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Geometry and Direct Kinematics to MP3R with 4×4 Operators

[1]Relly Victoria Petrescu, [2]Raffaella Aversa, [3]Bilal Akash, [2]Antonio Apicella and [1]Florian Ion Tiberiu Petrescu

[1]*ARoTMM-IFToMM, Bucharest Polytechnic University, Bucharest, (CE), Romania*
[2]*Advanced Material Lab, Department of Architecture and Industrial Design, Second University of Naples, 81031 Aversa (CE), Italy*
[3]*Dean of School of Graduate Studies and Research, American University of Ras Al Khaimah, UAE*

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Corresponding Author:
Florian Ion Tiberiu Petrescu
ARoTMM-IFToMM, Bucharest Polytechnic University, Bucharest, (CE), Romania
E-mail: scipub02@gmail.com

Abstract: Anthropomorphic mechatronic systems are the most widely used robotics systems worldwide today in industry and in all automated environments. These systems are best suited to the modern automation and mechatronisation needs of the modern world, being mobile, dynamic, light, robust, complex, technologically simple, easy to design and manufactured, implemented, maintained and used in almost any industrial site, both in machine building and in special environments, such as chemical, toxic, dyeing, underwater, nuclear, in space.... Anthropomorphic robots are flexible, dynamic, stable, lightweight, fast, fast, inexpensive, easy-to-install, mechanical, mechanical, mechanical and mechanical systems with a pleasant appearance, modern industrial design and easy to design and implement in any workplace, imposed. In this study we will present the 4×4 operators and the way they can be implemented and used in the complex matrix calculations in order to simplify the complex matrix algebraic numerical methods. 4×4 operators are designed to simplify algebraic matrix calculations by making difficult matrix operations simpler and easier to approach. The method of using 4×4 operators is meant to introduce a degree in addition to the matrices to facilitate algebraic operations. The kinematics of serial manipulators and robots will be exemplified for the 3R kinematic model. The fixed coordinate system was denoted by $x_0O_0y_0z_0$. The mobile systems (rigid) of the three mobile elements (1, 2, 3) have indices 1, 2 and 3. Their orientation has been chosen conveniently. Known kinematic parameters in the direct kinematics are the absolute rotation angles of the three moving elements: φ_{10}, φ_{20}, φ_{30}, angles related to the rotation of the three actuators (electric motors) mounted in the kinematic rotation couplers. The output parameters are the three absolute coordinates x_M, y_M, z_M of point M, i.e., the kinematic parameters (coordinates) of the endeffector (the actuator element (the final), which can be a grasping hand, a solder tip, painted , cut, etc ...). The 3×3 matrix is transformed into 4×4 (it is a mathematical operator) by adding two zero vectors (formed from three elements 0), one line and the other column, and adding one element 1 to the main diagonal (the last element). The matched T_{01} matrix becomes T_{01}^4. The column vector matrix (consisting of three elements) undergoes a minimal transformation receiving a fourth fixed value 1 if it is only used for matrix products. The convenient form of matrix A_{12} is A_{12}^c.

Keywords: Anthropomorphic Mechatronic Systems, Robots, Geometry, Kinematics, 4×4 Operators

