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# Nearly Partial Derivations on Banach Ternary Algebras

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**Abstract: Problem statement:** In this study, we introduce the concept of a partial ternary derivation from $A_1 \times\cdots\times A_n$ into B, where $A_1, A_2\cdots, A_n$ and B are ternary algebras. **Conclusion/Recommendations:** We prove the generalized Hyers-Ulam stability of partial ternary derivations in Banach ternary algebras.

**Key words:** Banach ternary algebra, ternary derivation, generalized Hyers-Ulam stability, natural generalization, scalar field, stability of homomorphisms, stability of derivations, approximate derivations

## INTRODUCTION

A ternary (associative) algebra (A, []) is a linear space A over a scalar field F = R or C equipped with a linear mapping, the so-called ternary product, [ ]: $A\times A\times A \to A$ such that [[abc] de] = [a [bcd]e] = [ab [cde]] for all a, b, c, d, e $\in$ A. This notion is a natural generalization of the binary case. Indeed if (AΘ,) is a usual (binary) algebra then [abc]:= (aΘb) Θc induced a ternary product making A in to a ternary algebra which will be called trivial. By a Banach ternary algebra we mean a ternary algebra equipped with a complete norm ||.|| such that ||[abc]|| _ kakkbkkck for all a, b, c $\in$ A.

Ulam (1960) gave a talk before the Mathematics Club of the University of Wisconsin in which he discussed a number of unsolved problems. Among these was the following question concerning the stability of homomorphisms.

We are given a group G and a metric group G' with metric ρ(.,.). Given $\in>$ 0, does there exist a $\delta>$ 0 such that if f: G→G' satisfies:

$$\rho(f(xy), f(x)f(y)) < \delta$$

For all x, y∈G, then a homomorphism h: G→G' exists with $\rho(f(x), h(x)) < \in$ for all x∈G?

In other words, we are looking for situations when the homomorphisms are stable, i.e., if a mapping is almost a homomorphism, then there exists a true homomorphism near it. In Hyers (1941) considered the case of approximately additive mapping in Banach spaces 1and satisfying the well-known weak Hyers inequality controlled by a positive constant. The famous Hyers stability result that appeared in (Hyers, 1941) was generalized in the stability involving a sum of powers of norms by Rassias (1978).

**Theorem 1.1:** (Th. M. Rassias). Let f: E→F be a mapping from a normed vector space E into a Banach space F subject to the inequality:

$$\| f(x+y) - f(x) - f(y) \| \le \in (\| x \|^P + \| y \|^P) \quad (1)$$

For all x, y∈E, where $\in$ and p are constant with $\in>$ 0 and p<1. Then the limit:

$$L(x) = \lim_{n\to\infty} \frac{f(2^n x)}{2^n}$$

For all x∈E and L : E→F is the unique additive mapping which satisfies:

$$\| f(x) - L(x) \| \le \frac{2\in}{2-2^P} \| x \|^P \quad (2)$$

For all x∈E. If p<0 then inequality (1.1) holds for x, y ≠ 0 and (1.2) for x≠0. Also, if the mapping t→f (tx)

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is continuous in $t\in R$ for each fixed $x\in X$, then L is R-linear.

