Chlodowsky Type $(\lambda, q)$-Bernstein Stancu Operator of Korovkin-Type Approximation Theorem of Rough I-Core of Triple Sequences

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Abstract: In this study, we obtain a Korovkin-type approximation theorem for Chlodowsky type $(\lambda, q)$-Bernstein Stancu operator of rough I-convergent of triple sequences of positive linear operators of two variables from $H_u(K)$ to $C_u(K)$. We introduce and study some basic properties of Korovkin-type approximation theorem for Chlodowsky type $(\lambda, q)$-Bernstein Stancu operator of rough I-convergent of triple sequence spaces and also study the set of all Korovkin-type approximation theorem for Chlodowsky type $(\lambda, q)$-Bernstein Stancu operator of rough I-limits of triple sequence spaces and the relation between analyticness and Korovkin-type approximation theorem for Chlodowsky type $(\lambda, q)$-Bernstein Stancu operator of rough I-core of triple sequence spaces.

Keywords: Chlodowsky Type $(\lambda, q)$-Bernstein Stancu Operator, Ideal, Triple Sequences, Rough Convergence, Closed and Convex, Cluster Points and Rough Limit Points, Korovkin-type Approximation.

Introduction

The idea of rough convergence was first introduced by (Phu, 2001; 2002; Xuan Phu, 2003) infinite-dimensional normed spaces. He showed that the set $\text{LIM}^r_\lambda$ is bounded, closed, and convex; and he introduced the notion of a rough Cauchy sequence. He also investigated the relations between rough convergence and other convergence types and the dependence of $\text{LIM}^r_\lambda$ on the roughness of degree $r$.

Aytar (2008a) studied rough statistical convergence and defined the set of rough statistical limit points of a sequence and obtained two statistical convergence criteria associated with this set and prove that this set is closed and convex. Also, Aytar (2008b) studied that the $r$-limit set of the sequence is equal to the intersection of these sets and that the $r$-core of the sequence is equal to the union of these sets. Dündar and Çakan (2014) investigated rough ideal convergence and defined the set of rough ideal limit points of a sequence the notion of I-convergence of triple sequence spaces is based on the structure of the ideal $I$ of subsets of $\mathbb{N} \times \mathbb{N} \times \mathbb{N}$, where $\mathbb{N}$ is the set of all natural numbers, is a natural generalization of the notion of convergence and statistical convergence.

Our primary interest in the present paper is to obtain a general Korovkin-type approximation theorem for triple sequences of positive linear operators of two variables from $H_u(K)$ to $C_u(K)$ via statistical A-summability.

Let $A$ be a three-dimensional summability matrix. For a given triple sequence $x = (x_{mnk})$, the $A$-transform of $x$, denoted by $Ax : x(\{Ax\}_{m,n,k})$, given by:

$$\{Ax\}_{i,j,\ell} = \sum_{(m,n,k)\in\mathbb{N}^3} a_{i,j,\ell,m,n,k} x_{mnk} \quad (1.1)$$

provided the triple series converges in Pringsheim’s sense for every $(i,j,\ell) \in \mathbb{N}^3$.

A three dimensional matrix $A = (a_{i,j,\ell,m,n,k})$ is said to be RH-regular if it maps every bounded $P$-convergent sequence into a $P$-convergent sequence with the same $P$-limit. A three dimensional matrix $A = (a_{i,j,\ell,m,n,k})$ is RH-regular if and only if:

(i) $P - \lim_{i,j,\ell} a_{i,j,\ell,m,n,k} = 0$ for each $(m, n, k) \in \mathbb{N}^3$
(ii) $P - \lim_{i,j,\ell} \sum_{(m,n,k)\in\mathbb{N}^3} a_{i,j,\ell,m,n,k} = 1$
(iii) $P - \lim_{i,j,\ell} \sum_{(m,n,k)\in\mathbb{N}^3} a_{i,j,\ell,m,n,k} = 0$ for each $n, k \in \mathbb{N}$
(iv) $P - \lim_{\{i,j, \ell \in \mathbb{N}\}} \sum_{m,n,k \in \mathbb{N}^3} a_{i,j,m,n,k} = 0$ for each $m, k \in \mathbb{N}$

(v) $P - \lim_{\{i,j, \ell \in \mathbb{N}\}} \sum_{m,n,k \in \mathbb{N}^3} a_{i,j,m,n,k} = 0$ for each $m, n \in \mathbb{N}$

(vi) $\sum_{m,n,k \in \mathbb{N}^3} |a_{i,j,m,n,k}|$ is $P$-convergent for every $(i, j, \ell) \in \mathbb{N}^3$

(vii) There exist finite positive integers $A$ and $B$ such that $\sum_{m,n,k \in \mathbb{N}^3} |a_{i,j,m,n,k}| < A$ holds for every $(i,j, \ell) \in \mathbb{N}^3$.