Introduction

Anthropomorphic mechatronic systems are the most widely used robotics systems worldwide today in industry and in all automated environments. These systems are best suited to the modern automation and mechatronisation needs of the modern world, being mobile, dynamic, light, robust, complex, technologically simple, easy to design and manufactured, implemented, maintained and used in almost any industrial site, both in machine building and in special environments, such as chemical, toxic, dyeing, underwater, nuclear, in space.... Anthropomorphic robots are flexible, dynamic, stable, lightweight, fast, fast, inexpensive, easy-to-install, mechanical, mechanical, mechanical and mechanical systems with a pleasant appearance, modern industrial design and easy to design and implement in any workplace, imposed. In this study we will present the 4×4 operators and the way they can be implemented and used in the complex matrix calculations in order to simplify the complex matrix algebraic numerical methods. 4×4 operators are designed to simplify algebraic matrix calculations by making difficult matrix operations simpler and easier to approach. The method of using 4×4 operators is meant to introduce a degree in addition to the matrices to facilitate algebraic operations. (Antonescu and Petrescu, 1985; 1989; Antonescu *et al*., 1985a; 1985b; 1986; 1987; 1988; 1994; 1997; 2000a; 2000b; 2001; Aversa *et al*., 2017a; 2017b; 2017c; 2017d; 2017e; 2016a; 2016b; 2016c; 2016d; 2016e; 2016f; 2016g; 2016h; 2016i; 2016j; 2016k; 2016l; 2016m; 2016n; 2016o; Berto *et al*., 2016a; 2016b; 2016c; 2016d; Cao *et al*., 2013; Dong *et al*., 2013; Comanescu, 2010; Franklin, 1930; He *et al*., 2013; Lee, 2013; Lin *et al*., 2013; Liu *et al*., 2013; Mirsayar *et al*., 2017; Padula and Perdereau, 2013; Perumaal and Jawahar, 2013; Petrescu, 2011; 2015a; 2015b; Petrescu and Petrescu, 1995a; 1995b; 1997a; 1997b; 1997c; 2000a; 2000b; 2002a; 2002b; 2003; 2005a; 2005b; 2005c; 2005d; 2005e; 2011; 2012a; 2012b; 2013a; 2013b; 2016a; 2016; 2016c; Petrescu *et al*., 2009; 2016; 2017a; 2017b; 2017c; 2017d; 2017e; 2017f; 2017g; 2017h; 2017i; 2017j; 2017k; 2017l).

Materials and Methods

The kinematics of serial manipulators and robots will be exemplified for the 3R kinematic model (Fig. 1).

The fixed coordinate system was denoted by $x_0O_0y_0z_0$. The mobile systems (rigid) of the three mobile elements (1, 2, 3) have indices 1, 2 and 3. Their orientation has been chosen conveniently. Known kinematic parameters in the direct kinematics are the absolute rotation angles of the three moving elements: φ_{10}, φ_{20}, φ_{30}, angles related to the rotation of the three actuators (electric motors) mounted in the kinematic rotation couplers. The output parameters are the three absolute coordinates x_M, y_M, z_M of point M, ie the kinematic parameters (coordinates) of the endeffector (the actuator element (the final), which can be a grasping hand, a solder tip, painted , cut, etc ...).

The 3×3 matrix is transformed into 4×4 (it is a mathematical operator) by adding two zero vectors (formed from three elements 0), one line and the other column, and adding one element 1 to the main diagonal (the last element). The matched T_{01} matrix becomes T_{01}^4 (relation 1):

$$T_{01}=\begin{bmatrix}\alpha_x & \beta_x & \gamma_x\\ \alpha_y & \beta_y & \gamma_y\\ \alpha_z & \beta_z & \gamma_z\end{bmatrix}=\begin{bmatrix}\cos\phi_{10} & -\sin\phi_{10} & 0\\ \sin\phi_{10} & \cos\phi_{10} & 0\\ 0 & 0 & 0\end{bmatrix}\Rightarrow$$
$$\Rightarrow T_{01}^4=\begin{bmatrix}\alpha_x & \beta_x & \gamma_x & 0\\ \alpha_y & \beta_y & \gamma_y & 0\\ \alpha_z & \beta_z & \gamma_z & 0\\ 0 & 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}\cos\phi_{10} & -\sin\phi_{10} & 0 & 0\\ \sin\phi_{10} & \cos\phi_{10} & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix} \tag{1}$$

The column vector matrix (consisting of three elements) undergoes a minimal transformation receiving a fourth fixed value 1 if it is only used for matrix products.

The convenient form of matrix A_{12} is A_{12}^c (2):

$$A_{12}=\begin{bmatrix}d_1\\ a_2\\ 0\end{bmatrix}\Rightarrow A_{12}^c=\begin{bmatrix}d_1\\ a_2\\ 0\\ 1\end{bmatrix} \tag{2}$$

The resulting product is also a 4×1 column vector (3):

$$T_{01}^4\cdot A_{12}^c=\begin{bmatrix}\cos\phi_{10} & -\sin\phi_{10} & 0 & 0\\ \sin\phi_{10} & \cos\phi_{10} & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\cdot\begin{bmatrix}d_1\\ a_2\\ 0\\ 1\end{bmatrix}=$$
$$=\begin{bmatrix}d_1\cdot\cos\phi_{10} \quad -a_2\cdot\sin\phi_{10}\\ d_1\cdot\sin\phi_{10} \quad +a_2\cdot\cos\phi_{10}\\ 0\\ 1\end{bmatrix} \tag{3}$$

