  (4)

- \(\varepsilon_0\) = The vacuum permittivity
- \(G\) = The gravitational constant
- \(m_e\) = The mass of the electron
- \(m_p\) = The mass of the proton

In the last century, Paul Dirac popularized this \(\beta\) ratio in a cosmological context (Dirac, 1938; 1974; 1979). The value of the \(\beta\) ratio depends on the masses of the interacting particles. This study sticks to the natural choice made by Dirac (the electron and the proton). In this context, these two particles are regarded as two sides of the same coin: The electron-proton couple (subscript \(ep\)).

The gravitational force is tiny compared to other forces, so the \(\beta\) ratio is sometimes called a dimensionless “Large Number”. It still gets attention nowadays (Ray et al., 2019; Cetto et al., 1986; Berman, 1992; Lau and Prokhovnik, 1986).

Calculations Using the \(\beta\) Ratio

Gravitational and electrostatic forces are conservative, which implies that the \(\beta\) ratio applies to both forces and energies. An energy \(E_{ep}\) can thus be associated with the electron-proton couple:

\[
E_{ep} = \beta E_{min} = \beta \hbar H
\]  (5)

That could be calculated precisely using the assumed relationship:

\[
E_{ep}^2 = m_e m_p \ c^4
\]  (6)

where, \(c\) is the speed of light in a vacuum. Using the 2018 CODATA recommended values, we get:

\[
E_{ep} = \sqrt{m_e m_p \ c^2} \approx 3.5082 \times 10^{-12} \ J, \text{ or } 21.9 \text{ MeV}
\]  (7)

Since \(E_{ep} = \beta \hbar H\), we deduce that:

\[
H = \frac{E_{ep}}{\beta \hbar} \approx 2.3338 \times 10^{-18} \text{ s}^{-1}, \text{ or } 72.013 \text{ km s}^{-1} \text{ Mpc}^{-1}
\]  (8)

Halfway between low and high observed values. This means that a frequency \(f_{ep}\) can be associated with the electron-proton couple:
Expanding Eq. 8, we note that physical constants would be interlinked:

\[ H = 4\pi e_0 G (m_e m_p)^{3/2} c^2 / h e^2 \]  

(10)

The value of the minimum energy would be:

\[ E_{\text{min}} = \hbar H \approx 1.5464 \times 10^{-51} \text{ J} \]  

(11)

and that of the minimum mass:

\[ M_{\text{min}} = \hbar H / c^2 \approx 1.7206 \times 10^{-68} \text{ kg} \]  

(12)

The very definition of the \( \beta \) ratio suggests that this tiny mass might be that of the graviton, provided such a particle exists.

**Previous Calculations of Minimum Values**

Wesson (2004) suggests a quantum of mass \( (m_q \approx 2 \times 10^{-68} \text{ kg}) \) that coincides with \( M_{\text{min}} \). This tiny mass is calculated using the constants \( c \), \( \hbar \), and \( A \) (the cosmological constant). According to Wesson, \( m_q = (\hbar/c) (\Lambda/3)^{2/3} \), compared to \( M_{\text{min}} \approx H / c^2 \) under the present hypothesis. We deduce that:

\[ A = 3H^2 / c^2 \approx 1.818 \times 10^{-52} \text{ m}^2 \]  

(13)

according to Wesson's \( A \) hypothesis. In the conformal cosmological model (ΛCDM), this value is multiplied by a factor \( \Omega_\Lambda \approx 0.7 \) (Lusso et al., 2019; Shuntov, 2018).

Valev (2010; 2013; 2015), suggests a minute mass \( (m_H \approx \hbar H / c^2) \) using dimensional analysis. This tiny mass might be that of the graviton and is of the same order of magnitude as the minimum mass \( M_{\text{min}} \).

Likewise, using dimensional analysis, (Rushdi and Rushdi, 2016) deduced a dimensionless product \( (\pi_{ad} \approx \hbar H/m_2c^2) \) where \( m_2 \) equals \( M_{\text{min}} \) when \( \pi_{ad} = 1 \).

Talbot (2021) is not specifically aimed at the research community, but this book contains several equations, including those used for the calculation of \( H \) and \( M_{\text{min}} \).

