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Calculation of the Hubble Constant, the Minimum Mass and the Proton Charge Radius Using the Dirac's Hypothesis on the Ratio of the Electrostatic Force to the Gravitational Force

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Abstract: Currently, several physical constants are determined by observation. This is the case for the Hubble constant and the proton charge radius whose observed values involve large uncertainties. This publication suggests that these values could be calculated more precisely using algebraic equations involving other physical constants. To do so, some assumptions must be put forward, namely, the Dirac's hypothesis on the observed ratio of the electrostatic force to the gravitational force. The approach used also allows calculating the value of a minimum mass. The calculated value of the Hubble constant is: $H \approx 72.013$ km s^{-1} Mpc^{-1}, and that of the minimum mass: $M_{min} \approx 1.7206\times10^{-68}$ kg. Recent observations suggest that the proton charge radius could also be calculated using an additional but related assumption: $r_p \approx 0.8264$ fm.

Keywords: Cosmology, Hubble Constant, Minimum Mass, Graviton, Proton Charge Radius, Dimensionless Constants

Introduction

The local value of the Hubble constant is determined by observation. The observed values are more or less precise and oscillate between 70 and 74 km s^{-1} Mpc^{-1}, depending on the techniques used (Riess *et al*., 2022; Freedman, 2021; Khetan *et al*., 2021; Liao *et al*., 2019; Pesce *et al*., 2020; Yang *et al*., 2020).

As for the proton charge radius, large uncertainties are also observed (Gao and Vanderhaeghen, 2022).

To calculate these values more precisely, we must assume some inherent relationships with others physical constants (Fig. 1). The proposed relationships in this study rely on the Dirac's hypothesis on the observed ratio of the electrostatic force to the gravitational force.

Materials and Methods

We know that some physical quantities are limited by minimum values. For example, the quantum of action h is the minimum value of the action. The elementary charge e can also be regarded as the minimum charge. Quarks are associated with a fractional charge, but they are confined inside hadrons, which do have an integer charge (Perl *et al*., 2004).

Hypothesis About Minimum Values

The working hypothesis is that other physical quantities would also be limited by minimum values (subscript $_{min}$).

The Hubble constant H (of dimension T^{-1}) would correspond to the minimum frequency:

$$f_{min} = H \tag{1}$$

Since $E = hf$, the minimum energy would be:

$$E_{min} = h\, f_{min} = hH \tag{2}$$

Since $M = E/c^2$, the minimum mass would be:

$$M_{min} = E_{min} / c^2 = hH / c^2 \tag{3}$$

To calculate H, another, but more precise value must be involved. Since May 2019, the value of the elementary charge is fixed at $e = 1.602\,176\,634\times10^{-19}$ C.

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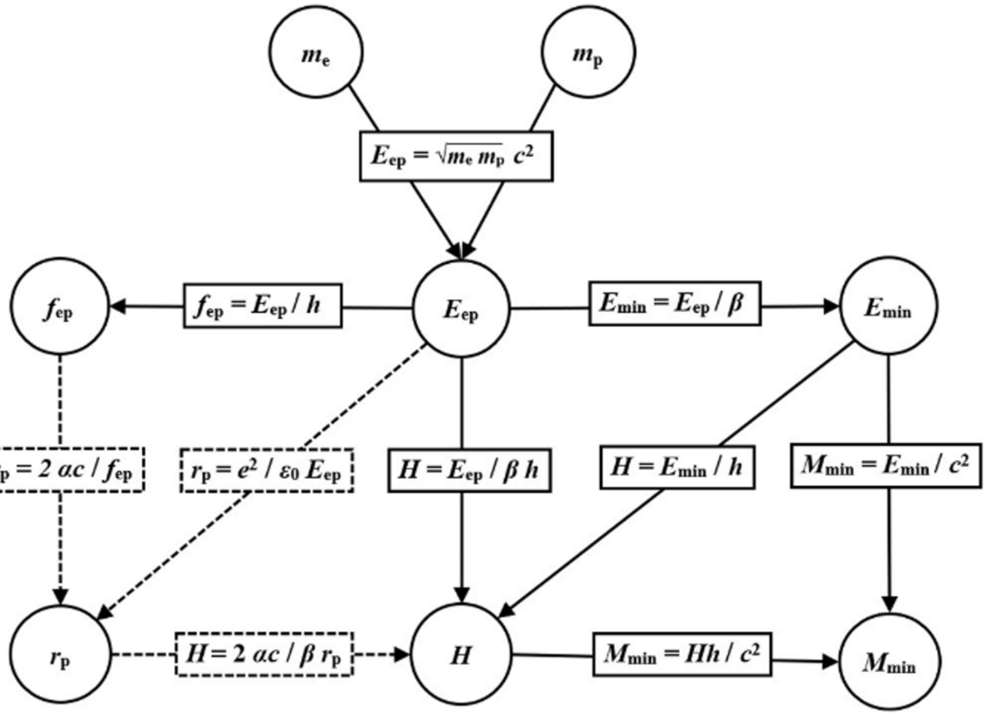