When the vector matrices multiply, it is sufficient to transform them into the vector matrix 4×1 column. However, if a vector matrix has to be assembled to convert the sum (from the space of 3 dimensions 3×3 or 3×1) into a multiplication operation (produced in space with 4 dimensions 4×4 or 4×1) of matrices, no 4×1 forms are allowed only 4×4 (4-7):

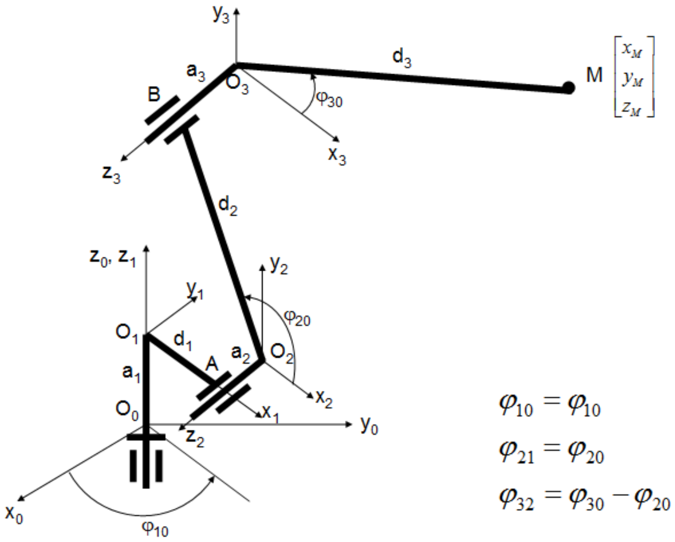

Fig. 1: The geometry and cinematic of a MP3R

$$T_{01}^4 \cdot T_{12}^4 = \begin{bmatrix} \cos\phi_{10} & -\sin\phi_{10} & 0 & 0 \\ \sin\phi_{10} & \cos\phi_{10} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\phi_{10} & 0 & \sin\phi_{10} & 0 \\ \sin\phi_{10} & 0 & -\cos\phi_{10} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

$$T_{01}^4 \cdot T_{12}^4 \cdot A_{23}^c = \begin{bmatrix} \cos\phi_{10} & 0 & \sin\phi_{10} & 0 \\ \sin\phi_{10} & 0 & -\cos\phi_{10} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} d_2 \cdot \cos\phi_{20} \\ d_2 \cdot \sin\phi_{20} \\ -a_3 \\ 1 \end{bmatrix} = \begin{bmatrix} d_2 \cdot \cos\phi_{10} \cdot \cos\phi_{20} - a_3 \cdot \sin\phi_{10} \\ d_2 \cdot \sin\phi_{10} \cdot \sin\phi_{20} + a_3 \cdot \cos\phi_{10} \\ d_2 \cdot \sin\phi_{20} \\ 1 \end{bmatrix} \tag{5}$$

$$T_{01}^4 \cdot T_{12}^4 \cdot T_{23}^4 = \begin{bmatrix} \cos\phi_{10} & 0 & \sin\phi_{10} & 0 \\ \sin\phi_{10} & 0 & -\cos\phi_{10} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{6}$$

$$T_{01}^4 \cdot T_{12}^4 \cdot T_{23}^4 = \begin{bmatrix} \cos\phi_{10} & 0 & \sin\phi_{10} & 0 \\ \sin\phi_{10} & 0 & -\cos\phi_{10} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{6'}$$

$$T_{01}^4 \cdot T_{12}^4 \cdot T_{23}^4 \cdot X_{3M}^c = \begin{bmatrix} \cos\phi_{10} & 0 & \sin\phi_{10} & 0 \\ \sin\phi_{10} & 0 & -\cos\phi_{10} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} d_3 \cdot \cos\phi_{30} \\ d_3 \cdot \sin\phi_{30} \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} d_3 \cdot \cos\phi_{10} \cdot \cos\phi_{30} \\ d_3 \cdot \sin\phi_{10} \cdot \cos\phi_{30} \\ d_3 \cdot \sin\phi_{30} \\ 1 \end{bmatrix} \tag{7}$$