Now let $A = (a_{i,j,m,n,k})$ be a non-negative RH-regular summability matrix, and $K \subset \mathbb{N}^3$. Then the $A$-density of $K$ is given by:

$$\delta_A^K = P - \lim_{(i,j,\ell) \to (\infty, \infty, \infty)} \sum_{m,n,k \in \mathbb{N}^3} a_{i,j,m,n,k}$$

where:

$$K(\epsilon) = \{(m,n,k) \in \mathbb{N}^3 : \mid x_{mnk} - L \mid \geq \epsilon \}$$

provided that the limit on the right-hand side exists in Pringsheim’s sense. A real triple sequence $x = (x_{mnk})$ is said to be $A$-statistically convergent to a number $L$ if, for every $\epsilon > 0$:

$$\delta_A^K = \{(m,n,k) \in \mathbb{N}^3 : \mid x_{mnk} - L \mid \geq \epsilon \} = 0.$$

In this case, we write $st A - \lim_{i,j,\ell} = L$.

In this study, we investigate some basic properties of the Korovkin-type approximation theorem for rough $L$-convergence of triple sequence spaces in three-dimensional matrix spaces which are not earlier. We study the set of all rough $L$-limits of triple sequence spaces and also the relation between analytic ness and rough $L$-core of a Korovkin-type approximation theorem for triple sequence spaces. We recommend the reader to refer to (Arqub, 2015; Abu-Arqub et al., 2013; Al-Smadi et al., 2012; 2015; Momani et al., 2016) and (Shawagfeh et al., 2014) references to see the different approaches.

Let $K$ be a subset of the set of positive integers $\mathbb{N} \times \mathbb{N} \times \mathbb{N}$ and let us denote the set $K_{i\ell j} = \{(m,n,k) \in K : m \geq i, n \geq j, k \geq \ell \}$. Then the natural density of $K$ is given by:

$$\delta(K) = \lim_{i,j,\ell \to \infty} \frac{|K_{i\ell j}|}{ij\ell}$$

where, $|K_{i\ell j}|$ denotes the number of elements in $K_{i\ell j}$.

In this study, we construct Chlodowsky type $(\lambda, q)$-Bernstein Stancu operators of triple sequence space is defined as:

$$B_{r,s,t}^{\alpha,\beta,\lambda, q}(f;x) = \sum_{m,n,k \in \mathbb{N}^3} b_{mnk}(x; q) f \left( \frac{[mnk]_q + \alpha}{[rst]_q + \beta} b_{mnk} \right), \quad (1.2)$$

where, $r,s,t \in \mathbb{N}$, $0 < q \leq 1$, $0 \leq x \leq b_{rst}$ and $b_{rst}$ is a sequence of positive numbers such that

$$\lim_{r,s,t \to \infty} b_{rst} = \infty, \lim_{r,s,t \to \infty} \frac{b_{rst}}{[rst]_q} = 0,$$

and $\alpha, \beta \in \mathbb{R}$ and $0 \leq \alpha, \beta \leq 1$. For $\alpha = \beta = 0$ we obtain the Chlodowsky type $(\lambda, q)$-Bernstein Stancu polynomials.

Throughout the paper, $\mathbb{R}$ denotes the real of three-dimensional space with metric $(X, d)$. Consider a triple sequence of Chlodowsky type $(\lambda, q)$-Bernstein Stancu operators $\left\{B_{r,s,t}^{\alpha,\beta,\lambda, q}(f;x)\right\}$ such that

$$\left\{B_{r,s,t}^{\alpha,\beta,\lambda, q}(f;x)\right\} \in \mathbb{R}, m, n, k \in \mathbb{N}.$$

