Modified Human Development Index using Data Envelopment Analysis Approach

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Abstract: Composite Indicator is considered the mathematical aggregation which has wide usage for monitoring performances, conducting benchmarks, analyzing policies, and communicating publicly. Human Development Index (HDI) is the most popular index which measures human development through average achievement in its main dimensions: Health status, education status, and living standard but it is faced with several critiques, positive and negative. Moreover, HDI was tested to have a positive and significant correlation with natural resource abundance. Therefore, based on Mathematical Programming approaches, previously tested for Composite Indicators' development, this research proposes a new calculated HDI using a Data Envelopment Analysis approach based on the Goal Programming model; including missing values' estimation. This new proposed HDI was validated through Sensitivity Analysis of Normalization and Weighting methods; in addition to Wilcoxon Signed Rank Test. The first test shows a positive high correlation between the proposed HDIs and the United Nations HDI. Those tests ensure that HDI rankings are highly correlated and that they are unchanged given the different normalization and weighting techniques. Moreover, they reflect that the paired sample mean is not the same. This highlights the advantageous property of the proposed HDI; preserving both the advantages of Goal Programming and Data Envelopment Analysis approaches, in addition to others.

Keywords: Composite Indicator (CI), Human Development Index (HDI), Goal Programming (GP), Data Envelopment Analysis (DEA), Missing Values

Introduction

Composite Indicator (CI) is considered the mathematical aggregation which has wide usage for monitoring the performances, conducting benchmarks, analyzing policies, and communicating publicly in various fields and sectors; including but not limited to society, economy and environment. There is a wide set of CIs, where the United Nation's (UN) Human Development Index (HDI) is highlighted globally which advises countries' strategies/policies. HDI's calculation is through the geometric mean of selected normalized indices. Those indices are concerned with the main human development dimensions. Such indices measure the average achievement in health status, education status, and living standard. This is about the Human Development Report's (HDR) (2016) definition (HDI, 2016; Hudrliková, 2013; Sayed et al., 2017; Zhou et al., 2007).

Changes in HD, which is proxied by HDI's changes, are correlated significantly and positively with abundance

in natural resources; in particular to its dimensions concerned with non-income. These results proved that natural resources' dimensions may have a blessing effect rather than a curse one for HD (Pineda and Rodríguez, 2010).

HDI is criticized positively and negatively, including critiques related to statistical quality and methodological soundness. Examples of such critiques include measurement errors, biases, and increasing reliance on mathematical interpolations, imputations, and modeling. Moreover, the choice of aggregation's and weighting strategy's arbitrariness are one of the highlighted critiques as well (Kovacevic, 2010).

Data Envelopment Analysis (DEA) is considered an alternative method used in the field of constructing CIs; which overcomes some of the limitations of constructing CIs. DEA is a type of nonparametric Mathematical Programming (MP)-based technique, that is used for converting a set of multiple inputs into a set of multiple outputs. DEA approach is used to evaluate Decision-Making Units (DMUs); such as a set of peer kind of entities' performance. There are various applications of



DEA that are used to evaluate the performances of different entities across the years (Cooper *et al.*, 2011; Sayed *et al.*, 2015; Thanassoulis *et al.*, 2008).

But still, these introduced DEA models have advantages as well as disadvantages, including cases of missing data. Such an issue is considered a chronic disease in applications of DEA; as variables have insufficient coverage which leads to having DMUs failing to report all required statistics (Kuosmanen, 2014). Therefore, further research should be exerted to construct CIs through this highly-influential methodology; leading to better results (Sayed *et al.*, 2015).

Moving to GP as one of the MP-based techniques highlights, that its goal is attained through minimizing the absolute deviation. This goal may not be fully achieved, but this method optimizes the results that are closely possible to this set goal. This GP model characteristic permits an opportunity of allowing multiple conflicting goals in the same model (Ahmad *et al.*, 2005; Schniederjans, 1995). Moreover, the GP achievement function can be formed through several priorities associated with each objective, along with the deviation variables (Ignizio, 1976).

Furthermore, this GP model is used as a method for estimating regression model parameters. This is mostly used when outliers exist in the analysis when compared with the least-squares method; which has a frequent usage for estimating regression model parameters (Ahmad *et al.*, 2005).

Reference to all the above, taking into consideration the importance of HDI in decision making at the countries' levels and its calculations' criticism, therefore, HDI's calculation should be as accurate as possible to give a good indication for decision-makers. Moreover, DEA and GP models have various positive characteristics that work on overcoming most of these critiques as well as missing values estimation. Accordingly, this study is aiming to present a modified model using DEA methodologies for calculating *CI*, concentrating on HDI.

Composite Indicators and the Development of "Human Development Index"

Composite Indicators

CI is a mathematical aggregation that is defined by Organisation for Economic Co-operation and Development (OECD) as a single index compiling selected individual indicators. The selection of individual indicators is determined by reference to the multi-dimensional concept's underlying model being measured. Although its usage is usual debate, CI has wide usage for monitoring performances, conducting benchmarks, analyzing policies, and communicating publicly in different fields and sectors including but not limited to society, economy, and environment. That is because CI facilitates the results' interpretation because of reducing the indicators' numbers without losing information. On the other hand, in case of poor construction of CI, it can provide misleading information. That's why; CI's usefulness depends mainly on the weighting and aggregation formulas; leading to its subjectivity (Hudrliková, 2013; Zhou *et al.*, 2007).

Due to the importance of the CIs, much literature is provided in the CIs' construction field. In addition, various methodologies have been studied in every step of CIs' construction; such that the "weighting and aggregation" are the most influential ones. Therefore, one of the biggest problems faced is selecting the best weighting methodology. These weights are important as they might be favoring countries over others. There are different methodologies; mainly summarized in Principal Components Analysis/Factor Analysis (PCA/FA), Equal Weights (EW), User-Weighting,... etc. (Sayed et al., 2015) As for the aggregation methodologies, additive and geometric aggregations are the mostly-wide used ones. The additive aggregation method is the most-widely spread is the linear aggregation; which is the summation of weighted and normalized sub-indicators: $CI = \sum_{i=1}^{I} w_i sub - CI_i$ with $\sum_{i=1}^{I} w_i = 1$ and $0 \le w_i \le 1$, for all $i = 1, \dots, I$, while the geometric aggregation is defined as $CI = \prod_{i=1}^{I} sub - CI_i^{wi}$ is an in-between solution (OECD, 2008).

Human Development Index

UN's HDI is a CI that is used popularly and widely. The HDI was introduced, through the first HDR, for advancing human well-being. This indicator is different as it doesn't mostly concentrate on the economy's richness but is concerned with the expansion of human life's richness. Mahbub Ul Haq is the economist who developed the HDI (HDI, 2016a).

HDI's calculation is through the geometric mean of selected normalized indices. Those indices are concerned with main human development dimensions, including health status, education status, and living standard. This is about the Human Development Report's HDR (2016) definition. Health status is assessed by birth's life expectancy, education status is assessed by years of schooling's mean for adults who are aged 25 years and more, and schooling's expected years for children at school's entry age and finally, the living standard is assessed by per capita Gross National Income (GNI). Based on the HDI's calculations, the HDR lists policy recommendations at the national level to what societies should do to advance human development for everyone. Since the HDI formulation, its critics have stimulated adjustments to the index. Therefore, the index was subject to some refinements and modifications (HDI, 2016a; Sayed et al., 2017).

Human Development Index Methodology

Calculation's Steps

HDI is calculated through two steps. Starting with Step 1: "*Creation of the Dimension Indices*":

Goalposts are set, as determined in Table 1. These goalposts are used to change the indicators' different units of measurement into a 0 to 1 scale. By that, these indicators were standardized (HDI, 2016b).

There is the justification behind the goalposts' choice. As for the life expectancy, the selection of 20 years is the minimum value referenced to historical evidence; which is that the life expectancy is not less than 20 years in any of the countries. For the schooling's expected years' minimum values and the years of schooling's mean, they are set to 0; as formal education is essential for societies. The schooling's expected years' maximum value is set to 18, which is equivalent to most countries' expected years of the master's degree achievement. While years of schooling's mean's maximum value is set to 15, which is a projected value by 2025 (HDI, 2016b).

Unmeasured subsistence and nonmarket production considerable amount in close to the minimum economies justifies the per capita GNI minimum value of \$100. This value is not referenced in the official data. While the \$75,000 maximum value was justified based on what was shown, by Kahneman and Deaton (2010), that for the per capita income above \$75,000, there is no gain in well-being and human development virtually (HDI, 2016b).

The dimension indices are calculated, through the minimum and maximum values, as follows (HDI, 2016b):

$$Dimension index = \frac{actual value - minimum ualue}{maximum value - minimum value}$$
(1)

As listed before, since the education dimension was measured through two indicators, therefore, Eq. 1 was calculated for each indicator separately. Afterward, the arithmetic mean was calculated for the two resulting indices (HDI, 2016b).

Based on the above calculations, each dimension index is a proxy for the corresponding dimension's capabilities. As a result, the income indicator should reflect the transformation of income into human capabilities' diminishing returns. To take this transformation into consideration, the real per capita Gross Domestic Product (GDP) logarithm was used for the income indicator (Anand and Sen, 2000; HDI, 2016b).

Table	1:	Indicators'	Goa	lposts ^a
_	_			

Next moving to Step 2: "Aggregation of the Dimensional Indices to Calculate the HDI":

The HDI is calculated through the three dimension indices' geometric mean, as calculated through Eq. 2 (HDI, 2016b):

$$HDI = \left(I_{Health} \cdot I_{Education} \cdot I_{Income}\right)^{\frac{1}{3}}$$
(2)

Missing Values' Estimation

Cross-country regression models are used to estimate the missing values, by the HDR Office, for specific countries with any of its indicators with missing values. In HDR, for ten countries, schooling's expected years were estimated and, for eleven countries, mean years of schooling were estimated (HDI, 2016b).

