Stochastic Model for Pricing Normal Bonds when Maturity Periods Cross Over to Pandemic Period

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Abstract: In this study, Ito form for normal bonds trading where maturity periods cross over to COVID-19 pandemic period is presented. It is shown that normal bonds in this period experience path reversals respectively to their canonical paths. The criterion used in arriving at this striking result is also presented. As a key recommendation, it is necessary that bondholders enact flexible pricing laws that strengthen the issuer to continue trading in the present COVID-19 pandemic time through the reverse path identified in this study.

Keywords: CAT Bond, Sub-Exponential Shock, Noise, Wiener Process, Issuer

Introduction

The advent of COVID-19 pandemic has altered the dynamics of many stochastic processes. As at the 6th of February 2023, the number of infected persons stands at 671,706,853 to which 6,844,645 deaths were recorded1. The expectation that global bond traders are a significant fraction of these figures is not only likely but realistic. Liang (2020) asserts that if a significant chunk of bond traders lies in the COVID-19 infected class then the bond market incurs huge losses due to rapid sales at giveaway prices. Thus, if there are many COVID-19 infected bond traders in countries known for raising bonds, economic breakdown and meltdown may happen unless adequate trade precautions are taken. One precaution is the adoption of new hybrid bond pricing models that can curb emerging pandemic arbitrage in markets. For more on the subject of hybrid models for optimization purposes, we refer the reader to the works of (Alhawarat et al., 2021; Salleh et al., 2022).

Similarly, if many issuers die in a small time as in the case of the dreaded COVID-19 pandemic figures above, holders will struggle in receiving proportionate coupon payments. Under this condition, bond corporations will be unable to make capital payments on both secured and unsecured loans and will face serious challenges such as total collapse leading to loss of resources (Brown, 2006). The unfortunate collapse of silicon valley bank USA is a good example in this respect. Another realistic instance when bondholders will not receive payment from issuers is when bond-raising countries operate under lockdowns and other forms of COVID-19 restrictions. In this case, issuers are not fully paid salaries and wages due to changing work forms leading to a rising rate of defaults (Buchheit and Gulat, 2002). Here, transaction strains are developed and can lead to acute financial losses. To mitigate these losses and many others, newer bond pricing models for the COVID-19 pandemic time are needed to save investment resources.

For instance on 23rd March 2020, the announcement for investment grade bonds was made. This announcement pushed the price of the bond by 7% on exercise with no signal effect on stock prices. This shocking bond-stock price path resulted in investors flocking to stocks instead of bonds (Haddad et al., 2021) because of additionally envisaged risks attached to bond dynamics sequel to the pandemic. At the moment, global bond market operates under additional risk that increases both rates and rise of defaults (Zaghini, 2023). Since COVID-19 pandemic has created additional risk measures in reality, then the need to incorporate this realism where maturity periods take longer than expected default times to wade away price informativeness (Dávila and Parlatore, 2018), or feedback effect (Sani et al., 2020; Edmans et al., 2015; Cofnas, 2016) and ripple effect (George and Beard, 2023) cannot be over-emphasized. In this case, the feedback effect will change speculators’ decision on bonds which eventually lead to mispricing in both the short and long terms. For instance in the long term, bond markets will lose their fair share of profit margins to stocks since profit margins depend on the correctness of pricing models employed in both time and space (Joyner, 2006). Consequently, it is crucial to develop newer bond pricing models especially where maturity times cross over to the pandemic time. In this respect, we construct a new stochastic bond pricing model for cross-over bonds. First, the maturity time is

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1Data retrieved from the John Hopkins University Dashboard on the 6th of February 2023@9.10 am

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redefined in view of lengthy default tendencies (Goldie and Kluppelberg, 1988; Mikosh, 1999; Foss et al., 2013).

Chao and Zou (2018) studied flood related Catastrophe (CAT) bond prices and established the CIR-copula-POT model with stochastic rates showing that maturity dates affect bond prices. Shao (2015) considered both seismic and nuclear catastrophes and showed that bond prices are affected by extended maturities and defaults (Shao et al., 2015). Shao et al. (2017) studied CAT bond prices as a sequence of compound inhomogeneous Poisson processes disturbed by a diffusion process and came up with a semi-Markov model that examined the claim's inter-arrival times. Numerical analysis showed that the price of CAT bonds will decline when the threshold level is decreased, the time to maturity is increased and the likelihood of default also increased. Lee and Yu (2002) explored how CAT bonds relate to the financial market in the presence of rising default risks and showed that both moral hazards and baseline risks significantly can lower bond trade.