Fig. 1: Chart of the assumed relationships between physical values; Physical values are represented by circles and relationships between them by arrows, each with a boxed formula. Each arrow indicates the direction of the relationship expressed by the formula linking two quantities. This direction is arbitrary and can be reversed by changing the formula. For instance, the relationship ($H = 2\alpha c/\beta r_p$) can be written ($r_p = 2\alpha c/\beta H$). The dotted lines depict the additional assumption (Eq. 16) about the calculated value of r_p.

The Ratio of the Electrostatic Force to the Gravitational Force

The ratio, say β, of the electrostatic force to the gravitational force between an electron and a proton is:

$$\beta = e^2 / 4\pi \varepsilon_0 G m_e m_p \approx 2.26866 \times 10^{39} \quad (4)$$

ε_0 = The vacuum permittivity
G = The gravitational constant
m_e = The mass of the electron
m_p = The mass of the proton

In the last century, Paul Dirac popularized this β ratio in a cosmological context (Dirac, 1938; 1974; 1979). The value of the β ratio depends on the masses of the interacting particles. This study sticks to the natural choice made by Dirac (the electron and the proton). In this context, these two particles are regarded as two sides of the same coin: The electron-proton couple (subscript $_{ep}$).

The gravitational force is tiny compared to other forces, so the β ratio is sometimes called a dimensionless "Large Number". It still gets attention nowadays (Ray *et al*., 2019; Cetto *et al*., 1986; Berman, 1992; Lau and Prokhovnik, 1986).

Calculations Using the β Ratio

Gravitational and electrostatic forces are conservative, which implies that the β ratio applies to both forces and energies. An energy E_{ep} can thus be associated with the electron-proton couple:

$$E_{ep} = \beta E_{min} = \beta h H \quad (5)$$

That could be calculated precisely using the assumed relationship:

$$E_{ep}{}^2 = m_e m_p\ c^4 \quad (6)$$

where, c is the speed of light in a vacuum. Using the 2018 CODATA recommended values, we get:

$$E_{ep} = \sqrt{m_e m_p}\ c^2 \approx 3.5082 \times 10^{-12}\ \text{J, or } 21.9\ \text{MeV} \quad (7)$$

Since $E_{ep} = \beta h H$, we deduce that:

$$H = E_{ep} / \beta h \approx 2.3338 \times 10^{-18}\ \text{s}^{-1}\text{, or } 72.013\ \text{km s}^{-1}\ \text{Mpc}^{-1} \quad (8)$$

Halfway between low and high observed values. This means that a frequency f_{ep} can be associated with the electron-proton couple:

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$$f_{ep} = \beta H = E_{ep} / h \approx 5.2945 \times 10^{21} \text{ s}^{-1} \quad (9)$$

Expanding Eq. 8, we note that physical constants would be interlinked:

$$H = 4\pi\varepsilon_0 G (m_e m_p)^{3/2} c^2 / h e^2 \quad (10)$$

The value of the minimum energy would be:

$$E_{min} = hH \approx 1.5464 \times 10^{-51} \text{ J} \quad (11)$$

and that of the minimum mass:

$$M_{min} = hH / c^2 \approx 1.7206 \times 10^{-68} \text{ kg} \quad (12)$$

The very definition of the β ratio suggests that this tiny mass might be that of the graviton, provided such a particle exists.

Previous Calculations of Minimum Values

Wesson (2004) suggests a quantum of mass ($m_E \approx 2\times10^{-68}$ kg) that coincides with M_{min}. This tiny mass is calculated using the constants c, h, and Λ (the cosmological constant). According to Wesson, $m_E = (h/c)(\Lambda/3)^{1/2}$, compared to $M_{min} = H h / c^2$ under the present hypothesis. We deduce that:

$$\Lambda = 3H^2 / c^2 \approx 1.818 \times 10^{-52} \text{ m}^2 \quad (13)$$

according to Wesson's Λ hypothesis. In the conformal cosmological model (ΛCDM), this value is multiplied by a factor $\Omega_\Lambda \approx 0.7$ (Lusso *et al*., 2019; Shuntov, 2018).