Country's Categorization

The same HDI cutoff points are used by HDR to group countries introduced in the Report by 2014, as in Table 2 (HDI, 2016b).

Human Development Index and Natural Resources

Changes in HD, which is proxied by HDI's changes, are correlated positively and significantly with abundance in natural resources; in particular to its dimensions concerned with non-income. These results proved that natural resources' dimensions may have a blessing effect rather than a curse one for HD.

Results also proved that net importers have HDI with higher levels; along with all of its components. However, when changing the variables, revealed a different scenario. Life expectancy's changes are roughly the same across all groups of countries, i.e., net importers' and net exporters' countries. While for growth in GDP, net exporter countries have smaller values. However, literacy and gross enrolment changes are larger, on average, for net exporter countries. Mostly, all the abovementioned changes are greater for net exporter countries than net importer countries except for growth in per capita GDP and that HD is affected primarily by natural resources through channels other than income.

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Indicator	Minimum value	Maximum value
Life expectancy in years	20	85
Schooling's expected years	0	18
Years of schooling's mean	0	15
Per capita GNI as per Purchasing Power Parity (PPP) \$ in 2011	100	75,000
otes, the united nations development program, human development	report 2016	
Points ^a		
lopment		0.800 and above
	Indicator Life expectancy in years Schooling's expected years Years of schooling's mean Per capita GNI as per Purchasing Power Parity (PPP) \$ in 2011 potes, the united nations development program, human development r Points ^a Hopment	Indicator Minimum value Life expectancy in years 20 Schooling's expected years 0 Years of schooling's mean 0 Per capita GNI as per Purchasing Power Parity (PPP) \$ in 2011 100 otes, the united nations development program, human development report 2016 Points ^a dopment

very nigh human development	0.000 ulu ubove
High human development	0.700-0.799
Medium human development	0.550-0.699
Low human development	Below 0.550

a. Source: Technical notes, the United Nations development program, human development report 2016

In summary, natural resources proved to be development assets that require adequate physical and human capital in addition to appropriate policies. Moreover, natural resources can be properly employed to create economic growth and development sustainably through physical and human capital investment, proper export diversification, volatility, and real exchange rate control (Pineda and Rodríguez, 2010).

Composite Indicators' Criticism (Concentrating on Human Development Index)

Any index is faced with positive and negative critiques, similar to the HDI. These critiques comprise statistical quality and methodological soundness. As for the positive critiques, composite indicators overcome the cons of the multiple indicators. Multiple indicators are used to allow observing the multiple angles of the object of interest; however, it doesn't allow a parsimonious understanding of the phenomenon being under consideration, which composite indicators can allow. Composite indicators are used to assist decision-makers such that they can recap multidimensional or complex issues. Moreover, they are used to rank countries in terms of complex issues. Therefore, these indicators attract public interest. Furthermore, it is quite clear that using composite indicators reduces the size of the indicators' list (Kovacevic, 2010).

Talking about the negative critiques, their construction is judgmental/subjective, as it depends on the choice of component indices, choice of the functional model, choice of measurement error estimation model, choice of including and/or excluding indices' mechanisms, choice of transformation and/or trimming of indicators, choice of weights, choice of normalization scheme's type, choice of aggregation system, choice of imputation algorithm type and missing data amount, ... etc. (EC, 2018; Kovacevic, 2010).

As well, HDI's calculation is faced with several critiques: First, HDI was criticized for the use of equal weights to all its component indices. Second, each of its component indices is calculated differently from its raw components, with different implicit weights. Third, measurement errors, which are a major statistical suffer for all statisticians, are represented in incomplete coverage, estimated data sets, and lack of census data. These errors may end up with excessive variability in each component index. Fourth, HDI's goalposts' values weren't realistic in the HDI dataset by 2012. In addition, the highest observed values in the time series (1980–2012) were the estimates of the maximum values.

 $\underbrace{ \begin{array}{c} \text{Moreover, the normalization method} \\ \underbrace{ \begin{array}{c} actual \ value \ -minimum ualue \\ \hline maximum \ value \ -minimum value \\ \end{array} } \end{array} }_{\text{leads the numerator to} }$

approach*zero* when the actual value approaches the minimum. This is considered not compatible with the method of geometric aggregation where there is a need for positive indices. Moreover, fifth, the credibility *CIs* is

limited by the unfavorable dependence of countries' rankings on methods of weighting, aggregation, and normalization. As a result of these negative critiques, alternative indices were calculated to modify the HDI's calculation (Aguña and Kovacevic, 2011; Kovacevic, 2010; Pinar *et al.*, 2013; Sayed *et al.*, 2017).

Mathematical Programming

MP is defined as a tool to theoretically manage science and economics. MP has different types: Linear programming where linear algebraic equations' form is the basic description. It can also be nonlinear programming; in case of a requirement for more complex forms. MP has various usages; such as Calculation of economic growth, planning of production schedules, ... etc. (EEB, 2017). DEA and Goal Programming Model (GP) are examples of the MP-based techniques; which are the main core of this study.

Data Envelopment Analysis

In 1978, Charnes, Cooper, and Rhodes (CCR) developed DEA. DEA is the nonparametric and MP-based technique. Moreover, it is a data-oriented approach that transforms a set of multiple inputs into a set of multiple outputs; to evaluate a set of peer entities' performance, called DMUs. DEA model aims to calculate a DMU's efficiency; such that if the efficiency score is 1, then this unit is an efficient one. On the other hand, if the efficiency score is positive and less than 1, then this unit is an inefficient one (Cooper *et al.*, 2011; Sayed *et al.*, 2015; Thanassoulis *et al.*, 2008).

Various DEA applications had been used in various entities' performance evaluations, conducting different contexts of activities in different countries, during these recent years. Examples of the different entities are hospitals, universities, business firms, courts, ... etc.; along with countries' performances, regions' performances, ... etc. (Cooper *et al.*, 2011; Sayed *et al.*, 2015; Thanassoulis *et al.*, 2008).

There are several advantages of DEA summarized in its empirical orientation and fewer assumptions compared to other approaches. Therefore, DEA is used as a good replacement in case of resistance to alternative approaches. This resistance is a result of the type of relations between the set of outputs and the set of inputs; which may be complex, and mostly unknown (Cooper *et al.*, 2011; Thanassoulis *et al.*, 2008).

Composite Indicators through Data Envelopment Analysis

As mentioned earlier, construction of CIs is one of the growing literature; including its different steps' methodologies. "Weighting and Aggregation" are considered the most influential steps. One of the various methods used for calculating weights, through MP, is the DEA methodologies (Sayed *et al.*, 2015).

As previously mentioned, there are limitations, to constructing *CIs*, in terms of EW and PCA/FA, therefore, DEA techniques were one of the highlights in *CI* construction. DEA endogenously calculates objective weights which provide more perception about the dataset's variables. This technique highlights the more significant variables (Sayed *et al.*, 2015).

DEA model is designed with an objective function to maximize:

$$Z_0 = \frac{\sum_{i=1}^{I} w_i y_{i0}}{\sum_{j=1}^{J} v_j x_{j0}}$$
(3)

where, z_0 is the objective variable that denotes the efficiency of the DMU_0 . DMU is efficient when $z_0 = 1$ and is inefficient when $z_0 < 1$. The below model is repeated *N* times, once for every DMU_n , where n = 1, 2, ..., N; such that DMU_0 is the DMU under investigation (Saved *et al.*, 2015):

$$\max = \frac{\sum_{i=1}^{I} w_i y_{i0}}{\sum_{j=1}^{J} v_j x_{j0}}$$
(4)

Subject to:

$$\frac{\sum_{i=1}^{I} w_i y_{in}}{\sum_{j=1}^{J} v_j x_{jn}} \le 1, n = 1, 2, ..., N$$
(5)

$$w_i, v_j \ge 0, i = 1, 2, ..., I; j = 1, 2, ..., J$$
 (6)

where:

- *y_{in}* is *DMU_n* ' *s ith* output and *w_i* is its corresponding weight, where *i* = 1,2,..., *I* output
- x_{jn} is DMU_n ' $s j^{th}$ input and v_j is its corresponding weight, where j = 1, 2, ..., J inputs and
- *w_i* and *v_j* are the decision variables for this model

This model is an optimization technique that evaluates the efficiency of DMUs through a set of inputs and sets of outputs on the weighted input and output ratio strength, whereas the DEA frontier is obtained through linear fractional programming (Dar *et al.*, 2016). In general, DEA has assisted in creating CIs; as it doesn't require weights that are previously agreed upon or weights that are uniquely set. This is a result that the model assigns the weights from the available data (Sayed *et al.*, 2015).

Missing data is a continuing challenge in DEA applications, mostly, to insufficient coverage of significant input and/or output variables, or failing of DMUs to report needed statistics. DEA approach mostly needs large numbers of DMUs to provide meaningful and statistical results. This is due to its multidimensional and

nonparametric nature. As a result, DEA is most vulnerable to the problems of the data. Yet, the most traditional approach to addressing missing data problems is deleting blank entries from matrices of the data, by deleting the entire rows or columns. As a result, valuable and important information is lost (Kuosmanen, 2014).