We have prepared the arrays necessary for the summation, now we can go directly to their assembly in the form of column vectors (3×1 or 4×1). In this way the direct result is obtained, and the operators we have complicated do not use anymore (apparently). We will still use the form with operators first to see how they work, and then we will resume the algorithm intelligently to understand the role of the operators. To make the addition in multiplication (matrix product) by operators, we must have a 4×4 matrix. In this case, whether we have a product or a sum, we perform the product of the 4×4 operator matrices (so using matrixes expanded to 4×4 we perform only matrix product regardless of whether it is a sum in 3×3 or a multiplication). A 4×1 vector array is operationally written 4×4 by completing the 3×3 unit matrix below it with a 1×3 vector zero line (0, 0, 0) and right with the original vector 4×1.

When making the actual amount, things get complicated and at first sight this complication seems useless, but its role is essential (as we will see later) to work directly with transfer matrices. This is the real role of operators.

The assembly to be performed is (between the operator arrays the sign is + instead of +):

$$A_{01}^4 + (T_{01}^4 \cdot A_{12}^c)^4 + (T_{01}^4 \cdot T_{12}^4 \cdot A_{23}^c)^4 + (T_{01}^4 \cdot T_{12}^4 \cdot T_{23}^4 \cdot X_{3M}^c)^4$$

(see relationship (8):

$$A_{01}^{4}+(T_{01}^{4}\cdot A_{12}^{c})^{4}+(T_{01}^{4}\cdot T_{12}^{4}\cdot A_{23}^{c})^{4}+(T_{01}^{4}\cdot T_{12}^{4}\cdot T_{23}^{4}\cdot X_{3M}^{c})^{4}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&a_1\\0&0&0&1\end{bmatrix}\cdot\begin{bmatrix}1&0&0&(d_1\cdot\cos\phi_{10}-a_2\cdot\sin\phi_{10})\\0&1&0&(d_1\cdot\sin\phi_{10}+a_2\cdot\cos\phi_{10})\\0&0&1&0\\0&0&0&1\end{bmatrix}\cdot$$
$$\cdot\begin{bmatrix}1&0&0&(d_2\cdot\cos\phi_{10}\cdot\cos\phi_{20}-a_3\cdot\sin\phi_{10})\\0&1&0&(d_2\cdot\sin\phi_{10}\cdot\cos\phi_{20}+a_3\cdot\cos\phi_{10})\\0&0&1&d_2\cdot\sin\phi_{20}\\0&0&0&1\end{bmatrix}\cdot \tag{8}$$
$$\cdot\begin{bmatrix}1&0&0&(d_3\cdot\cos\phi_{10}\cdot\cos\phi_{30})\\0&1&0&(d_3\cdot\sin\phi_{10}\cdot\cos\phi_{30})\\0&0&1&d_3\cdot\sin\phi_{30}\\0&0&0&1\end{bmatrix}=\begin{bmatrix}1&0&0&(d_1\cdot\cos\phi_{10}-a_2\cdot\sin\phi_{10})\\0&1&0&(d_1\cdot\sin\phi_{10}+a_2\cdot\cos\phi_{10})\\0&0&1&a_1\\0&0&0&1\end{bmatrix}\cdot$$
$$\begin{bmatrix}1&0&0&(d_2\cdot\cos\phi_{10}\cdot\cos\phi_{20}-a_3\cdot\sin\phi_{10})\\0&1&0&(d_2\cdot\sin\phi_{10}\cdot\cos\phi_{20}+a_3\cdot\cos\phi_{10})\\0&0&1&d_2\cdot\sin\phi_{20}\\0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&(d_3\cdot\cos\phi_{10}\cdot\cos\phi_{30})\\0&1&0&(d_3\cdot\sin\phi_{10}\cdot\cos\phi_{30})\\0&0&1&d_3\cdot\sin\phi_{30}\\0&0&0&1\end{bmatrix}$$