Let $f$ be a continuous function defined on the closed interval $[0,1]$. A triple sequence of Bernstein polynomials $\left\{B_{r,s,t}^{\alpha,\beta,\lambda, q}(f;x)\right\}$ is said to be statistically convergent to $0$ in $\mathbb{R}$, written as $st - \lim x = 0$, provided that the set:

$$K_{\epsilon} := \{(m,n,k) \in \mathbb{N}^3 : \mid B_{r,s,t}^{\alpha,\beta,\lambda, q}(f;x) - f(x) \mid \geq \epsilon \}$$

has natural density zero for any $\epsilon > 0$. In this case, $0$ is called the statistical limit of the triple sequence of Bernstein polynomials, i.e., $\delta(K_{\epsilon}) = 0$. That is:

$$\lim_{r,s,t \to \infty} \frac{1}{rst} \left| \sum_{m \leq r, n \leq s, k \leq t} \mid B_{r,s,t}^{\alpha,\beta,\lambda, q}(f;x) - f(x) \mid \geq \epsilon \right| = 0.$$

In this case, we write $\delta - \lim B_{r,s,t}^{\alpha,\beta,\lambda, q}(f;x) = f(x)$ or $B_{r,s,t}^{\alpha,\beta,\lambda, q}(f;x) \to S f(x)$.

Throughout the paper, $\mathbb{N}$ denotes the set of all positive integers, $\chi_a$ the characteristic function of $A \subset \mathbb{N}$, and $\mathbb{R}$ the set of all real numbers. A subset $A$ of $\mathbb{N}$ is said to have asymptotic density $d(A)$ if:

$$d(A) = \lim_{i,j,\ell \to \infty} \frac{1}{ij\ell} \sum_{m,n,k \in \mathbb{N}^3} A(\{m,n,k\}).$$

A triple sequence (real or complex) can be defined as a function $x : \mathbb{N} \times \mathbb{N} \times \mathbb{N} \to \mathbb{R}(\mathbb{C})$, where $\mathbb{N}$, $\mathbb{R}$, and $\mathbb{C}$ denote the set of natural numbers, real numbers, and complex numbers respectively. The different types of notions of the triple sequence were introduced and investigated at the initial by (Sahiner et al., 2007; Sahiner and Tripathy, 2008; Esi, 2014; Esi and Catalbas, 2014; Esi and Savas, 2015; Esi et al., 2016; Datta et al., 2013; Subramanian and Esi, 2015; Esi et al., 2022, Debnath et al., 2015) and many others.
The space of all triple analytic sequences is usually denoted by $\Lambda^3$.

**Definitions and Preliminaries**

Throughout the paper, $\mathbb{R}^3$ denotes the real three-dimensional case with the metric. Consider a triple sequence $x = (x_{m,n,k})$ such that $x_{m,n,k} \in \mathbb{R}^3$; $m, n, k \in \mathbb{N}$. The following definition is obtained.

**Definition 1**

Let $f$ be a continuous function defined on the closed interval $[0, 1]$. A triple sequence of Chlodowsky type $(\lambda, q)$-Bernstein Stancu operators $B_{(r,s,t)}^{\lambda,q}(f:x)$ of real numbers and $A = (a_{ij,m,n,k})$ to be a non-negative RH-regular summability matrix is said to be rough statistically $A$-summable to $f(x)$ if for every $\epsilon > 0$:

$$\delta_1 \sum \left\{ (i,j,t) \in \mathbb{N}^3 : \left| B_{(r,s,t)}^{\lambda,q}(f;Ax) - f(x) \right| \geq r + \epsilon \right\} = 0,$$

i.e.:

$$P - \lim_{mnk} \frac{1}{mnk} \sum \left\{ i \leq m, j \leq n, t \leq k : \left| B_{(r,s,t)}^{\lambda,q}(f;Ax) - f(x) \right| \geq r + \epsilon \right\} = 0,$$

where, $(Ax)_{ij}$ is as in (1.1).

**A Korovkin-Type Approximation Theorem**

Let $C_0(K)$ the space of all continuous and bounded real-valued functions on $K = [0, \infty) \times [0, \infty) \times [0, \infty)$. This space is equipped with the supremum norm:

$$\|f\| = \sup_{(x,y,z) \in K} B_{(r,s,t)}^{\lambda,q}(f;Ax), (f \in C_0(K)).$$

Consider the triple space of $H_\infty(K)$ of all real-valued functions of Chlodowsky type $(\lambda, q)$-Bernstein Stancu operators of $f$ on $K$ satisfying:

$$\left| B_{(r,s,t)}^{\lambda,q}(f;u,v,w) - B_{(r,s,t)}^{\lambda,q}(f;u,y,z) \right| \leq w \left( \frac{u}{1+u} - \frac{x}{1+x} \right) \left( \frac{u}{1+u} - \frac{y}{1+y} \right) \left( \frac{w}{1+w} - \frac{z}{1+z} \right)$$

where, $w$ be a function of the type of the modulus of continuity given by, for $\delta, \delta_1, \delta_2, \delta_3 > 0$:

1. $w$ is non-negative increasing function on $K$ with respect to $\delta_1, \delta_2, \delta_3$
2. $w(\delta, \delta_1 + \delta_2 + \delta_3) \leq w(\delta, \delta_1) + w(\delta, \delta_2) + w(\delta, \delta_3)$
3. $w(\delta_1 + \delta_2 + \delta_3, \delta_1) \leq w(\delta_1, \delta_1) + w(\delta_2, \delta) + w(\delta_3, \delta)$
4. $\lim_{\delta, \delta_1, \delta_2, \delta_3 \to 0} w(\delta_1, \delta_2, \delta_3) = 0$

The Chlodowsky type $(\lambda, q)$-Bernstein Stancu operators $B_{(r,s,t)}^{\lambda,q}(f;Ax) \in H_\infty(K)$ satisfies the inequality:

$$B_{(r,s,t)}^{\lambda,q}(f,(x,y,z)) \leq B_{(r,s,t)}^{\lambda,q}(f,(0,0,0)) + w(1,1,1), x, y, z \geq 0$$

and hence it is bounded on $K$. Therefore:

$$H_\infty(K) \subset C_0(K).$$

We also use the following Chlodowsky type $(\lambda, q)$-Bernstein Stancu operators of test functions:

$$B_{(r,s,t)}^{\lambda,q}(f_{000},(u,v,w)) = 1, B_{(r,s,t)}^{\lambda,q}(f_{111},(u,v,w)) = \frac{u}{1+u},$$

$$B_{(r,s,t)}^{\lambda,q}(f_{222},(u,v,w)) = \frac{u}{1+u}, B_{(r,s,t)}^{\lambda,q}(f_{333},(u,v,w)) = \frac{w}{1+u}$$

and:

$$B_{(r,s,t)}^{\lambda,q}(f_{444}(u,v,w)) = \left( \frac{u}{1+u} \right)^2 + \left( \frac{v}{1+u} \right)^2 + \left( \frac{w}{1+u} \right)^2.$$

**Results**

**Theorem 1**

Let $f$ be a continuous function defined on the closed interval $[0, 1]$. A triple sequence of Chlodowsky type $(\lambda, q)$-Bernstein Stancu operators $B_{(r,s,t)}^{\lambda,q}(f;Ax)$ of real numbers from $H_\infty(K)$ into $C_B(K)$ and let $A = (a_{ij,m,n,k})$ be a nonnegative RH-regular summability matrix. Assume that the following conditions hold:

$$B_{(r,s,t)}^{\lambda,q} - \lim \sum_{(u,v,w) \in K} a_{(u,v,w)} B_{(r,s,t)}^{\lambda,q}(f_m) - f_m = 0, r,s,t = 0,1,2,3...$$

Then, for any $f \in H_\infty(K)$:

$$B_{(r,s,t)}^{\lambda,q} - \lim \sum_{(u,v,w) \in K} a_{(u,v,w)} B_{(r,s,t)}^{\lambda,q}(f) - f_m = 0. \quad (4.2)$$

**Proof**

Assume that (4.1) holds. Let $B_{(r,s,t)}^{\lambda,q}(f,(x,y,z)) \in H_\infty(K)$ and $f(x, y, z) \in K$ be fixed.
Since $B_{r,s,t,q}^\alpha\beta\rho(f,(u,v,w)) \in H_\alpha(K)$ for all $(u,v,w) \in K$ be fixed, we write:

$$\left| B_{r,s,t,q}^\alpha\beta\rho(f,(u,v,w)) - B_{r,s,t,q}^{\alpha\beta}(f,(x,y,z)) \right| \leq r + \epsilon + C \sum_{m,n,k} \left| a_{r,s,t,q}^\rho B_{r,s,t,q}^{\alpha\beta}(f_{m,n,k}) - f_{m,n,k} \right|$$

where $N_r = \|\mathcal{N}\|$. Using the linearity of Chlodowsky type $(\lambda, q)$-Bernstein Stancu operators $B_{r,s,t,q}^{\alpha\beta}(f;x,y,z)$, we obtain:

$$\left| \sum_{m,n,k} a_{r,s,t,q} B_{r,s,t,q}^{\alpha\beta}(f_{m,n,k}) - f(x,y,z) \right| \leq r + \epsilon + C \left| \sum_{m,n,k} a_{r,s,t,q} B_{r,s,t,q}^{\alpha\beta}(f_{m,n,k}) - f(x,y,z) \right| + C \sum_{m,n,k} \left| a_{r,s,t,q} B_{r,s,t,q}^{\alpha\beta}(f_{m,n,k}) - f_{m,n,k} \right|$$

then taking supremum over $f(x,y,z) \in K$ we get:

$$\left| \sum_{m,n,k} a_{r,s,t,q} B_{r,s,t,q}^{\alpha\beta}(f_{m,n,k}) - f \right| \leq r + \epsilon + C \sum_{m,n,k} \left| a_{r,s,t,q} B_{r,s,t,q}^{\alpha\beta}(f_{m,n,k}) - f_{m,n,k} \right|$$

where:

$$C := \max \left\{ r + \epsilon + N, \frac{2N}{\delta^3} \left( \frac{A}{1+A} \right)^2 + \frac{B}{1+B} \right\}$$

$$6N \frac{A}{1+A} \frac{6N}{\delta^3} \left( \frac{B}{1+B} \right) \frac{6N}{\delta^3} \left( \frac{C}{1+C} \right) \frac{2N}{\delta^3}$$

From (4.1), we obtain (4.2). This completes the proof.

### Corollary 1

Let $f$ be a continuous function defined on the closed interval $[0,1]$. A triple sequence of Chlodowsky type $(\lambda, q)$-Bernstein Stancu operators $B_{r,s,t,q}^{\alpha\beta}(f;x)$ of real numbers from $H_\alpha(K)$ into $C_B(K)$. Assume that the following conditions hold:

$$B^{\alpha\beta} - \lim \left\| B_{r,s,t,q}^{\alpha\beta}(f_{m,n}) - f_{m,n} \right\| = 0, r,s,t = 0,1,2,3\ldots (4.4)$$

Then, for any $f \in H_{\alpha}(K)$:

$$B^{\alpha\beta} - \lim \left\| \sum_{m,n,k} B_{r,s,t,q}^{\alpha\beta}(f_{m,n,k}) - f \right\| = 0 (4.5)$$

In Corollary 1, if the statistical convergence ([C,1,1] statistical convergence) replace with Pringsheim convergence, we obtain the following classical version of Theorem 1.

### Corollary 2

Let $f$ be a continuous function defined on the closed interval $[0,1]$. A triple sequence of Chlodowsky type $(\lambda, q)$-Bernstein Stancu operators $B_{r,s,t,q}^{\alpha\beta}(f;x)$ of real numbers from $H_\alpha(K)$ into $C_B(K)$. Assume that the following conditions hold:

$$P - \lim \left\| \sum_{m,n,k} B_{r,s,t,q}^{\alpha\beta}(f_{m,n,k}) - f_{m,n} \right\| = 0, r,s,t = 0,1,2,3\ldots (4.6)$$

Then, for any $f \in H_{\alpha}(K)$:

$$P - \lim \left\| \sum_{m,n,k} B_{r,s,t,q}^{\alpha\beta}(f) - f \right\| = 0 (4.7)$$

Remark 1. We now show that our result Theorem 1 is stronger than its classical version Corollary 2 and
statistical version Corollary 1. To see this first consider the following Bleimann, Butzer, and Hahn operators of three variables of Chlodowsky type ($\lambda$, q)-Bernstein Stancu operators $\left(B_{(r,s,t)_{,\lambda},q}^{\alpha,\beta}(f;x)\right)$ are:

$$\left(B_{(r,s,t)_{,\lambda},q}^{\alpha,\beta}(f;x,y,z)\right) = \frac{1}{(1+y)^{r}(1+z)^{s}(1+x)^{t}} \sum_{m,n,k=0}^{\infty} \sum_{\ell=0}^{\infty} B_{(r,s,t)_{,\lambda},q}^{\alpha,\beta} \left( \begin{array}{c}
m - i + 1 \ 
 n - j + 1 \\
 k - \ell + 1
\end{array} \right) \binom{m}{i} \binom{n}{j} \binom{k}{\ell} x^{i}y^{j}z^{\ell},$$

where $f \in H_{m}(K)$ and $K = [0, \infty) \times [0, \infty) \times [0, \infty)$. We have:

$$B_{(r,s,t)_{,\lambda},q}^{\alpha,\beta}(f_{000}(x,y,z)) = B_{(r,s,t)_{,\lambda},q}^{\alpha,\beta}(f_{100}(x,y,z)) = \frac{m}{m+11+x},$$

$$B_{(r,s,t)_{,\lambda},q}^{\alpha,\beta}(f_{222}(x,y,z)) = \frac{n}{n+11+y} B_{(r,s,t)_{,\lambda},q}^{\alpha,\beta}(f_{100}(x,y,z)) = \frac{k}{k+11+z},$$

$$B_{(r,s,t)_{,\lambda},q}^{\alpha,\beta}(f_{s0k}(x,y,z)) = \frac{m(n-2)(m-1)}{(m+1)(m-1)} \left( \frac{x}{1+x} \right) + \frac{m(n-1)}{(m+1)} \left( \frac{y}{1+y} \right) + \frac{n(n-2)}{(n+1)} \left( \frac{z}{1+z} \right) + \frac{k(k-2)}{(k+1)} \left( \frac{z}{1+z} \right) + \frac{k(k-1)}{(k+1)} \left( \frac{z}{1+z} \right) + \frac{k}{k+11+z},$$

Now take $A = [C,1,1]$ and define a triple sequence $u: = \left(u_{mnk}\right)$ by:

$$u_{mnk} = (-1)^{m+n+k}$$

we observe that:

$$B^{n+1} \lim_{[1,1,1]}(u) = 0$$

However, the Chlodowsky type ($\lambda$, q)-Bernstein Stancu operators $\left(B_{(r,s,t)_{,\lambda},q}^{\alpha,\beta}(f;x)\right)$ of the triple sequence of $u$ is not P-convergent and statistical convergent. Now using (4.10) and (4.11), we define the following double-positive linear operators on $H_{m}(K)$ as follows:

$$B_{(r,s,t)_{,\lambda},q}^{\alpha,\beta}(f_{,000}(x,y,z)) = \left(1 + u_{mnk} \right) B_{(r,s,t)_{,\lambda},q}^{\alpha,\beta}(f_{,000}(x,y,z))$$

Then, observe that the Chlodowsky type ($\lambda$, q)-Bernstein Stancu operators $\left(B_{(r,s,t)_{,\lambda},q}^{\alpha,\beta}(f;x)\right)$ of a triple sequence $\left(B_{(r,s,t)_{,\lambda},q}^{\alpha,\beta}(f;x)\right)$ defined by (4.12) satisfy all hypotheses of Theorem 1. Hence, by (4.9) and (4.11), we have, for all $f \in H_{m}(K)$:

$$B^{n+1} \lim_{[1,1,1]} \sum_{a,b,c} a_{g_{a,b,c}} B_{(r,s,t)_{,\lambda},q}^{\alpha,\beta}(f) = 0$$

Since $u$ is not P-convergent and statistical convergent, the sequence $\left(B_{(r,s,t)_{,\lambda},q}^{\alpha,\beta}(f)\right)$ cannot uniformly converge to $f$ on $K$ or statistical sense.

Example 1 with the help of Matlab, we show comparisons and some illustrative graphics for the convergence of operators (1.2) to the function $f(x) = 1 - x^{2}e^{-x^{2}}$ under different parameters.

**Fig. 1:** Chlodowsky type ($\lambda$, q)-Bernstein-Stancu operators
From Fig. 1(a), it can be observed that as the value the q approaches towards 1 provided 0 < q ≤ 1, Chlodowsky type (λ, q)-Bernstein-Stancu operators given by (1.2) converges towards the function \( f(x) = 1 - xe^{-x^2} \). From Fig. 1(a), it can be observed that for \( \alpha = \beta = 0 \), as the value the \((r, s, t)\) increases, Chlodowsky type (λ, q)-Bernstein-Stancu operators given by (1.2) converges towards the function. Similarly from Fig. 1(b), it can be observed that for \( \alpha = \beta = 1 \), as the value the \((r, s, t)\) increases, Chlodowsky type (λ, q)-Bernstein-Stancu operators given by (1.2) converges towards the function. From Fig. 1(b), it can be observed that as the value the \([r, s, t]\) increases, Chlodowsky type (λ, q)-Bernstein-Stancu operators given by (1.2) converges towards the function.

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Author’s Contributions

All authors equally contributed in this study.

Ethics

This article is original and contains unpublished material. The corresponding author confirms that all of the other authors have read and approved the manuscript and no ethical issues involved.

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