Goal Programming Methodology and Applications

In 1961, GP was introduced by A. Charnes and W. W. Cooper. Goal attainment is obtained by minimization of their absolute deviation. Such goals can be not fully achievable, but through GP, the results are achieved closely possible to the agreed goals. These deviation variables are placed in the objective function to be minimized. This permits an opportunity of allowing multiple conflicting goals in the same model. While formulating this model, decision and deviational variables are used and structural/system constraints and goal constraints are the defined categories of constraints. A generally accepted model is (Ahmad *et al.*, 2005; Schniederjans, 1995):

$$Minimize: Z\sum_{n \in \mathbb{N}} \left(d_n^+ + d_n^- \right) \tag{7}$$

Subject to:

$$\left(\sum_{j\in J}\beta_{j}x_{jn}\right) + d_{n}^{-} - d_{n}^{+} = b_{n}, j = 1, 2, ..., J \text{ and } n = 1, 2, ..., N$$
 (8)

$$d_n^{+} * d_n^{-} = 0 (9)$$

$$\beta_j, d_n^-, d_n^+ \ge 0, j = 1, 2, ..., J \text{ and } n = 1, 2, ..., N$$
 (10)

where:

- Z: Objective function
- *x_{jn}*: The coefficient linked with variable *j* in the *n*th goal
- β_i : The *j*th decision variable
- b_n : The linked value of the right-hand side
- d_n : Negative deviational variable from the n^{th} goal. This is considered an underachievement
- d_n^+ : Positive deviational variable from the n^{th} goal. This is considered overachievement

There are a set of procedures to build a GP model; as per Ignizio, 1976, which are as follows:

- Goals Setting: From the above model, it is clear that the goals in the GP model are equivalent to constraints, as shown in Eq. 9. Goals are discriminated by deviation variables; which represent the over-achievement and the underachievement per each goal
- Aspiration Level Determination: Based on the study's objective, the aspiration level differs from one GP model to another. Accordingly, each goal is

followed by a specific aspiration level that connects the objective to a real target

• Achievement Function: Each goal is associated with deviation variables, where the achievement function is minimizing those unwanted deviation variables. Those unwanted variables are set to reference the mathematical relation between the goal and the targeted level

There are several types of GP, and the main two forms are the lexicographic and the weighted GP; as per Tamiz and Jones (1995). The lexicographic GP is also known as the pre-emptive GP in literature. This type has the remarkable feature of the existence of priority levels for the different goals. Goals with the highest priority are solved first; then the results are involved when solving other priority levels; along with their associated deviation variables to be minimized (Ahmed, 2017).

Achievement function is formed through those priorities associated with each objective, along with the deviation variables; as follows:

$$\begin{aligned} \text{Minimize} Z &= \\ \left\{ P_1 \left[g_1 \left(\overline{s^+}, \overline{s^-} \right) \right], P_2 \left[g_2 \left(\overline{s^+}, \overline{s^-} \right) \right], \dots, P_k \left[g_k \left(\overline{s^+}, \overline{s^-} \right) \right] \right\} \end{aligned} \tag{11}$$

where:

- $g_k(\overline{S^+}, \overline{S^-})$ is a linear function of the deviation variables
- \bar{a} dimension represents the preemptive priority levels number (K) among the objectives
- P_k is the priority associated with $g_k(\overline{S^+}, \overline{S^-})$

 $K \le m$, i.e., the pre-emptive priorities number is equal to or less than the objectives total number

• This leads to the general GP model formulation, with multiple objectives, as follows (Ignizio, 1976):

Minimize Z =

$$\left\{P_{1}\left[g_{1}\left(\overline{s^{+}},\overline{s^{-}}\right)\right],P_{2}\left[g_{2}\left(\overline{s^{+}},\overline{s^{-}}\right)\right],\dots,P_{k}\left[g_{k}\left(\overline{s^{+}},\overline{s^{-}}\right)\right]\right\}$$
(12)

Subject to:

$$\left(\sum_{j=1}^{J} v_{j} x_{jn}\right) + s_{n}^{-} - s_{n}^{+} = b_{n}, n = 1, 2, \dots, Nj = 1, 2, \dots, J$$
(13)

$$v_j, s_n^-, s_n^+ \ge 0, j = 1, 2, ..., J, n = 1, 2, ..., N$$
 (14)

Estimating Missing Values Through Goal Programming Model

Multiple regression is one of the most popular methods for estimating missing values, however, sometimes, the data do not satisfy some of its required assumptions (linearity, constant variance, normality, independence, and large sample size). Accordingly, if those assumptions are satisfied, then the least square method is used to estimate this model's parameters for missing values estimation. On the other side, if those assumptions are not met in the data, particularly with a small sample size, then the results can be misleading. Therefore, there needs to be an alternative method for estimating the model's parameters. One such alternative is the GP method; which is used as a method for estimating regression model parameters. GP was proved to be better than the least square method; with Mean Square Error (MSE) proved to be the smallest for all sample sizes, significant parameters, and significant overall fitting of the model in all sample sizes (Ahmad *et al.*, 2005; Hussain and Ali, 2019).

Moreover, GP is mostly used when outliers exist in the analysis when compared with the least-squares method, which is mostly used for regression model parameter estimation. However, biasness for the least squares method occurs in the case of outliers. Analysis proved that when using the GP approach, the sum of absolute residuals is minimized rather than the sum of the squares of the residuals as in the case of the least squares technique. Accordingly, the generally accepted model is used (Ahmad *et al.*, 2005; Schniederjans, 1995).

The model parameters are estimated from the complete datasets (NM). Afterward, the imputation process takes place, which occurs via substituting a missing value with a particular one. Imputation provides assumptions about relationships among or between the variables; along with the relationships in the analytic model itself. Accordingly, those model parameters are used to estimate the missing values of the remaining datasets through the below prediction equation (Ahmad *et al.*, 2005; CBHSQ, 2018; Schniederjans, 1995):

$$\hat{X}_{(j\in J)_{\text{missing-var}P}} = \left(\sum_{j\in J} \hat{\beta}_j x_{(j\in J)\text{complete-var}P}\right), j = (15)$$

$$1, 2, \dots, J \text{ and } p = NM + 1, NM + 2, \dots, N$$

Proposed Model using Goal Programming for a "Modified Human Development Index using Data Envelopment Analysis Approach"

As previously mentioned, one of the most popular and widely used *CI* is the UN HDI. Based on the HDI's calculations, the HDR lists policy recommendations at the national level as to what societies should do to advance human development for everyone. Therefore, HDI's calculation should be as accurate as possible to give a good indication for decision-makers.

HDI's calculation, as any other *CI*, is faced with several critiques, as mentioned in the previous sections and DEA methodologies have various positive characteristics that work on overcoming most of these critiques. Therefore, DEA methodologies are used, by many researchers, as a good replacement for the CI's calculations. Various DEA methodologies were used in the evaluation of different

entities' performances. But still, these DEA models are criticized negatively, and there should be an adjustment to these models as a replacement for CI's calculations.

After listing the importance of the HDI for decisionmakers, its methodological criticism, DEA's methodologies for calculating CIs, and lastly these methodologies' criticisms, this study is aiming to present GP modified model using DEA methodologies for calculating CI, concentrating on HDI.

Accordingly, the proposed model is based on the generally accepted GP with two main objectives: (1) Estimating missing values through the GP model; and (2) Calculating Weights for HDI using the DEA approach.

Estimating Missing Values through Goal Programming Model

In 1961, GP was introduced by A. Charnes and W. The first objective is designed for estimating the missing values through the GP model and estimating regression model parameters; since missing data is a continuing challenge in DEA applications, mostly insufficient coverage of significant input and/or output variables, or failure of DMU to report needed statistics. DEA approach mostly needs large numbers of DMUs to provide meaningful and statistical results. This is due to its multidimensional and nonparametric nature. As a result, DEA is most vulnerable to the problems of the data.

Since there usually exists some connection between some variables that can be used to make a prediction and in case of the existence of outliers, therefore the priority level is designed by minimizing the sum of absolute residuals. Moreover, goal constraint is defined as follows:

$$\left(\sum_{(j\in J)complete-vat's} \beta_j x_{jm}\right) + d_m^{-} - d_m^{+} = x_{(j\in J)missing-vat''}, j = 1, 2, ..., J and m = 1, 2, ..., NM$$
(16)

To estimate missing values and achieve the first objective, the below goal constraint is defined, for the imputation process, limited to the set NM + 1, ..., NM; which is a defined set of *x*'s corresponding to the missing values; as follows:

$$\hat{X}_{(j\in J)\text{missing-var}P} = \left(\sum_{j\in J} \hat{\beta}_j X_{(j\in J)\text{complete-var}sP}\right), j = (17)$$

$$1, 2, \dots, J \text{ and } p = NM + 1, NM + 2, \dots, N$$

Similarly, the case for estimating the missing values for the output variables through the below two constraints:

$$\left(\sum_{(i \in I) complete-var's} \alpha_i y_{im}\right) + d_o^{-} - d_o^{+} = y_{(i \in I) missing-var'}, i = (18)$$
1,2,..., *I* and $o = 1, 2, ..., NO$

$$\hat{y}_{(i\in I)missing-\operatorname{var} q} = \left(\sum_{i\in I} \hat{\alpha}_i \; y_{(i\in I)complete-\operatorname{var} sq}\right), i = 1, 2, \dots, I \text{ and } q = NO+1, NO+2, \dots, N$$
(19)

Data Envelopment Analysis Model used for Calculating Weights for Human Development Index

The second objective is formulated to maximize $\left(\frac{\sum_{i=1}^{I} w_i y_{in}}{\sum_{j=1}^{J} v_j x_{jn}}\right)$ per each *DMU*; in this case, it reflects the

country, to be equal to 1. This reflects the efficiency of the DMU. The below equation is repeated N times, once for every DMU_n , where n = 1, 2, ..., N. In addition, to avoid non-optimality or infeasibility, two nonnegative decision variables are created. These variables are designed to capture positive and negative deviations. Finally, the constraint is created as follows to maximize the efficiency per each DMU; i.e.: Country:

$$\frac{\sum_{i=1}^{I} w_i y_{in}}{\sum_{j=1}^{J} v_j x_{jn}} + s_n^{-} - s_n^{+} = 1;$$

$$n = 1, 2, ..., N, \ i = 1, 2, ..., I, \ j = 1, 2, ..., J$$
(20)