The relationship (8) continues with (8 '):

$$\begin{bmatrix}1&0&0&(d_1\cdot\cos\phi_{10}-a_2\cdot\sin\phi_{10}+d_2\cdot\cos\phi_{10}\cdot\cos\phi_{20}-a_3\cdot\sin\phi_{10})\\0&1&0&(d_1\cdot\sin\phi_{10}+a_2\cdot\cos\phi_{10}+d_2\cdot\sin\phi_{10}\cdot\cos\phi_{20}+a_3\cdot\cos\phi_{10})\\0&0&1&(a_1+d_2\cdot\sin\phi_{20})\\0&0&0&1\end{bmatrix}\cdot$$
$$\cdot\begin{bmatrix}1&0&0&(d_3\cdot\cos\phi_{10}\cdot\cos\phi_{30})\\0&1&0&(d_3\cdot\sin\phi_{10}\cdot\cos\phi_{30})\\0&0&1&d_3\cdot\sin\phi_{30}\\0&0&0&1\end{bmatrix}= \tag{8'}$$
$$\begin{bmatrix}1&0&0&(d_1\cos\phi_{10}-a_2\sin\phi_{10}+d_2\cos\phi_{10}\cos\phi_{20}-a_3\sin\phi_{10}+d_3\cos\phi_{10}\cos\phi_{30})\\0&1&0&(d_1\sin\phi_{10}+a_2\cos\phi_{10}+d_2\sin\phi_{10}\cos\phi_{20}+a_3\cos\phi_{10}+d_3\sin\phi_{10}\cos\phi_{30})\\0&0&1&(a_1+d_2\cdot\sin\phi_{20}+d_3\cdot\sin\phi_{30})\\0&0&0&1\end{bmatrix}$$

Next step will be to step-by-step determination of the transfer matrix from left to right, which was not possible in the 3×3 system. The relationship (9) is written in the form (9'); we see how the sum turns into a product due to 4×4 operators, which allows us to carry out the operation between matrices from left to right because we do not add or multiply (9-12).

$$X_{0M}=A_{01}+T_{01}\cdot X_{1M} \tag{9}$$

$$X_{0M}^{4}=A_{01}^{4}\cdot T_{01}^{4}\cdot X_{1M}^{4}=D_{01}\cdot X_{0M}^{4} \tag{9'}$$

$$D_{01}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&a_1\\0&0&0&1\end{bmatrix}\cdot\begin{bmatrix}\cos\phi_{10}&-\sin\phi_{10}&0&0\\\sin\phi_{10}&\cos\phi_{10}&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}=\begin{bmatrix}\cos\phi_{10}&-\sin\phi_{10}&0&0\\\sin\phi_{10}&\cos\phi_{10}&0&0\\0&0&1&a_1\\0&0&0&1\end{bmatrix} \tag{10}$$

$$\begin{cases}X_{1M}=A_{12}+T_{12}\cdot X_{2M}\\X_{1M}^{4}=A_{12}^{4}\cdot T_{12}^{4}\cdot X_{2M}^{4}=D_{12}\cdot X_{2M}^{4}\Rightarrow\\\Rightarrow D_{12}=A_{12}^{4}\cdot T_{12}^{4}\end{cases} \tag{11}$$

$$D_{12}=\begin{bmatrix}1&0&0&d_1\\0&1&0&a_2\\0&0&1&0\\0&0&0&1\end{bmatrix}\cdot\begin{bmatrix}1&0&0&0\\0&0&-1&0\\0&1&0&0\\0&0&0&1\end{bmatrix}=\begin{bmatrix}1&0&0&d_1\\0&0&-1&a_2\\0&1&0&0\\0&0&0&1\end{bmatrix} \tag{12}$$

It found the crossings from 0 to 1 and from 1 to 2; at this point we do not go any further until we set the transition from 0 to 2 (13-14).

$$X_{0M}^4 = D_{01} \cdot X_{1M}^4 = D_{01} \cdot D_{12} \cdot X_{2M}^4 = D_{02} \cdot X_{2M}^4 \\ \Rightarrow D_{02} = D_{01} \cdot D_{12} \tag{13}$$

$$D_{02} = \begin{bmatrix} \cos\phi_{10} & -\sin\phi_{10} & 0 & 0 \\ \sin\phi_{10} & \cos\phi_{10} & 0 & 0 \\ 0 & 0 & 1 & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & d_1 \\ 0 & 0 & -1 & a_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \\ = \begin{bmatrix} \cos\phi_{10} & 0 & \sin\phi_{10} & (d_1\cos\phi_{10} - a_2\sin\phi_{10}) \\ \sin\phi_{10} & 0 & -\cos\phi_{10} & (d_1\sin\phi_{10} + a_2\cos\phi_{10}) \\ 0 & 1 & 0 & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{14}$$