Nowak and Romaniuk (2017) studied CAT bonds related to investor risk-taking and showed that adjusted bond pricing models can evade away arbitrage tendencies better than non-adjusted bond pricing models (Romaniuk, 2003). Ma and Ma (2013) studied CAT bonds with non-homogeneous Poisson losses given interest rate uncertainty, loss severity, and claim arrival intensities and showed that interest rates rise bond prices when the maturity time bracket is [0.25, 2.5]. Hofer et al. (2020) studied CAT bond pricing where associated risk is defined in a fixed interval and showed that threshold losses and expiration times have significant roles in arbitrage-free pricing. Tao et al. (2009) studied pricing models for earthquake-related catastrophes and stated the needed techniques for estimating personal insurance under seismic related deductibles. Vaugirard (2004) stated that the subject of pricing CAT bonds boils down to computing the first passage times for some jump diffusion stochastic processes under certain risk assumptions.

Egami and Young (2008) studied CAT bonds where inter-arrival claims follow the Poisson diffusion process and showed that prices are indifferent. Ma et al. (2017) developed a bond pricing model for zero coupon CAT bonds with stochastic Poisson arrivals and showed that hazards and interest rate risks affect zero coupon bond prices. In general, it is imperative to note that CAT bonds are hedging financial options as in Galeotti et al. (2013); Bodoff and Gan (2009); Lai et al. (2014); Gürtier et al. (2016). De Spiegeleer and Schoutens (2011); Baz and Chacko (2004); Rebonato (2018) described how existing bond models devoid of extra-uncertainties (Szczygelski et al., 2021; Lyócsa et al., 2020; Albulescu and Grecu 2023) are distinct from those ones where coupon payments assumed the paths of long-tailed processes (Norman et al., 2020). Thus for completeness, linking the times to maturity with the pandemic time is important for aided efficiency especially where default times assumed the path of the COVID-19 pandemic.

Materials and Methods

We consider a bond \( \mathbb{B} \) whose non-pandemic trading time price at maturity \( p(\mathbb{B}, T) \) is quoted as:

\[
p(\mathbb{B}, T) = e^{\left( 1 - \left( \frac{1}{T} \right)^{\frac{\phi}{\chi}} \right) + \frac{\phi}{\chi^{\phi}}} \quad \text{(1)}
\]

where, \( T \geq 0 \) is the time to maturity parameter of \( \mathbb{B} \), \( e \) is the ratio of monthly coupon payment to prevailing monthly market interest rate, \( \chi \) is one on prevailing monthly market interest rate and \( \phi \) is the face value of \( \mathbb{B} \). We suppose that the same \( \mathbb{B} \) is to be traded and exercised in the present COVID-19 pandemic period with \( p(\mathbb{B}, T) \rightarrow p(\mathbb{B}, \bar{F}(T)) \) such that:

\[
p(\mathbb{B}, F(T)) = e^{\left( 1 - \left( \frac{1}{\bar{F}} \right)^{\frac{\phi}{\chi}} \right) + \frac{\phi}{\chi^{\phi}}} \quad \text{(2)}
\]

Here, \( \bar{F} \in \mathbb{S} \) is a bridge-in-price function sequel to changing trading intervals generated by the COVID-19 pandemic on \( \mathbb{B} \) with the property that:

\[
\lim_{n \to \infty} \frac{\bar{F}(\mathbb{B}, T)}{F(\mathbb{B}, T)} = n, n \geq 2 \quad \text{(3)}
\]

If the path of \( \bar{F}(\mathbb{B}, T) \) assumes the path of a wiener process \( \{W(\mathbb{B}, T): T \geq 0\} \) through the maturity parameter \( T \leq t \), then the guarantee of the existence of at least one natural homomorphism \( \phi \) such that:

\[
\phi : \mathbb{T} \times \Omega \to \mathbb{R} \quad \text{(4)}
\]

is established. In this case, \( \phi \) is a homomorphism between rings with identities onto \( \mathbb{R} \).

Proposition 1

The homomorphism \( \phi \) in (4) such that:

\[
\phi : \mathbb{T} \times \Omega \times F(T) \to \bar{F}(T, \Omega, W(\mathbb{B}, \bar{F})) \quad \text{(5)}
\]

is sub-exponential with parameter \( N \geq n \) provided that \( \phi \) is measurable.