Moving to the weights, this model avoids the zero weights; including the below positivity constraints. Moreover, the sum of sub-indicators' weights adds up to one for a country/DMU. This provides a comparison ability between this model's weights and those calculated through another weighting method(s):

$$\sum_{i=1}^{I} w_i = 1; i = 1, 2, \dots, I$$
(21)

$$\sum_{j=1}^{J} v_j = 1; \ j = 1, 2, ..., J$$
(22)

$$w_i, v_j \ge \varepsilon; \ i = 1, 2, ..., I; \ j = 1, 2, ..., J$$
 (23)

New Proposed Model

Finally, the newly suggested goal programming model is designed to result in a modified HDI using the DEA approach; with an achievement function that is formed through three priorities associated with each of the above objectives, along with the deviation variables; as follows:

$$Minimize Z = \left\{ \sum_{m=1}^{NM} \left(d_m^{+} + d_m^{-} \right), \sum_{o=1}^{NO} \left(d_o^{+} + d_o^{-} \right), \sum_{n=1}^{N} s_n^{-} \right\} (24)$$

Subject to:

$$\left(\sum_{(j \in J) complete-vat's} \beta_j X_{jm}\right) + d_m^{-} - d_m^{+} = x_{(j \in J) missing-vat''}, j = 1, 2, ..., J and m = 1, 2, ..., NM$$
(25)

$$\hat{x}_{(j\in J)\text{missing}-\text{var }P} = \left(\sum_{j\in J} \hat{\beta}_j x_{(j\in j)\text{ complete}-\text{var'} sP}\right),$$

$$j = 1, 2, \dots, J \text{ and } p = NM + 1, NM + 2, \dots, N$$
(26)

$$\left(\sum_{(i=1)complete-var's} \alpha_{i} y_{im}\right) + d_{o}^{-} - d_{o}^{+} = y_{(i\in I)missing-var^{o}},$$

$$i = 1, 2, ..., I \text{ and } o = 1, 2, ..., NO$$
(27)

$$\hat{y}_{(i=1)\text{missing-var}^q} = \left(\sum_{i \in I} \alpha_i \ y_{(i\in I)\text{complete-var}^{'s^q}}\right),$$

$$i = 1, 2, \dots, I \text{ and } q = NO + 1, NO + 2, \dots, N$$
(28)

$$\frac{\sum_{i=1}^{I} w_i y_{in}}{\sum_{j=1}^{J} v_j x_{jn}} + s_n^{-} - s_n^{+} = 1;$$

$$n = 1, 2, ..., N, \ i = 1, 2, ..., I, \ j = 1, 2, ..., J$$
(29)

$$\sum_{i=1}^{I} w_i = 1; i = 1, 2, \dots, I$$
(30)

$$\sum_{j=1}^{J} v_i = 1; \ j = 1, 2, \dots, J$$
(31)

$$w_i, v_j \ge \varepsilon; \ i = 1, 2, ..., I; \ j = 1, 2, ..., J$$
 (32)

$$d_n^{+} * d_n^{-} = 0 \tag{33}$$

$$s_n^{-} * s_n^{+} = 0 \tag{34}$$

$$d_n^{-}, d_n^{+} \ge 0, n \in N$$
 (35)

$$s_n^- * s_n^+ \ge 0, n = 1, 2, ..., N$$
 (36)

where:

- Z is the objective function
- y_{in} is DMU_n 's i^{th} output and w_i is its corresponding weight, where i = 1, 2, ..., I outputs,
- x_{jn} is DMU_n 's j^{th} input and v_j is its corresponding weights, where j = 1, 2, ..., J inputs,
- *NM* is a set of *x*'s corresponding to the complete values
- *NO* is a set of *y*'s corresponding to the complete values
- E is a small positive value set to be equal to 0.0001. The selection of E value was determined based on several computations with different values till this proposed model yielded the best results based on the objective function and
- d_m^+ , d_m^- , d_o^+ , d_o^- , s_n^+ , s_n^- , β_j , α_j , w_i , and v_j are the decision variables for this model

Data and Results

To calculate a modified HDI using the DEA approach, the model should have a defined set of input as well as

 2 Please refer to Table 3 in the appendix.

output variables. The output variables will consider the standardized indices (2016)¹ which measures the average achievement in human development key dimensions: Health status, education status, and living standard (Anand and Sen, 2000; HDI, 2016b).

As for the input variables, natural resources are measured, by most scholars, as primary product export'(s) share, including fuel, food, ores, agricultural raw materials, and metals to GDP. However, this mostly reflects measures of dependence on natural resources rather than measures of resource abundance. Therefore, the selection of input variables will follow Lederman and Maloney's net exports measure², which applies Leamer's (1999) primary goods groups. Resource abundance is measured through natural resource exports per worker. This proxy is a tradebased multi-commodity proxy, which allows for countries' larger coverage and is correlated positively with per worker natural resource endowment. Those are considered key advantages.

On the other side, there could be two flaws related to consumption with this proxy; as recognized by Lederman and Maloney. The first flaw is that income growth increases consumption, which could result in biasness when estimating the relationship between per labor net exports and income. This is confirmed by a positive correlation between income and exports among net exporters. The second flaw is that a decrease in exports of natural resources and an increase in its imports is linked with capital endowments' rise. To overcome those flaws, additional input variables will be identified as: Per worker imports of natural resources³. The indirect effect of natural resources on human development will be reflected (Pineda and Rodríguez, 2010).

The input variables will constitute standardized World Bank Development Indicators (2016) for Natural Resources⁴ for 189 countries. Out of those 189 countries, there are almost 43 records (23%) fully missing from all natural resource's variables, 4 records from the fuel exports input indicator, and 14 records missing from the labor force (WB, 2017).

To fully estimate the missing records "43 records", the countries were classified by income as per the World Bank classification. This assists to shed the light on the different groups of countries are doing. The categorization is based on different characteristics, such as average level of income, fragility, lending eligibility, geography, and fragility. Accordingly, the income classification was followed which is based on a measure of per person national income, or per capita

¹Data is extracted on 01/02/2020, from: "Human Development Index (HDI) | Human Development Reports (undp.org)"

³Please refer to Table 3 in the appendix.

⁴Data is extracted on 02/02/2020, from "Indicators | Data (worldbank.org)".

GNI, calculated through the Atlas method⁵. The world's economies were categorized into four groups of income: Low, lower-middle, upper-middle, and high (WB, 2019a). Data users are assisted, through this classification, to the aggregate, group and compare of interest statistical data and for key statistics presentation (WB, 2019b). Per each record and reference to the country classification group, the median imputation method was used to substitute missing values per each natural resource input variable. The median imputation method was selected given its robustness in outliers' presence in the data observed (Salgado *et al.*, 2016).

Accordingly, input and output variables are ready to be processed under the new proposed GP model; with "4 missing records" for the fuel exports variable. As previously mentioned, this model has two main goals, missing data estimation and modified HDI calculation including input variables as well as output ones. Consequently, the modified HDI is calculated in Table 4 in the appendix.

For the sake of outputting the new proposed model's results, the General Algebraic Modeling System (GAMS)⁶ the software package is used. This software satisfies meeting the objectives of this new proposed model, minimizing the achievement function through its three priorities.

Validation of the Proposed Human Development Index

HDI, like any other CI, includes subjective calculations; including normalization techniques, weighting techniques, and aggregation techniques. Those techniques are the most that affect the CIs' calculation. Accordingly, subjectivity assessment is crucial to measure the reliability of the CI (Ahmed, 2017).

Sensitivity Analysis of Normalization and Weighting Methods

As highlighted earlier, one of the tests applied for validation of the proposed HDI is the sensitivity analysis test, which is used to assess the normalization and weighting techniques' effect on the rankings of the countries, along with the difference between the proposed HDI versus HDI calculated through UN. The sensitivity analysis test is based on the Spearman''s Rank Correlation Coefficient ρ to assess the effect of the normalization and weighting techniques on the results of the CI.

The Sensitivity test is applied by comparing the UN's calculated HDI versus the proposed HDI; as H_0 : $\rho = 0$ versus H_1 : $\rho \neq 0$; with $\rho = 0.946^7$. The ρ 's value shows a positive high correlation between both HDIs, with a p-value less than 0.001. Therefore, the test reflects significant results which ensure that HDI rankings are highly correlated and that they are unchanged given the different normalization and weighting techniques.

Wilcoxon Signed Rank Test

Before applying the Wilcoxon Signed Rank test, the average absolute rank difference, between UN's values and the new proposed model's values, was calculated. This value, of 13.016⁸, shows that the proposed calculation of HDI has no great effect on the rankings of the HDI with an average absolute rank difference. Through checking the ranking difference, there were some highlights for some of the countries. Norway and Switzerland were on top of the list for HDI calculated through the UN, while through the proposed model: they come on the second 10th batch of the countries' list, while Australia remains on the top. On the other side, New Zealand and Canada move from the second 10th batch countries to the top of the list for HDI through the new proposed model. It was clear that the ranking of ten countries (Sweden, Trinidad and Tobago, Paraguay, Indonesia, Nicaragua, Kingdom of Eswatini, Côte d'Ivoire, Yemen, Sierra Leone, and Niger) hadn't changed.

Afterward, the Wilcoxon Signed Rank test is conducted. This test is a non-parametric method of analyzing the paired sample to compare the test's performance. The Wilcoxon Signed Rank tests the hypothesis about the median. At a fixed significance level of 5%, H_0 : $\mu_d = 0$ was set, i.e., the same mean from the paired sample against H_1 : $\mu_d \neq 0$, i.e., a different mean for the paired sample. Here, μ_d represents the average value of the deviation between the paired sample two sets from a normal distribution of the sample size of interest; i.e., the proposed HDI versus HDI calculated through UN (Imam *et al.*, 2014).