Meanwhile and independently, (Wilmot, 2021) was posted on ResearchGate. Although the reasoning differs and implies a fictitious particle called «mason», the calculations and the resulting values of \( H \) and \( M_{\text{min}} \) are almost identical. The anteriority of the assumed calculations of these values goes to Mr. Wilmot.

Among related research, some calculated results come close to the suggested value of \( H \):

<table>
<thead>
<tr>
<th>This study</th>
<th>(Wilmot, 2021)</th>
<th>(Mercier, 2019)</th>
<th>(Kritov, 2021)</th>
<th>(Wolf, 2022)</th>
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<tr>
<td></td>
<td>(72.013 km s^{-1} Mpc^{-1})</td>
<td>(72.095 km s^{-1} Mpc^{-1})</td>
<td>(71.995 km s^{-1} Mpc^{-1})</td>
<td>(71.994 km s^{-1} Mpc^{-1})</td>
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**The Charge Radius of the Proton**

In addition to the agreement of the calculated values with observation and previous works, there is something else peculiar about this value of \( E_{\text{cp}} \). Indeed, we calculate that:

\[ e^2 / \epsilon_0 E_{\text{cp}} \approx 0.8264 \text{ fm} \approx r_\beta \]  

(14)

where, \( r_\beta \) is the proton charge radius. When measured by an electron-proton interaction, the calculated value of \( r_\beta \) matches (within 1\sigma) many recent observed values (Atou et al., 2023; Xiong et al., 2019; Bezginov et al., 2019; Beyer et al., 2017; Djukanovic et al., 2021). It is unlikely that an arbitrary value of \( E_{\text{cp}} \) could lead to such consistent results, so this fact makes up an additional but related assumption.

Replacing \( E_{\text{cp}} \) by \( \beta H \) (Eq. 5), this new assumption can be written:

\[ r_\beta = (e^2 / \epsilon_0 h) / \beta H \]  

(15)

Using the fine-structure constant \( (\alpha = e^2 / 2 \epsilon_0 h c) \), Eq. 15 then becomes:

\[ r_\beta = 2\alpha c / \beta H = 2\alpha c / f_{\text{cp}} \]  

(16)

where, \( \alpha c \) is the speed of the electron in the ground state of the hydrogen atom. Inversely:

\[ H = 2\alpha c / \beta r_\beta \]  

(17)

This last equation might be a clue to the enigmatic values of the dimensionless constants \( \alpha \) and \( \beta \).

It is worth highlighting that in agreement with Eq. 14 and Eq. 16, we do observe:

\[ r_\beta \approx e^2 / \epsilon_0 E_{\text{cp}} \approx 2\alpha c / \beta H \approx 0.83 \text{ fm} \]  

(18)

According to known physics, \( r_\beta \) should neither be related to \( E_{\text{cp}} \) nor to \( H \). It is like winning the lottery twice in a row with the same combination.

Using the known relationship \( \epsilon_0 \mu_0 = 1/c^2 \), \( r_\beta \) can then be expressed in relation to the vacuum permeability \( (\mu_0) \):

\[ r_\beta = \mu_0 e^2 / (m_e m_p)^{1/2} \]  

(19)

**Results and Discussion**

The calculated value of the Hubble constant is:

\[ H \approx 72.013 \text{ km s}^{-1} \text{ Mpc}^{-1} \], that of the minimum mass:
\[ M_{\text{min}} \approx 1.7206 \times 10^{-68} \, \text{kg}, \text{ and that of the proton charge} \]
\[ r_p \approx 0.8264 \, \text{fm}. \]

It is unlikely that an arbitrary value of \( E_{cp} \) could lead to such consistent results, so there may indeed be some inherent properties associated with the electron-proton couple, as Dirac assumed.

Moreover, these results suggest that some physical quantities, like mass, energy and frequency could be limited by minimum values.

**Conclusion**

Using the observed ratio of the electrostatic force to the gravitational force, the assumed relationship Eq. 6 allows precisely calculating the Hubble constant and the minimum mass. These values agree with observation and previous works. Recent observations suggest that the proton charge radius could also be calculated using an additional but related assumption (Eq. 16). More precise observed values of \( H \) and \( r_p \) could confirm or refute some of these hypotheses.

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**Ethics**

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**References**