Now you can go further on the chain to determine D23 (15-16):

$$\begin{cases} X_{2M} = A_{23} + T_{23} \cdot X_{3M} \; trece\, în \\ X_{2M}^4 = A_{23}^4 \cdot T_{23}^4 \cdot X_{3M}^4 = D_{23} \cdot X_{3M}^4 \Rightarrow D_{23} = A_{23}^4 \cdot T_{23}^4 \end{cases} \tag{15}$$

$$D_{23} = \begin{bmatrix} 1 & 0 & 0 & d_2\cos\phi_{20} \\ 0 & 1 & 0 & d_2\sin\phi_{20} \\ 0 & 0 & 1 & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \\ = \begin{bmatrix} 1 & 0 & 0 & d_2\cos\phi_{20} \\ 0 & 1 & 0 & d_2\sin\phi_{20} \\ 0 & 0 & 1 & -a_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{16}$$

The transfer input matrix D03 can now easily be found (17-18):

$$X_{0M}^4 = D_{02} \cdot X_{2M}^4 = D_{02} \cdot D_{23} \cdot X_{3M}^4 = D_{03} \cdot X_{3M}^4 \Rightarrow \\ \Rightarrow D_{03} = D_{02} \cdot D_{23} \tag{17}$$

$$D_{03} = \begin{bmatrix} \cos\phi_{10} & 0 & \sin\phi_{10} & (d_1\cos\phi_{10} - a_2\sin\phi_{10}) \\ \sin\phi_{10} & 0 & -\cos\phi_{10} & (d_1\sin\phi_{10} + a_2\cos\phi_{10}) \\ 0 & 1 & 0 & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & d_2\cos\phi_{20} \\ 0 & 1 & 0 & d_2\sin\phi_{20} \\ 0 & 0 & 1 & -a_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ = \begin{bmatrix} \cos\phi_{10} & 0 & \sin\phi_{10} & (d_2\cos\phi_{10}\cos\phi_{20} - a_3\sin\phi_{10} + d_1\cos\phi_{10} - a_2\sin\phi_{10}) \\ \sin\phi_{10} & 0 & -\cos\phi_{10} & (d_2\sin\phi_{10}\cos\phi_{20} + a_3\cos\phi_{10} + d_1\sin\phi_{10} + a_2\cos\phi_{10}) \\ 0 & 1 & 0 & (d_2\sin\phi_{20} + a_1) \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{18}$$

The formula (17) can be simplified by reducing the X (4×4) matrices to the 4×1 column vector, because practically there is only a multiplication operation between the DX transfer matrix 4×4 and the vector X3M; as an observation (both forms may be used, but Xx vector type 4×1 is not required to matrix form 4×4, it is preferable to working with the simpler form), (19-20):

$$X_{0M}^4 = D_{03} \cdot X_{3M}^4 \Rightarrow X_{0M}^c = D_{03} \cdot X_{3M}^c \tag{19}$$

$$X_{0M}^c = D_{03} \cdot X_{3M}^c \Rightarrow \begin{bmatrix} x_{0M} \\ y_{0M} \\ z_{0M} \\ 1 \end{bmatrix} = D_{03} \cdot \begin{bmatrix} x_{3M} \\ y_{3M} \\ z_{3M} \\ 1 \end{bmatrix} = D_{03} \cdot \begin{bmatrix} d_3 \cdot \cos\phi_{30} \\ d_3 \cdot \sin\phi_{30} \\ 0 \\ 1 \end{bmatrix} = \\ \begin{bmatrix} d_3\cos\phi_{10}\cos\phi_{30} + d_2\cos\phi_{10}\cos\phi_{20} - a_3\sin\phi_{10} + d_1\cos\phi_{10} - a_2\sin\phi_{10} \\ d_3\sin\phi_{10}\cos\phi_{30} + d_2\sin\phi_{10}\cos\phi_{20} + a_3\cos\phi_{10} + d_1\sin\phi_{10} + a_2\cos\phi_{10} \\ d_3\sin\phi_{30} + d_2\sin\phi_{20} + a_1 \\ 1 \end{bmatrix} \tag{20}$$