The data revealed a calculated mean of 8977.5, a variance of 567078.75, and a standard deviation of 753.0469. From those statistics, a z-score of 9.97910 is calculated, which yields a p-value of 0 (two-tail test) that is less than $\alpha = 0.05$. Here, the test results in H0 rejection reflect that the mean is not the same for the paired sample.

Discussion on the Proposed Human Development Index Findings

As previously illustrated, the new proposed model is developed through the GP model to produce a modified HDI using the DEA approach. Accordingly, the new proposed HDI is preserving both the advantages as well as limitations of GP and DEA approaches, in addition to others.

⁵ Please refer to The World Bank Atlas method - detailed methodology

⁻ World Bank Data Help Desk

⁶ GAMS - Cutting Edge Modeling

⁷This value is approximated to three decimal places.

⁸This value is approximated to three decimal places.

⁹ This value is approximated to three decimal places.

¹⁰This value is approximated to three decimal places.

Advantages of the Proposed Human Development Index Findings

The new proposed methodology is preserving the essence of the GP and DEA approaches; with additional features adding to such advantages. Those advantages are listed below.

The CI values equal maximization for all countries: The proposed methodology preserves the DEA approach's core benefit. This methodology is favoring all countries equally.

Weights' endogenous calculation: The weights of *CI* are calculated from the data which doesn't need any weights' prior information and limits the subjectivity in determining those weights. Moreover, it preserves the effect of natural resources variables, i.e., input variables' effect on the calculated output variables' weights.

Weights' electronic calculation: Weights are entirely electronically calculated through the proposed methodology. This adds to the advantages of the DEA approach where the CI values are maximized and eliminate the researcher or experts' weight bias subjectivity.

Highlighting influential sub-indicators: The proposed methodology calculated unequal and/or equal weights that are calculated based on the data. Accordingly, influential sub-indicators are more highlighted achieving higher weights and may affect decision making.

A common basis for countries comparison: The proposed methodology is planned to evaluate weights' one set for all countries for comparison on one scale. This offers easy interpretation and more intuitive weights' set, which can be compared to another weighting method(s).

Avoidance of zero weights: The proposed methodology endogenously estimates a lower bound of ε on weights.

Minimal assumptions: Inefficiency is the reason for all deviations from the frontier by DEA; however, none of the assumptions are required by DEA rather than convexity and free disposal as well as accommodation of several outputs and inputs. Moreover, the production frontier with DEA doesn't need any explicit functional form.

Priority levels for the different goals: The lexicographic model is characterized by achieving several goals with varying priorities.

Single step: The proposed methodology to calculate HDI is achieved through one step; while achieving more than one goal. This reduces random errors per each estimation step, as well as is time-saving and more convenient.

No intermediate sub-indicators along with input sub-indicators: As per the Spearman correlation test, the proposed HDI is highly correlated with the UN-HDI. This high correlation is examined even though the proposed HDI is calculated through the four normalized output and input sub-indicators and the UN-HDI is calculated through the three normalized sub-indicators. This, as well, contributes to the reduction of errors. Moreover, the proposed model contributes to the convenience of calculating weights.

Sum of absolute residuals minimization: The proposed methodology includes missing values estimation through the GP approach, which was proved to be more accurate than those obtained using the method of leastsquares. Using the GP approach, the problem can be restated to minimize the sum of absolute residuals rather than the sum of the squares of the residuals as in the case of the least squares technique.

Limitations of the Proposed Human Development Index Findings

As with any other new proposed methodology, it has its advantages as well as its limitations. Therefore, it is the analysts' objective to select the methodology that satisfies their needs. Those limitations pave the road for future research and further developments in this field. Those limitations are listed below:

- 1. Issues related to the GP model: As with any other model, errors can be accompanied by GP model formulations; which vary from one researcher's call to another. Moreover, there are some controversial issues as follows: Efficiency in GP solutions, inferiority in GP solutions, dominance in GP solutions, naive prioritization in GP models, naive relative weighting in GP models. redundancy, incommensurability, and others. Adding to the above, there are some arbitrary issues related to predetermined goals or targets' inappropriateness, which may result in issues such as dominance or just limits the information provided by the GP model. There may also occur failure of GP to identify unbounded solutions in GP models; in case of misformulation
- 2. Issues related to the DEA model: The DEA efficiency results are sensitive to the sample's changes. As for the frontier, it can only be composed of the sample's units, which could be efficient relative to the others in the sample but inefficient compared to the sample's outside units.
- 3. Issues related to the non-linear model: The proposed model has non-linear goals; which entails the use of a non-linear solving algorithm
- 4. Issues related to deviation variables: The proposed model includes a large number of deviation variables in case of large sample sizes; i.e., countries' number. As the countries' number increases, the deviation variables' number increases substantially. This has a direct effect on the processing time and complexity of the model

Conclusion

CI has wide usage for monitoring performances, benchmarks, analyzing policies, conducting and communicating publicly in several fields and sectors (Hudrliková, 2013; Zhou et al., 2007). HDI is one of the most popular and widely used CI, which is the geometric mean of selected normalized indices: Health status, education status, and living standard (HDI, 2016a; Sayed et al., 2017). HDI is significantly and positively correlated with natural resource abundance; specifically in its non-income dimensions (Pineda and Rodríguez, 2010). Given HDI's popularity, it is faced with some critiques, related to errors in measurement and inheritance of biases in the international data, the evidence-based character of the HDI violation by reliance increase on mathematical imputations, interpolations, and modeling, and the arbitrariness of weighting and aggregation strategy choice (Kovacevic, 2010).

In order to overcome such critiques, many types of research have sought the DEA to construct HDI, which is an optimization technique that evaluates the efficiency of DMUs through multiple outputs and multiple inputs on the weighted input and output ratios' strength (Dar *et al.*, 2016). However, missing data is a continuing challenge in DEA applications, mostly, to insufficient coverage of significant input and/or output variables, or failure of DMUs to report needed statistics (Kuosmanen, 2014). One of the methods to solve the missing data issue is GP, which is used as a method for estimating regression model parameters (Ahmad *et al.*, 2005).

Accordingly, our research is a combination of all the above technicalities proposing a modified HDI using the DEA approach; with a defined output and input variables' set: The average achievement in its main dimensions of human development and natural resources.

The proposed calculation for HDI includes subjective calculations; including normalization techniques, weighting techniques, and aggregation techniques. Therefore, subjectivity assessment is crucial to measure the reliability of the HDI. Those assessments include Sensitivity Analysis of Normalization and Weighting methods in addition to the Wilcoxon Signed Rank Test. The first test shows a positive high correlation between the proposed HDI and the UN HDI. Moreover, the proposed calculation of HDI has no great effect on the rankings of HDI with an average absolute rank difference of 13.016¹¹, in addition, the Wilcoxon Signed Rank test reflects that the mean is different for the paired sample.

Furthermore, this new proposed model is preserving both the advantages as well as limitations of GP and DEA approaches, in addition to others. The advantages include equal maximization of the CI values for all countries, endogenous calculation of weights, electronic calculation of weights, highlighting influential subindicators, countries comparison on a common basis, avoiding zero weights, minimal assumptions, priority levels for the different goals, single stepped model, no intermediate sub-indicators along with input subindicators and the sum of absolute residuals minimization. As for the limitation, they include issues related to the GP model, issues related to the DEA model, issues related to the non-linear model, and issues related to deviation variables. Those limitations can be addressed for future research.

Areas of Future Research

Below is the list of areas of future research, where academics can invest more for further development:

- 1. Input variables are set from the natural resources, while other input variables can be examined in correlation with the previously defined output variables. Such input variables may affect the calculated HDI and the countries' rankings
- 2. One of the goal constraints is a non-linear one which is computationally complex. Therefore, a linear transformation of this model is preferred, which is computationally simpler, compared to the non-linear one. Linear models are the main objectives and preferences of main researchers

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Author's Contributions

All authors equally contributed to this study.

Ethics

This article is original and contains unpublished material. The corresponding author confirms that all of the other authors have read and approved the manuscript and no ethical issues involved.

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¹¹ This value is approximated to three decimal places.