Valev (2010; 2013; 2015), suggests a minute mass ($m_H \approx \hbar H / c^2$) using dimensional analysis. This tiny mass might be that of the graviton and is of the same order of magnitude as the minimum mass M_{min}.

Likewise, using dimensional analysis, (Rushdi and Rushdi, 2016) deduced a dimensionless product ($\pi_{\alpha 4} = hH/m_2c^2$) where m_2 equals M_{min} when $\pi_{\alpha 4} = 1$.

Talbot (2021) is not specifically aimed at the research community, but this book contains several equations, including those used for the calculation of H and M_{min}.

Meanwhile and independently, (Wilmot, 2021) was posted on ResearchGate. Although the reasoning differs and implies a fictitious particle called «*mason*», the calculations and the resulting values of H and M_{min} are almost identical. The anteriority of the assumed calculations of these values goes to Mr. Wilmot.

Among related research, some calculated results come close to the suggested value of H:

This study	(72.013 km s^{-1} Mpc^{-1})
(Wilmot, 2021)	(72.008 km s^{-1} Mpc^{-1})
(Mercier, 2019)	(72.095 km s^{-1} Mpc^{-1})
(Kritov, 2021)	(71.995 km s^{-1} Mpc^{-1})
(Wolf, 2022)	(71.994 km s^{-1} Mpc^{-1})

The Charge Radius of the Proton

In addition to the agreement of the calculated values with observation and previous works, there is something else peculiar about this value of E_{ep}. Indeed, we calculate that:

$$e^2 / \varepsilon_0 E_{ep} \approx 0.8264 \text{ fm} \approx r_p \quad (14)$$

where, r_p is the proton charge radius. When measured by an electron-proton interaction, the calculated value of r_p matches (within 1σ) many recent observed values (Atoui *et al*., 2023; Xiong *et al*., 2019; Bezginov *et al*., 2019; Beyer *et al*., 2017; Djukanovic *et al*., 2021). It is unlikely that an arbitrary value of E_{ep} could lead to such consistent results, so this fact makes up an additional but related assumption.

Replacing E_{ep} by βhH (Eq. 5), this new assumption can be written:

$$r_p = (e^2 / \varepsilon_0 h) / \beta H \quad (15)$$

Using the fine-structure constant ($\alpha = e^2 / 2 \varepsilon_0 h c$), Eq. 15 then becomes:

$$r_p = 2\alpha c / \beta H = 2\alpha c / f_{ep} \quad (16)$$

where, αc is the speed of the electron in the ground state of the hydrogen atom. Inversely:

$$H = 2\alpha c / \beta r_p \quad (17)$$

This last equation might be a clue to the enigmatic values of the dimensionless constants α and β.

It is worth highlighting that in agreement with Eq. 14 and Eq. 16, we do observe:

$$r_p \approx e^2 / \varepsilon_0 E_{ep} \approx 2 \alpha c / \beta H \approx 0.83 \text{ fm} \quad (18)$$

According to known physics, r_p should neither be related to E_{ep} nor to H. It is like winning the lottery twice in a row with the same combination.

Using the known relationship $\varepsilon_0 \mu_0 = 1/c^2$, r_p can then be expressed in relation to the vacuum permeability (μ_0):

$$r_p = \mu_0 e^2 / (m_e m_p)^{1/2} \quad (19)$$

Results and Discussion

The calculated value of the Hubble constant is: $H \approx 72.013$ km s^{-1} Mpc^{-1}, that of the minimum mass:

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$M_{min} \approx 1.7206\times10^{-68}$ kg, and that of the proton charge radius: $r_p \approx 0.8264$ fm.

It is unlikely that an arbitrary value of E_{ep} could lead to such consistent results, so there may indeed be some inherent properties associated with the electron-proton couple, as Dirac assumed.

Moreover, these results suggest that some physical quantities, like Mass, Energy and Frequency could be limited by minimum values.

Conclusion

Using the observed ratio of the electrostatic force to the gravitational force, the assumed relationship Eq. 6 allows precisely calculating the Hubble constant and the minimum mass. These values agree with observation and previous works. Recent observations suggest that the proton charge radius could also be calculated using an additional but related assumption (Eq. 16). More precise observed values of H and r_p could confirm or refute some of these hypotheses.