Proof

Fix \( (\omega \in \Omega) \to x \) and apply the isomorphism of (Arnautov and Ermacova, 2014; Kulabokhov, 2019) on the right-hand side of (5). Consequently:
\[
\lim_{t \to \infty} \frac{F'(T,x,W(T,B))}{F(T,x,W(T,B))} \leq \gamma \lim_{t \to \infty} \frac{F'(W(T,B))}{F(W(T,B))} \leq N
\]  

(6)

Since, \( N \geq 2 \) in (7), the bond price \( p(B, T) \) in (1) must admit some sub-exponentially due to induced COVID-19 pandemic effects as in (2). In this case, the quoted price \( p(B, F(T)) \) in (2) holds good whenever \( B \) is within the dreaded\(^2\) pandemic period with the representation\(^3\) as in (7) and such that:

\[
p(B, F(T)) = \left(1 - \left(\frac{1}{2}\right)^T\right) + \frac{\theta}{\gamma}
\]

(8)

Equation (8) is an Ito process (Øksendal and Øksendal, 2003). Let:

\[
p(B, F(T)) = p_T = g(T, x, W(T, B))
\]

(9)

such that:

\[
g(F(T,x,W(T,B))) \in C^1([0, \infty)) \times \mathbb{R}
\]

(10)

Then by Ito’s formula on \( g(.) \), one obtains that:

\[
\frac{\partial p(B, F(T))}{\partial T} = -\ln \varepsilon \frac{\partial F(T,W(T,B))}{\partial T} p(B,F(T))
\]

(11)

\[
C_1 = -\ln \varepsilon \frac{dF(T,W(T,B))}{dT}
\]

(12)

and that:

\[
\frac{\partial p(B, F(T))}{\partial W(T, B)} = -\ln \varepsilon \frac{\partial F(T,W(T,B))}{\partial W(T,B)} p(B,F(T))
\]

(13)

Here:

\[
\varepsilon = \frac{C_1}{p_T}
\]

(14)

Finally:

\[
\frac{\partial^2 p(B, F(T))}{\partial W^2(T,B)} = \left(C_2 + C_1\right) p(B,F(T))
\]

(15)

Here again:

\[
C_3 = -\ln \varepsilon \frac{\partial^2 F(T,W(T,B))}{\partial W^2(T,B)}
\]

(16)

And that:

\[
C_4 = \ln \varepsilon \frac{\partial^2 F(T,W(T,B))}{\partial W^2(T,B)}
\]

(17)

Hence, by combining (11), (13), and (15), one obtains\(^4\) that:

\[
dp_T = \left(C_0 + \frac{1}{2}\right) p_T \omega dT + C_1 p_T dW(T,B)
\]

(18)

Corollary 1

Under (3) and (4), the price of \( B \) fluctuates with drift \( \alpha \) and volatility \( \sigma \) given by:

\[
\alpha = C_0 + \frac{1}{2}(C_2 + C_1)
\]

(19)

\[
\sigma = C_1
\]

(20)

Proof

This result is a consequence of (18) as in (Øksendal and Øksendal, 2003). Thus, if the COVID-19 process hits the bond price process in (1), then the result is a transformed process (8) whose features are those of the geometric Brownian motion (18). The path of (18) \((T \times \omega \times W(T, B)) \rightarrow (p_0(T, W(T, B)))\) is measurable in view of (Øksendal and Øksendal, 2003). Solving for \( p_T \) gives:

\[
p_T = p_0 \exp\left([\alpha - \frac{1}{2} \sigma^2] T + \sigma W(T,B)\right)
\]

(21)

\(^2\varepsilon = 1 + r_a\)

\(^3\alpha = \left[C_0 + \frac{1}{2}(C_2 + C_1)\right] \]
Table 1: α, σ Trends versus T in COVID-19 Times

<table>
<thead>
<tr>
<th>T(years)</th>
<th>α</th>
<th>σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>11.51300000</td>
<td>0.00000000</td>
</tr>
<tr>
<td>1</td>
<td>21.34200000</td>
<td>3.41120000</td>
</tr>
<tr>
<td>2</td>
<td>5.92190000</td>
<td>0.85281000</td>
</tr>
<tr>
<td>3</td>
<td>1.62960000</td>
<td>0.20760000</td>
</tr>
<tr>
<td>4</td>
<td>0.64778000</td>
<td>0.06317100</td>
</tr>
<tr>
<td>5</td>
<td>0.33685000</td>
<td>0.02339700</td>
</tr>
<tr>
<td>10</td>
<td>0.05464200</td>
<td>0.00086790</td>
</tr>
<tr>
<td>35</td>
<td>0.00181910</td>
<td>1.7451e-06</td>
</tr>
<tr>
<td>50</td>
<td>0.00065513</td>
<td>2.9402e-07</td>
</tr>
<tr>
<td>70</td>
<td>0.00024677</td>
<td>5.4734e-08</td>
</tr>
<tr>
<td>85</td>
<td>0.00013987</td>
<td>2.0741e-08</td>
</tr>
<tr>
<td>100</td>
<td>8.67930e-05</td>
<td>9.2048e-09</td>
</tr>
</tbody>
</table>

Table 1 shows α and σ paths for selected T. From the said table, the impact of the noise parameters can be deduced. For instance, in 2023, bond prices will drift about 65% of their non-pandemic face values at 6% frequency due to the COVID-19 pandemic. Interestingly in 2029, the price of the same asset drifts only around 5% at a frequency lesser than 1%.