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Appendix

Table 3: Net Exports and Net Imports Input Variables (2016)^{ab}

		Not agriculture	1 Not orac				Orec and	
	Not food	net agriculture	and motols	Not Evol	Food		ores and	Engl
	Net lood	raw materials	and metals	Net Fuel	FOOd	Agricultural raw	immorta/	Fuel
G (exports/	exports/	exports/	exports/	imports/	materials imports/	imports/	1mports/
Country	labor force	labor force	labor force	labor force	labor force	labor force	labor force	labor force
Norway	1749.016	-37.899	1173.146	20207.950	2739.042	332.285	1289.036	1351.291
Switzerland	-900.501	-273.011	-120.078	-1748.862	3288.289	387.199	1399.667	2307.312
Australia	1494.162	397.298	5233.254	3837.322	1166.999	96.078	248.848	1869.289
Ireland	2112.867	38.841	-138.014	-1790.529	4057.106	203.887	466.947	2267.236
Germany	-329.853	-116.055	-266.775	-1466.481	2204.282	366.909	1081.485	2061.617
Iceland	6693.560	-96.171	8926.094	-3425.957	3185.578	285.984	708.975	3699.896
Hong Kong, China (SAR)	- 4230.483	-86.562	227.752	-3158.788	7443.305	255.926	1719.447	3344.579
Sweden	-1262.633	753.511	598.468	-1066.436	3086.077	314.041	751.220	2907.696
Singapore	-943.121	237.303	-273.147	-7927.318	3924.960	408.996	1455.964	22446.480
Netherlands	3113.451	1036.569	112.956	-1947.439	8273.466	1028.784	1553.505	8433.119
Denmark	2669.461	191.807	38.177	-295.112	4335.355	782.856	507.467	1729.573
Canada	866.555	813.592	976.925	3197.381	1829.362	178.072	571.871	1545.295
United States	69.180	83.144	-59.769	-243.394	894.933	133.983	317.257	1246.732
United Kingdom	-1030.005	-158.247	-68.268	-560.454	1927.599	232.124	568.650	1626.819
Finland	-1445.310	1422.912	134.098	-1364.618	2089.898	445.137	1299.190	3309.308
New Zealand	7152.242	1598.705	250.451	-1171.022	1714.060	110.373	180.179	1410.733
Belgium	828.211	93.858	-682.850	-2548.141	7854.255	974.932	2963.549	9382.921
Liechtenstein	-319.717	-58.325	-40.754	-1170.126	2383.344	246.928	537.562	2190.018
Japan	-911.792	-61.229	-370.200	-1946.428	1001.084	138.348	616.613	2103.001
Austria	-199.927	-191.649	-527.979	-2156 498	3010.972	746 407	1753.924	2846 358
Luxembourg	-4349 902	-721 260	-2218 165	-5201 778	9372 713	1722 234	4946 020	5252 192
Israel	-1096 834	-66 783	-85 930	-1745 847	1560 274	171 727	251 263	1964 483
Korea (Republic of)	-742 851	-47 532	-718 220	-2621 975	1009.274	236 837	1131 8/3	3883 589
France	223 556	-47.332	-89 888	-1520.001	1975 895	230.837	456 704	2008 294
Slovenia	1229.550	131 344	475 585	1410 634	2865 465	868 083	1003 443	3251 402
Spain	532 174	42.813	-475.585	1283 021	1731 307	170.054	588 260	1003 032
Czachia	221 517	-42.013	-/0.417	1027 256	1741 256	264 480	054 640	1622 072
Italy	-231.317	205 257	401.545	-1037.230	1/41.230	247 612	934.040	2120 542
Malta	-111.609	-203.337	-421.192	-1494.041	2170 254	62 970	120,000	2139.343
Fatania	-2219.977	-33.330	-92.023	-5260.055	2508 880	647 160	120.009	7444.701
Estollia	-496.902	0.210	132.093	-352.912	2398.880	047.100	350.025	2100.001
Gieece	-244.101	9.210	155.555	-790.270	1302.003	74 265	409.901	2696.579
Cyprus	-030.393	-7.309	1/3.01/	-930.990	2155.070	74.303	01.200	2052.980
Poland	342.347	-59.582	-40.201	-022.715	21(7,642)	200.948	449.647	957.009
United Arab Emirates	- 2139.978	-83.042	1/05.054	10028.740	5167.042	130.315	955.094	1058.725
Andorra	- 5004.476	284.458	487.982	-1/31.149	5158.388	72.971	85.280	1/33.123
	/64.962	20.448	-32.890	-1129.399	2///.300	555.804	384.417	4139.275
Qatar	-1659.701	-72.452	-/90.664	30579.485	1667.079	/3.851	938.152	243.475
Slovakia	-588.899	-86.049	-228.986	-1313.085	1/20.463	338.293	869.269	2305.621
Brunei Darussalam	- 2124.429	-12.319	-162.891	22283.898	21/3./94	21.630	230.399	1253.569
Saudi Arabia	-1305.541	-52.117	-123.290	-151.915	1568.564	69.109	348.386	224.575
Latvia	83.223	885.772	62.853	-933.507	2696.387	420.829	210.936	1665.667
Portugal	-778.550	15.137	-123.288	-942.421	2215.464	258.426	389.323	1750.525
Bahrain	-696.829	-/1.437	3/36.152	5587.073	1307.708	86.718	669.102	3309.940
Chile	1128.682	396.134	4021.743	-997.207	700.716	46.397	83.068	1066.916
Hungary	698.345	-127.432	-285.869	-1193.525	1287.875	281.542	608.847	1/91.730
Croatia	-591.164	219.412	-25.025	-881.464	1765.141	161.315	359.895	1816.481
Argentina	1679.963	5.870	29.410	-190.609	139.460	27.611	67.663	274.403
Oman	-929.574	-43.176	64.711	8762.848	1433.216	46.468	728.964	622.088
Russian Federation	-90.453	84.204	232.606	2309.553	354.122	30.723	51.523	26.877
Montenegro	-2078.316	88.352	518.852	-921.073	2287.823	42.087	95.303	1122.452
Bulgaria	346.046	-21.556	342.647	-555.517	1021.691	132.878	986.588	1413.049
Romania	-79.707	-67.508	-65.441	-324.769	866.372	171.091	228.289	616.081
Belarus	117.723	7.951	-140.105	-570.187	828.290	122.355	211.192	1914.056
Bahamas	-1249.928	-132.722	-277.911	-1260.588	1513.780	156.836	341.432	1390.989
Uruguay	2221.439	552.981	-43.960	-508.785	643.741	76.304	58.455	599.667
Kuwait	-1974.321	-55.125	-292.730	21689.962	2155.356	70.433	330.075	74.049
Malaysia	548.630	-32.914	-58.529	510.639	1043.773	264.579	628.356	1662.111
Barbados	-1112.364	-106.819	-53.374	-1853.561	2259.619	115.163	76.791	2016.819
Kazakhstan	-106.713	-2.951	685.031	3127.942	366.565	19.511	117.461	200.740
Iran (Islamic								
Republic of)	-168.178	-37.875	82.794	2416.207	369,685	49.095	27.440	12.337

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Table 3: Continue								
Palau	-10.397	-0.480	-0.334	-9.244	10.784	0.483	0.396	9.252
Seychelles	35.659	-3.135	-1.191	-61.014	95.305	3.135	1.634	61.026
Costa Rica	898.577	17.429	-63.849	-600.155	881.337	69.007	118.879	602.046
Turkey	127.131	-173.269	-317.378	-426.703	414.056	195.906	528.331	556.093
Mauritius	-610.208	-161.905	-65.492	-1419.169	2104.870	209.192	80.156	1480.347
Panama	-487.888	-41.692	-30.323	-918.259	1397.953	116.083	186.894	1324.172
Serbia	-2.537	-1.323	32.601	-432.869	861.416	71.530	115.164	815.952
Albania	-394.956	-27.424	65.876	-185.045	564.625	35.080	18.176	199.557
Trinidad and Tobago	316.555	16.410	91.790	-278.977	1025.347	106.232	231.266	942.176
Antigua and Barbuda	-5.594	-1.312	3.749	-26.592	37.824	2.519	0.734	26.592
Georgia	-124.399	-14.607	127.820	-527.427	557.955	30.959	212.728	584.860
Saint Kitts and Nevis	-13.598	-1.103	-0.486	-0.412	14.854	1.114	0.486	0.415
Cuba	-156.231	-13.096	-17.248	-195.541	251.116	20.852	33.572	237.862
Mexico	123.627	-59.655	18.234	-250.930	458.728	79.262	187.503	661.453
Grenada	-16.575	-1.379	-2.152	-16.542	18.254	1.516	2.440	17.291
Sri Lanka	17.823	-12.922	-26.106	-348.651	328.513	48.017	36.427	382.847
Bosnia and	-896.061	160.024	116.557	-/46./95	1328.346	91.8//	237.621	1137.100
Herzegovina								
Venezuela (Bolivarian	401.020	01 101	76742	26 125	112 206	11.002	11.500	121 004
Republic of)	401.929	21.181	/6./43	-26.425	113.386	11.093	11.522	121.804
Brazil	654.449	74.088	197.513	-34.688	99.384	19.519	50.330	216.340
Azerbaijan	-198./95	-20.386	20.856	2725.770	331.284	29.563	16.785	/6./18
Lebanon	-1884.859	1/.16/	35.707	-245.153	2788.995	12.337	19.941	253.992
The former Yugoslav	-260.420	-39.445	-909.472	-698.312	872.134	/1.023	1223.164	/8/.835
Republic of Macedonia	10 (07	24 752	(10.077	125 (50	557 007	24 510	76 647	400 002
Armenia Thailand	-19.007	-24.752	018.277	-435.058	220.275	34.519	/0.04/	498.803
Inailand	467.949	144.955	-158.085	-629.327	380.875	102.146	248.732	822.553
Algeria	-745.215	-37.480	-51.356	2599.137	//3.818	39.597	56.180	161.625
China	-/2.1/4	-/3.951	-234.624	-293.776	159.211	85.822	267.973	338.815
Ecuador	909.839	105.315	-/.446	5/0.369	245.663	23.890	38.593	269.002
Ukraine	657.669	7.604	115.134	-535.236	203.285	28.039	62.798	567.423
Peru	314.678	-5.312	13/3.276	-79.379	271.084	28.816	26.394	319.575
Colombia	13.013	42.688	-10.901	6/9.553	222.417	16.767	29.735	142.290
Saint Lucia	-1244.//1	-/5.106	-1.94/	-962.219	1631.259	/6.926	39.368	1034.355
Fiji	/10.8/1	17.101	-1/./54	-12/9.1/1	1251.868	31.214	56.839	1280.534
Mongolia	-294.818	248.027	2016.346	1513.157	413.589	11.454	9.413	/35.054
Dominican Republic	29.993	-38.892	3.304	-602.750	552.344	56.726	36.103	610.054
Jordan	-1029.898	-//.956	111.360	-1397.564	1563.589	91.045	133.624	1401.282
Tunisia	-194.888	-/1.808	-123.983	-458.969	565.732	90.949	188.027	668.466
Jamaica	-398.262	-42.115	66.38/	-/98.92/	636.045	44.291	13.424	965.290
Tonga	-691.928	-57.559	-89.465	-695.137	//1.194	64.038	103.102	/30.492
Saint Vincent and	-1010.272	-99.807	-39.218	-531.586	1601.077	100.023	40.592	531.586
the Grenadines	1751 477	1552.006	70.010	50 (70	066 024	56.014	20.240	51.055
Suriname	1/51.4//	1555.880	72.219	59.079	900.934	56.914	29.349	51.855
Botswana	-539.773	-40.510	62.684	-/44.95/	640.286	45.204	84.298	/53.1/3
Maldives	-027.040	-209.196	-350.932	-1222.729	1845.724	209.306	3/3.202	1222.747
Dominica	-7.430	-0.019	-0.948	-/.02/	8.000	0.715	1.150	8.151
Samoa	-1904.970	-252.000	-40.920	-1105.696	2017.234	255.550	37.090	1302.524
Uzbekistan	-1.105	30.243	05.027	117.081	85.042	23.033	20.228	49.004
Menshell Islanda	1141.995	-44.520	-20.917	-404.932	1081.005	97.795	29.152	030.079
Marshall Islands	-1.349	-0.110	-0.054	-2.932	4.050	0.380	0.022	4.405
Libya	031.033	50.878	131.029	-10.434	382.945	48.400	77.955	352.178
Turkmenistan	244.954	19.733	50.078	-2.285	222.549	18.480	29.755	210.804
Gabon	931.527	/5.584	1/6.893	198.165	433.605	36.006	57.969	410.720
Paraguay	12/1.090	10.665	-8.896	226.329	305.425	25.486	27.548	406.234
Moldova (Republic of)	690.607	-42.483	-49.175	-430.125	533.280	54.833	/6.946	431.260
Philippines	-110.861	-1.023	/.851	-241.119	262.244	11.820	83.003	265.451
South Africa	97.141	47.691	912.008	-1/1.011	356.571	44.741	96.009	669.416
Egypt	-229.115	-42.0//	-52.146	-1/3.42/	380.391	58.962	87.182	351.064
Indonesia Viot Nor-	160.562	30.466	21.365	0/.9/5	143.976	59.412	54.267	213.833
viet inam	159.468	-61.270	-114.829	-108.86/	300.834	116.404	14/.302	194.559
Boiivia (Plurinational	105.259	-4.191	584.526	38/.9/6	149.085	10.699	12.348	190.005
State of)	210 717	59 205	40.754	1170 100	0202.244	246.020	507 540	0100.010
Palestine,	-319./1/	-58.325	-40./54	-11/0.126	2585.544	246.928	537.562	2190.018
State of	200 466	21.059	02 624	159 504	662 777	55 110	00 741	(20 744
iraq El Salvador	390.400	51.058	92.034	-158.524	624.062	55.119	88./41	028.744
EI Salvador Kurguzatan	-252.259	-60.35/	-30.463	-435.6/6	034.003	/0.555	54.183	500.514
куrgyzstan	-107.456	15./52	123.827	-190.372	244.006	8.287	16.094	246.648