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Ethics

There are no ethical issues implied by the described research.

References

Atoui, M., Barbaro, M., Hoballah, M., Keyrouz, C., Lassaut, M., Marchand, D., . . . Voutier, E. (2023). Determination of the moments of the proton charge density. *Nuclear experiment (arXiv:2304.13521*, 9. https://doi.org/10.48550/arXiv.2304.13521

Berman, M. S. (1992). Large number hypothesis. *International Journal of Theoretical Physics*, *31*, 1447-1450. https://doi.org/10.1007/BF00673977

Beyer, A., Maisenbacher, L., Matveev, A., Pohl, R., Khabarova, K., Grinin, A., ... & Udem, T. (2017). The Rydberg constant and proton size from atomic hydrogen. *Science*, *358*(6359), 79-85. https://doi.org/10.1126/science.aah6677

Bezginov, N., Valdez, T., Horbatsch, M., Marsman, A., Vutha, A. C., & Hessels, E. A. (2019). A measurement of the atomic hydrogen Lamb shift and the proton charge radius. *Science*, *365*(6457), 1007-1012. https://doi.org/10.1126/science.aau7807

Cetto, A., de La Pena, L., & Santos, E. (1986). Dirac's large-number hypothesis revised. *Astronomy and Astrophysics (ISSN 0004-6361), vol. 164, no. 1, Aug. 1986, p. 1-5. Research Supported by the Ministerio de Educacion y Ciencia of Spain.*, *164*, 1-5. https://adsabs.harvard.edu/full/1986A%26A...164....1C

Dirac, P. A. M. (1938). A new basis for cosmology. *Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences*, *165*(921), 199-208. https://doi.org/10.1098/rspa.1938.0053

Dirac, P. A. M. (1974). Cosmological models and the large numbers hypothesis. *Proceedings of the Royal Society of London. A. Mathematical and Physical Sciences*, *338*(1615), 439-446. https://doi.org/10.1098/rspa.1974.0095

Dirac, P. A. M. (1979). The large numbers hypothesis and the Einstein theory of gravitation. *Proceedings of the Royal Society of London. A. Mathematical and Physical Sciences*, *365*(1720), 19-30. https://doi.org/10.1098/rspa.1979.0003

Djukanovic, D., Harris, T., von Hippel, G., Junnarkar, P. M., Meyer, H. B., Mohler, D., ... & Wittig, H. (2021). Isovector electromagnetic form factors of the nucleon from lattice QCD and the proton radius puzzle. *Physical Review D*, *103*(9), 094522. https://doi.org/10.1103/PhysRevD.103.094522

Freedman, W. L. (2021). Measurements of the Hubble constant: tensions in perspective. *The Astrophysical Journal*, *919*(1), https://doi.org/16. 10.3847/1538-4357/ac0e95

Gao, H., & Vanderhaeghen, M. (2022). The Proton Charge Radius. *Reviews of Modern Physics*, *94*, 49. https://doi.org/10.1103/RevModPhys.94.015002

Khetan, N., Izzo, L., Branchesi, M., Wojtak, R., Cantiello, M., Murugeshan, C., ... & Valenti, S. (2021). A new measurement of the Hubble constant using Type Ia supernovae calibrated with surface brightness fluctuations. *Astronomy & Astrophysics*, *647*, A72. https://doi.org/10.1051/0004-6361/202039196

Kritov, A. (2021). Explicit Values for Gravitational and Hubble Constants from Cosmological Entropy Bound and Alpha-Quantization of Particle Masses. *Progress in Physics*, *17*(2), 4-16. https://www.ptep-online.com/2021/PP-62-04.PDF

Lau, Y. K., & Prokhovnik, S. J. (1986). The large numbers hypothesis and a relativistic theory of gravitation. *Australian Journal of Physics*, *39*(3), 339-346. https://doi.org/10.1071/PH860339

Liao, K., Shafieloo, A., Keeley, R. E., & Linder, E. V. (2019). A model-independent determination of the Hubble constant from lensed quasars and supernovae using Gaussian process regression. *The Astrophysical Journal Letters*, *886*(1), L23. https://doi.org/10.3847/2041-8213/ab5308