Results and Discussion

For further numerical discussions, we simulate (21) to study the path of (2) in the light of the 80-20 Pareto distribution assuming that 80% of bond wealth is in the hands of 20% bondholders who are infected by the COVID-19 pandemic. We also simulate the bond price model in (1) to study the changes in the path between the two models. The following remarks hold good.

Remark 1

If T is Pareto 80-20 and positively increasing in R sequence to defaults, then the constants C_0, C_1, and C_3 decrease while C_2 increases.

Table 2 presents the constant values C_0, C_1, C_2 and C_3 against T≤100 years. From the said table, it is clear that C_0 values decrease slowly to 0.05 at T = 10 years. This trend is followed by a constant pattern of C_0 ≃ 0 for T>10 years. Clearly, the general trend for C_0 is that of a bond constant decreasing as T increases. In this case, C_0 represents the component of the prevailing interest rate during the pandemic. From Table 2 again, C_1 decreases to 0.02 where T = 5 years. This trend is followed by C_1 ≃ 0 afterward for all T>5 years. As a trend, C_1 decreases as T increases in the pandemic period. The same trend in path is taken by C_3 as a function of T. On the contrary, C_2 starts with a value of 3.4112 at T = 1 year and steadily increases to 0.04 at time T = 4 years. It can be conjectured that the decreasing C_1, C_3 emanated from the hidden effects of the COVID-19 pandemic on the existing model (1).

Remark 2

Suppose B is quoted where T is sub-exponentially 80-20 Pareto-tailed sequel to the COVID-19 pandemic or any of its arguments. Then p(β, F (T)) is astronomical in the short term.

Table 3 presents the path analysis of p(β, F (T)) as in (18) and that of p as in (1). Clearly, at T = 1 year, p_T skycracks from $1000 to an astronomical value of $16,729,000,000. Again, At T = 2 years and T = 3 years respectively, the bond price values are $370,310,000 and $232,080. Here, the COVID-19 pandemic created 80% uncertainties in the bond market causing investors to flee to other financial markets as in Ma and Ma (2013) under high-level uncertainty, trader loss severity, and high claim arrival intensities. Bond traders will choose to cash in to treat COVID-19 pandemic symptoms without minding the price. As a result, bond markets lose control over the short-term maturities. Afterward, bond prices show signs of stability in both time and space. For instance, in the year 2029, the bond exercised at $1000 will trade at $1494 via the canonical model and at $1742 via the stochastic model developed in this study.

Remark 3

Under the 80-20 pareto COVID-19 pandemic tail, the price of β reverses its path to a monotonically decreasing path within the same maturity interval.

This information is clear from Table 3 showing that the price of the bond p climbs consistently from $1000 at T = 0 to approximately $6000 at T = 100 years. On the other hand, p(β, F (T)) declines towards extremely low values close to $1000 as time to maturity T approaches 100 years. In this case, the paths assumed by the two models are opposite in direction and strictly monotonic agreeing with (Shao et al., 2015) that showed that the price of CAT bonds is most likely to reverse when the threshold level is decreased, the time to maturity is increased and the likelihood of default increases. This further proves the strength of the methodology designed and presented in this study.
for $T \geq 10$. Clearly here, the pattern of long maturity times is being much closer to the face value of the bond during the COVID-19 pandemic. Again, bond prices enter the normal region in volatility hence, normal under maturity periods $T \geq 10$.

**Conclusion**

We design a CAT bond pricing model for normal bond trading during the COVID-19 pandemic time and provide its Ito representation and analysis. The work discovered a major result showing the existence of a relationship between CAT bonds of the COVID-19 pandemic with that of earthquakes as in Romaniuk (2003). Again, we have shown the existence of path reversals for COVID-19 pandemic CAT bonds. This result agrees with (Shao et al., 2015) for earthquakes CAT bonds. As a recommendation, since CAT bonds with short-term maturity periods are astronomically expensive in the 80-20 pareto tail region containing the COVID-19 pandemic, it is better to issue bonds in the mid-term to wave foreseen uncertainties. Finally, the work posits that CAT bonds within 80-20 pareto tail containing COVID-19 trading times attained steadiness under long maturity periods with equilibrium points around the eleventh year of the COVID-19 pandemic.

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**Competing Interests and Declarations**

**Consent to Participate**

All authors have given their consent to participate in the entire process of this article.

**Consent for Publication**

All authors have consented to publish this article.

**Author’s Contributions**

**Sulaiman Sani:** Pens down the article and proves the remarks together with the methods.

**Siphelele Lushaba:** Provides the literature backing of the article and the sample path analysis leading to the tables and figures studied in this study.
Ethics

This article is original and contains unpublished material. The corresponding author confirms that all of the other authors have read and approved the manuscript and no ethical issues involved.

References