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Table 3: Continue								
Morocco	19.110	-42.511	11.460	-589.650	431.657	63.522	133.987	609.587
Nicaragua	570.148	-13.095	-5.131	-210.940	346.945	27.346	18.838	215.319
Cabo Verde	-758.656	-43.851	-12.886	-303.082	906.180	43.851	12.887	303.310
Guyana	1697.754	102.544	617.418	-1101.339	766.308	25.841	45.387	1101.452
Guatemala	374.646	11.093	26.441	-357.224	392.087	40.512	31.056	412.986
Tajikistan	-58.777	-9.330	1.112	-153.558	160.467	15.699	16.306	172.380
Namibia	787.869	8.557	136.736	-708.583	971.619	46.861	958.856	733.011
India	15.227	-9.909	-29.468	-195.155	54.469	17.668	52.352	267.458
Micronesia	-2.474	-0.293	-0.151	-3.943	3.920	0.384	0.398	4.211
(Federated States of)								
Timor-Leste	-508.555	-14.202	-16.976	-366.286	557.578	33.395	17.188	367.825
Honduras	860.726	-11.083	42.454	-388,792	491.102	38.350	12.783	406.395
Bhutan	-72.720	-18.575	15.289	-367.572	398.259	38.963	40.470	427,826
Kiribati	-1.916	-0.203	-0.164	-2.450	2.356	0.231	0.239	2.531
Bangladesh	-0.444	-3.959	7.704	-99.380	111.677	10.926	11.348	119,968
Congo	-236.933	4.095	33.528	1290.774	245.259	11.287	7.461	34.274
Vanuatu	-347.978	-36.852	-29.804	-444.728	427,735	41.847	43.465	459,490
Lao People's	38.918	-4.693	20.775	-172.872	202.525	19.814	20.580	217.561
Democratic Republic						-,		
Ghana	266 696	20.607	21.688	459.729	189,180	9.583	14.883	24 697
Equatorial Guinea	1162 863	94 332	221 480	238 790	558 420	46 370	74 656	528 948
Kenya	-9.392	28,900	5.040	-134.592	157.868	11.081	10.520	137,547
Sao Tome	-425 833	-31 988	-14 891	-404 827	619 545	33 985	22 051	404 829
and Principe	120.000	51.900	11.071	101.027	019.515	55.765	22.031	1011.02)
Eswatini	361 238	208 488	-26 511	-460 122	840 600	64 823	36 787	518 506
(Kingdom of)	501.250	200.400	20.511	400.122	040.000	04.025	50.707	510.500
Zambia	21.986	4 234	773 768	-164 895	64 282	5 234	155 585	176 232
Cambodia	35 922	-6.457	23 383	-213 264	247 329	24 197	25 133	265 690
Angola	-196 742	-10.633	84 045	2527.940	247.522	12 466	8 542	127 381
Myanmar	66 397	11 659	18 776	7 326	116.034	3 983	4 354	146 120
Nepal	-106 735	-7 /93	-25 261	-102 914	110.034	9 284	26 084	102 915
Pakistan	-39 214	-77 787	-25.846	-102.714	95 611	30 553	20.004	194 094
Cameroon	-42 781	56 023	8 557	61.052	108 630	8 066	5 100	68 300
Solomon Islands	-82 581	1202 617	71 102	-295 795	518 542	15 802	7 326	295 795
Papua New	647 882	34 535	122 926	-32 659	171 597	16 788	17.437	184 336
Guinea	047.002	54.555	122.920	52.057	171.577	10.700	17.457	104.550
Tanzania (United	71 558	6 391	3 347	-69 372	37 835	3 977	3 169	72 462
Republic of)	/1.550	0.571	5.517	07.572	37.055	5.977	5.10)	72.102
Svrian Arah	-92 947	-11 831	-4 027	-168 999	170 720	16 702	17 348	183 394
Republic	72.747	11.051	4.027	100.)))	170.720	10.702	17.540	105.574
Zimbabwe	107 578	11.013	209 706	-167.456	85 183	2 528	4.066	172 /32
Nigeria	-73 933	-3 192	-16 316	574.086	86 359	4 782	17 356	148 121
Rwanda	-7 307	-1.085	3 155	-66 572	74 281	5 711	3 396	68 272
Lesotho	-369 705	18 678	-1 429	-398 146	411 634	51 043	5 576	399.070
Mauritania	494 081	-2 745	703 650	-97 218	243 911	3 471	4 603	323 241
Madagascar	44 474	-3 258	30.692	-37 786	56 710	8 336	7 039	41 498
Uganda	79.630	4 569	-5 324	-61 461	54 189	6 252	6 199	67 203
Benin	-295 325	211 570	-1 467	-124 428	428 882	6 539	4 081	135 632
Senegal	-114 216	-10 523	37 350	-237 768	397 540	25 897	26.619	363 693
Comoros	-149 939	-12 085	-3 139	-201 555	221 264	17.012	10 115	203 365
Togo	-32 399	28 393	30.020	-80.687	98 922	8 134	5 083	87 004
Sudan	-38 875	-6 447	1 281	-108 532	113 970	11 150	11 581	122 431
Afghanistan	-84 174	-6 646	-2 689	-97 239	106 412	8 182	4 865	97 803
Haiti	-70 881	-6.057	0.813	-134 684	148 687	11 432	6 797	136 659
Côte d'Ivoire	626 351	133 521	-6 029	-1 021	248 210	6 874	14 315	197.001
Malawi	64 352	-5 797	-2.801	-32 922	39 978	7 696	3 038	33.015
Diibouti	-191 287	-21 223	-14 474	-268 846	262 543	25.686	26 679	282 035
Ethiopia	-36 126	-2 967	-0.390	-54 841	60 337	4 639	2 758	55 456
Gambia	-98 261	-2 716	-112 470	-223 184	210 200	3 584	112 593	223 188
Guinea	254 287	16 382	32 769	-128 995	151 557	11 653	6.928	139 297
Congo (Democratic	75 459	4 954	9 1 1 0	-27 739	33 181	2 551	1 517	30 497
Republic of the)	10.107	1.254	2.110	21.137	22.101	2.551	1.217	50.477
Guinea-Bissau	95 790	6.077	12,970	-59 327	69 102	5,313	3 1 5 9	63 512
Yemen	-117 059	-13 165	-8 506	-169.006	165 708	16.212	16 839	178 010
Fritrea	-5 054	-0.662	1 595	-35 970	40 104	3 083	1 833	36 850
Mozambique	-23 113	-2 532	74 337	95 677	60.810	7,708	43 029	102 255
Liberia	-44 314	-4 102	2.611	-120 277	133 321	10.251	6 095	122.235
Mali	57.246	94.646	1.285	-148.989	105.181	2.864	3.124	149.260

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Table 3: Continue								
Burkina Faso	82.493	146.189	66.327	-132.835	67.641	2.397	2.479	138.339
Sierra Leone	17.594	27.585	17.458	-2.792	165.750	7.515	5.630	2.933
Burundi	-1.145	-2.807	1.734	-31.728	30.657	2.939	1.057	31.728
Chad	18.445	0.566	6.528	-80.560	90.664	6.971	4.145	83.329
South Sudan	-24.340	-2.545	3.383	-99.489	110.629	8.506	5.057	101.680
Central African	-41.190	7.276	1.802	-1.525	42.257	2.232	1.064	1.530
Republic								
Niger	11.069	0.425	3.347	-38.559	43.458	3.341	1.987	39.943

^a These constitute the input variables for the new proposed model.