DOI: 10.3844/pisp.2023.1.5

Lusso, E., Piedipalumbo, E., Risaliti, G., Paolillo, M., Bisogni, S., Nardini, E. M. A. N. U. E. L. E., & Amati, L. O. R. E. N. Z. O. (2019). Tension with the flat ΛCDM model from a high-redshift Hubble diagram of supernovae, quasars and gamma-ray bursts. *Astronomy & Astrophysics*, *628*, L4. https://doi.org/10.1051/0004-6361/201936223

Mercier, C. (2019). Calculation of the Universal Gravitational Constant, of the Hubble Constant and of the Average CMB Temperature. *Journal of Modern Physics*, *10*(06), 641. https://doi.org/10.4236/jmp.2019.106046

Perl, M. L., Lee, E. R., & Loomba, D. (2004). A Brief review of the search for isolatable fractional charge elementary particles. *Modern Physics Letters A*, *19*(35), 2595-2610. https://doi.org/10.1142/S0217732304016019

Pesce, D. W., Braatz, J. A., Reid, M. J., Riess, A. G., Scolnic, D., Condon, J. J., ... & Lo, K. Y. (2020). The megamaser cosmology project. XIII. Combined Hubble constant constraints. *The Astrophysical Journal Letters*, *891*(1), L1. https://doi.org/10.3847/2041-8213/ab75f0

Ray, S., Mukhopadhyay, U., Ray, S., & Bhattacharjee, A. (2019). Dirac's large number hypothesis: A journey from concept to implication. *International Journal of Modern Physics D*, *28*(08), 1930014. https://doi.org/10.1142/S0218271819300143

Riess, A. G., Yuan, W., Macri, L. M., Scolnic, D., Brout, D., Casertano, S., ... & Zheng, W. (2022). A comprehensive measurement of the local value of the Hubble constant with 1 km s− 1 Mpc− 1 uncertainty from the Hubble Space Telescope and the SH0ES team. *The Astrophysical Journal Letters*, *934*(1), L7. https://doi.org/10.3847/2041-8213/ac5c5b

Rushdi, M. A., & Rushdi, A. M. (2016). On the Fundamental Masses Derivable by Dimensional Analysis. *Journal of King Abdulaziz University: Engineering Sciences*, *27*(1) https://doi.org/10.4197/Eng.27-1.3

Shuntov, M. (2018). Unveiling the Concordance Model of Cosmology. *Aix Marseille Universite, Marseille*. https://doi.org/10.13140/RG.2.2.30613.83688

Talbot, P. (2021). The Cosmopheric Principle St-Hubert, Qc, Canada, ISBN: 978-2-9819000-3-6, pp: 97-100 https://www.amazon.com/dp/B09G85GSPP

Valev, D. (2010). Phenomenological mass relation for free massive stable particles and estimations of neutrino and graviton masses. *arXiv preprint arXiv:1004.2449*. https://doi.org/10.48550/arXiv.1004.2449

Valev, D. (2013). Three fundamental masses derived by dimensional analysis. *Am. J. Space Sci*, *1*(2), 145-149. https://doi.org/10.3844/ajssp.2013.145.149

Valev, D. T. (2015). Estimations of Neutrino and Graviton Masses by a Phenomenological Mass Relation for Stable Particles. *Physics International*, *6*(2), 82-88. https://doi.org/10.3844/pisp.2015.82.88

Wesson, P. S. (2004). Is mass quantized?. *Modern Physics Letters A*, *19*(26), 1995-2000. https://doi.org/10.1142/S0217732304015270

Wilmot, R. G. (2021). Exact Value of the Hubble Constant-The Most Precise Value of the Hubble Constant Deduced from Other Constants of Physics. *Research Gate*, 28. https://doi.org/10.13140/RG.2.2.25508.60802

Wolf, C. G. (2022). Hubble constant H0 is derived from Newtonian gravitational constant G, speed of light in vacuum c, electron mass me, classical electron radius re squared and fine structure constant α. *ResearchGate*. https://doi.org/10.13140/RG.2.2.15487.48801

Xiong, W., Gasparian, A., Gao, H., Dutta, D., Khandaker, M., Liyanage, N., ... & Zhao, Z. W. (2019). A small proton charge radius from an electron-proton scattering experiment. *Nature*, *575*(7781), 147-150. https://doi.org/10.1038/s41586-019-1721-2

Yang, T., Birrer, S., & Hu, B. (2020). The first simultaneous measurement of Hubble constant and post-Newtonian parameter from Time-Delay Strong Lensing. *Monthly Notices of the Royal Astronomical Society: Letters*, *497*(1), L56-L61. https://doi.org/10.1093/mnrasl/slaa107