From the type 4×1 vector, we get the 3×1 vector, which we really care about, eliminating the final line, ie the element 1 (21).

$$X_{0M} = \begin{bmatrix} d_3\cos\phi_{10} \cdot \cos\phi_{30} + d_2\cos\phi_{10} \cdot \cos\phi_{20} - \\ a_3\sin\phi_{10} + d_1\cos\phi_{10} - a_2\sin\phi_{10} \\ d_3\sin\phi_{10} \cdot \cos\phi_{30} + d_2\sin\phi_{10} \cdot \cos\phi_{20} + \\ a_3\cos\phi_{10} + d_1\sin\phi_{10} + a_2\cos\phi_{10} \\ d_3\sin\phi_{30} + d_2\sin\phi_{20} + a_1 \\ \end{bmatrix} \tag{21}$$

Results

We can now write the coordinates of the M point taken separately, as functions of the independent rotation angles of the three movable elements (22).

$$\begin{cases} x_M = d_3\cos\phi_{10} \cdot \cos\phi_{30} + d_2\cos\phi_{10} \cdot \cos\phi_{20} - \\ -a_3\sin\phi_{10} + d_1\cos\phi_{10} - a_2\sin\phi_{10} \\ y_M = d_3\sin\phi_{10} \cdot \cos\phi_{30} + d_2\sin\phi_{10} \cdot \cos\phi_{20} + \\ +a_3\cos\phi_{10} + d_1\sin\phi_{10} + a_2\cos\phi_{10} \\ z_M = d_3\sin\phi_{30} + d_2\sin\phi_{20} + a_1 \end{cases} \tag{22}$$

Discussion

We have prepared the arrays necessary for the summation, now we can go directly to their assembly in the form of column vectors (3×1 or 4×1). In this way the direct result is obtained, and the operators we have complicated do not use anymore (apparently). We will still use the form with operators first to see how they work, and then we will resume the algorithm intelligently to understand the role of the operators. To make the addition in multiplication (matrix product) by operators, we must have a 4×4 matrix. In this case, whether we have a product or a sum, we perform the product of the 4×4 operator matrices (so using matrixes expanded to 4×4 we perform only matrix product regardless of whether it is a sum in 3×3 or a multiplication). A 4×1 vector array is operationally written 4×4 by completing the 3×3 unit matrix below it with a 1×3 vector zero line (0, 0, 0) and right with the original vector 4×1.

When making the actual amount, things get complicated and at first sight this complication seems useless, but its role is essential (as we will see later) to work directly with transfer matrices. This is the real role of operators.

Conclusion

In this study we will present the 4×4 operators and the way they can be implemented and used in the complex matrix calculations in order to simplify the complex matrix algebraic numerical methods. 4×4 operators are designed to simplify algebraic matrix calculations by making difficult matrix operations simpler and easier to approach. The method of using 4×4 operators is meant to introduce a degree in addition to the matrices to facilitate algebraic operations. The kinematics of serial manipulators and robots will be exemplified for the 3R kinematic model. The fixed coordinate system was denoted by $x_0O_0y_0z_0$. The mobile systems (rigid) of the three mobile elements (1, 2, 3) have indices 1, 2 and 3. Their orientation has been chosen conveniently. Known kinematic parameters in the direct kinematics are the absolute rotation angles of the three moving elements: φ_{10}, φ_{20}, φ_{30}, angles related to the rotation of the three actuators (electric motors) mounted in the kinematic rotation couplers. The output parameters are the three absolute coordinates x_M, y_M, z_M of point M, ie the kinematic parameters (coordinates) of the endeffector (the actuator element (the final), which can be a grasping hand, a solder tip, painted , cut, etc ...). The 3×3 matrix is transformed into 4×4 (it is a mathematical operator) by adding two zero vectors (formed from three elements 0), one line and the other column, and adding one element 1 to the main diagonal (the last element). The matched T_{01} matrix becomes T_{01}^4. The column vector matrix (consisting of three elements) undergoes a minimal transformation receiving a fourth fixed value 1 if it is only used for matrix products. The convenient form of matrix A_{12} is A_{12}^c.

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Author's Contributions

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Ethics

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