^b All numbers are approximated to the nearest third decimal place.

 Table 4: United Nations (UN) human development index versus modified human development index using data envelopment analysis approach^a

			Modified human	Modified human
			development index	development index
	UN's human	UN's human	using data envelopment	using data envelopment
Country	development index	development index rank	analysis approach	analysis approach rank
Norway	0.953	1	0.928	14
Switzerland	0.944	2	0.936	12
Australia	0.939	3	1.028	2
Ireland	0.938	4	0.946	10
Germany	0.936	5	0.976	6
Iceland	0.935	6	0.947	9
Hong Kong, China (SAR)	0.933	7	0.877	24
Sweden	0.933	7	0.967	7
Singapore	0.932	9	0.874	26
Netherlands	0.931	10	0.987	5
Denmark	0.929	11	0.951	8
Canada	0.926	12	1.006	3
United States	0.924	13	0.941	11
United Kingdom	0.922	14	0.920	16
Finland	0.920	15	1.006	4
New Zealand	0.917	16	1.061	1
Belgium	0.916	17	0.910	19
Liechtenstein	0.916	17	0.877	25
Japan	0.909	19	0.911	18
Austria	0.908	20	0.870	27
Luxembourg	0.904	21	0.799	44
Israel	0.903	22	0.920	17
Korea (Republic of)	0.903	22	0.885	22
France	0.901	24	0.858	29
Slovenia	0.896	25	0.883	23
Spain	0.891	26	0.795	45
Czechia	0.888	27	0.909	20
Italy	0.880	28	0.782	48
Malta	0.878	29	0.826	36
Estonia	0.871	30	0.921	15
Greece	0.870	31	0.825	37
Cyprus	0.869	32	0.857	30
Poland	0.865	33	0.868	28
United Arab Emirates	0.863	34	0.753	56
Andorra	0.858	35	0.764	52
Lithuania	0.858	35	0.885	21
Qatar	0.856	37	0.710	79
Slovakia Brunei	0.855	38	0.851	32
Darussalam	0.853	39	0.682	88
Saudi Arabia	0.853	39	0.712	78
Latvia	0.847	41	0.933	13
Portugal	0.847	41	0.737	64
Bahrain	0.846	43	0.712	77
Chile	0.843	44	0.816	40

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Table 4: Continue					
Hungary	0.838	45	0.819	38	
Croatia	0.831	46	0.818	39	
Argentina	0.825	47	0.764	53	
Oman	0.821	48	0 701	81	
Russian	0.021	10	0.701	01	
Enderation	0.816	40	0.912	41	
receration	0.810	49	0.813	41	
Montenegro	0.814	50	0.802	43	
Bulgaria	0.813	51	0.812	42	
Romania	0.811	52	0.765	51	
Belarus	0.808	53	0.831	34	
Bahamas	0.807	54	0.749	59	
Uruguay	0.804	55	0.732	66	
Kuwait	0.803	56	0.567	119	
Malaysia	0.802	57	0.723	71	
Barbados	0.800	58	0.752	57	
Kazakhstan	0.800	58	0.752	16	
Kazakiistaii	0.800	50	0.785	40	
Iran (Islamic Republic of)	0.798	60	0.721	12	
Palau	0.798	60	0.837	33	
Seychelles	0.797	62	0.691	86	
Costa Rica	0.794	63	0.703	80	
Turkey	0.791	64	0.622	105	
Mauritius	0.790	65	0.679	90	
Panama	0.789	66	0.726	68	
Serbia	0.787	67	0.780	49	
Albania	0.785	68	0.743	62	
Tripidad and Tobago	0.784	60	0.725	60	
Antique and Darbuda	0.784	09 70	0.725	80	
Antigua and Barbuda	0.780	70 70	0.679	89	
Georgia	0.780	70	0.854	31	
Saint Kitts and Nevis	0.778	72	0.637	99	
Cuba	0.777	73	0.830	35	
Mexico	0.774	74	0.658	95	
Grenada	0.772	75	0.672	93	
Sri Lanka	0.770	76	0.761	54	
Bosnia and Herzegovina	0.768	77	0.724	70	
Venezuela (Bolivarian					
Republic of)	0 761	78	0.735	65	
Brozil	0.750	70	0.631	103	
Agenhaiian	0.757	80	0.051	74	
Azerbaijan	0.757	80	0.718	74	
Lebanon	0.757	80	0.661	94	
The former Yugoslav					
Republic of Macedonia	0.757	80	0.692	85	
Armenia	0.755	83	0.784	47	
Thailand	0.755	83	0.619	108	
Algeria	0.754	85	0.622	106	
China	0.752	86	0.611	113	
Ecuador	0.752	86	0.678	91	
Ukraine	0.751	88	0.776	50	
Deru	0.750	80	0.674	92	
Colombia	0.750	00	0.674	100	
	0.747	90	0.055	100	
	0.747	90	0.655	96	
Fiji	0.741	92	0.743	61	
Mongolia	0.741	92	0.718	75	
Dominican Republic	0.736	94	0.597	114	
Jordan	0.735	95	0.714	76	
Tunisia	0.735	95	0.588	115	
Jamaica	0.732	97	0.698	83	
Tonga	0.726	98	0.760	55	
Saint Vincent and	0.720	20	0.700	55	
the Granadinas	0 723	00	0.610	107	
uic Orenaumes	0.725	77 100	0.019	72	
Suimaine	0.720	100	0./19	15	

Table 4: Continue				
Botswana	0.717	101	0.615	110
Maldives	0.717	101	0.519	126
Dominica	0.715	103	0.612	112
Samoa	0.713	104	0.696	84
Uzbekistan	0.710	105	0.751	58
Belize	0.708	106	0.700	82
Marshall Islands	0.708	106	0.740	63
Libva	0.706	108	0.565	120
Turkmenistan	0.706	108	0.631	102
Gabon	0.702	110	0.569	118
Paraguay	0.702	110	0.614	111
Moldova (Republic of)	0.700	112	0.749	60
Philippines	0.699	113	0.633	101
South Africa	0.699	113	0.648	97
Egypt	0.696	115	0 545	123
Indonesia	0.694	116	0.572	117
Viet Nam	0.694	116	0.619	109
Bolivia (Plurinational State of)	0.693	118	0.629	104
Palestine State of	0.686	110	0.646	98
Iraq	0.685	120	0.502	131
Fl Salvador	0.674	120	0.536	125
Kyrayzetan	0.672	121	0.729	67
Moroaco	0.072	122	0.729	126
Nicoragua	0.007	123	0.462	130
Caba Varda	0.038	124	0.337	124
Cabo verue	0.034	125	0.495	155
Guyana	0.034	125	0.572	110
	0.650	127	0.502	150
I ajikistan	0.050	127	0.084	8/
Inamibia	0.647	129	0.482	134
India Miserrania (Escherated States of)	0.640	130	0.483	135
Micronesia (Federated States of)	0.627	131	0.557	121
Timor-Leste	0.625	132	0.395	152
Honduras	0.617	133	0.500	132
Bhutan	0.612	134	0.331	16/
Kırıbatı	0.612	134	0.547	122
Bangladesh	0.608	136	0.467	140
Congo	0.606	137	0.446	142
Vanuatu	0.603	138	0.506	129
Lao People's	0.404	1.00	0.404	
Democratic Republic	0.601	139	0.401	151
Ghana	0.592	140	0.476	138
Equatorial Guinea	0.591	141	0.354	163
Kenya	0.590	142	0.477	137
Sao Tome and Principe	0.589	143	0.466	141
Eswatini (Kingdom of)	0.588	144	0.429	144
Zambia	0.588	144	0.474	139
Cambodia	0.582	146	0.403	150
Angola	0.581	147	0.371	156
Myanmar	0.578	148	0.375	154
Nepal	0.574	149	0.417	146
Pakistan	0.562	150	0.370	157
Cameroon	0.556	151	0.417	147
Solomon Islands	0.546	152	0.515	127
Papua New Guinea	0.544	153	0.355	162
Tanzania (United Republic of)	0.538	154	0.404	149
Syrian Arab Republic	0.536	155	0.393	153
Zimbabwe	0.535	156	0.507	128
Nigeria	0.532	157	0.357	161
Rwanda	0.524	158	0.353	164
Lesotho	0.520	159	0.372	155

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Table 4: Continue				
Mauritania	0.520	159	0.320	168
Madagascar	0.519	161	0.438	143
Uganda	0.516	162	0.409	148
Benin	0.515	163	0.318	169
Senegal	0.505	164	0.279	172
Comoros	0.503	165	0.364	158
Togo	0.503	165	0.358	160
Sudan	0.502	167	0.273	173
Afghanistan	0.498	168	0.305	171
Haiti	0.498	168	0.363	159
Côte d'Ivoire	0.492	170	0.312	170
Malawi	0.477	171	0.344	165
Djibouti	0.476	172	0.267	175
Ethiopia	0.463	173	0.243	180
Gambia	0.460	174	0.265	176
Guinea	0.459	175	0.214	185
Congo (Democratic				
Republic of the)	0.457	176	0.422	145
Guinea-Bissau	0.455	177	0.231	182
Yemen	0.452	178	0.258	178
Eritrea	0.440	179	0.272	174
Mozambique	0.437	180	0.253	179
Liberia	0.435	181	0.339	166
Mali	0.427	182	0.180	186
Burkina Faso	0.423	183	0.163	187
Sierra Leone	0.419	184	0.216	184
Burundi	0.417	185	0.241	181
Chad	0.404	186	0.143	188
South Sudan	0.388	187	0.260	177
Central African Republic	0.367	188	0.229	183
Niger	0.354	189	0.142	189

a. All numbers are approximated to the nearest third decimal place