In Gajda (1991) answered the question for the case p>1, which was raised by Th. M. Rassias. This new concept is known as Hyers-Ulam-Rassias stability or generalized Hyers-Ulam stability of functional equations. In 1994, a generalization of the Rassias' theorem was obtained by Gavruta (1999). The stability of functional equations was studied by a number of mathematicians, see (Savadkouhi *et al*., 2009; Czerwik, 2002), (Ebadian *et al*., 2010; Gordji, 2009; 2010; Gordji and Savadkouhi, 2009a; 2009b; 2009c; 2010a; 2010b; Gordji and Khodaei, 2009a; 2009b; Gordji *et al*., 2008; 2009a; 2009b; 2009c: 2009d; 2009e: 2009f; 2009g; 2009h; 2010a; 2010b; 2010c; 2010d; 2010e; Gordji and Najati, 2010; Gordji and Moslehian, 2010; Farokhzad and Hosseinioun, 2010; Gajda, 1991; Gavruta, 1999; Gavruta and Gavruta, 2010; Gilanyi, 2001; Gordji *et al*., 2009e; 2009f; 2009g; 2009h; Gordji and Savadkouhi, 2009a; 2009b; Gordji *et al*., Gordji and Savadkouhi, 2009; Gordji *et al*., 2010a; 2010b; Khodaei and Rassias, 2010), (Hyers *et al*., 1998; Jung, 2001),( Park, 2007; Park and Gordji, 2010; Park and Najati, 2010; Park and Rassias, 2010), (Rassias, 1990; 1998; 2000a; 2000b; 2000c; Rassias and Semrl, 1992; 1993; Rassias and Shibata, 1998) and references therein.

It seems that approximate derivations were first investigated Jun and Park (1996).

Recently, the stability of derivations has been investigated by some authors; (Badora, 2006; Chu *et al*., 2010; Gordji and Moslehian, 2010; Farokhzad and Hosseinioun, 2010) and references therein.

In this study, we introduce the concept of a partial ternary derivation from $A_1\times\cdot\cdot\cdot\times A_n$ into B, where $A_1, A_2, \cdot\cdot\cdot, A_n$ and B are ternary algebras. We prove the generalized Hyers-Ulam stability of the partial ternary derivation in Banach ternary algebras.

**Main results:** Let $A_1, A_2,...,A_n$ be normed ternary algebras over the complex field C and let B be a Banach ternary algebra over C. A mapping $\delta_k$ from $A_1\times\cdots\times A_n$ into B is called a k-th partial ternary derivation if there exists a mapping gk: Ak→B such that:

$$\begin{aligned}&\delta_k(x_{1,...,}[a_k b_k c_k],...,x_n)\\&=[g_k(a_k)g_k(b_k)\delta(x_1,...,c_k,...,x_k)]\\&+[g_k(a_k)\delta_k(x_1,...,b_k,...,x_n)g_k(c_k)]\\&+[\delta_k(x_1,...,|a_k,...,x_n)g_k(b_k)g_k(c_k)]\end{aligned}$$

And:

$$\begin{aligned}&\delta_k(x_1,...,\alpha\alpha_k+\beta b_k+\gamma_{ck},...,x_n)\\&=\alpha\delta(x_1,...,a_k,...,x_n)\\&+\beta\delta_k(x_1,...,b_k,...,x_n)\\&+\gamma\delta_k(x_1,...,c_k,...,x_n)\end{aligned}$$

For all $\alpha, \beta, \gamma\in C$, all $a_k, b_k, c_k\in A_k$ and all $x_i\in A_i$ $(i\neq k)$.

We denote that $0_k$, $0_B$ are zero elements of $A_k$, B, respectively.

**Remark 2.1:** Let $B = A_k$, $A_i = 0_i$ $(i\neq k)$ and $g_k = id_{Ak}$. Then the k-th partial ternary derivation can be considered as the ternary derivation of an original version.

**Theorem 2.2:** Let $l\in\{1,-1\}$ be fixed and let $F_k$: $A_1\times\cdots\times A_n\to B$ be a mapping with $F_k(x_1,\cdots, 0_k,\cdots,x_n) = 0_B$. Assume that there exist a function $\varphi_k: A_k^6\to[0,\infty)$ and an additive mapping $g_k$: $A_k\to B$ such that:

$$\lim_{m\to\infty}\frac{1}{3^{lm}}\varphi(3^{lm}a_k,3^{lm}b_k,3^{lm}c_k,3^{lm}d_k,3^{lm}e_k,3^{lm}f_k)=0$$

$$\begin{aligned}&\tilde{\varphi}(a_k,b_k,c_k,0_k,0_k,0_k):=\sum_{m=0}^{\infty}\frac{1}{3^l(m+1)}\\&\varphi_k(3^{lm}a_k,3^{lm}b_k,3^{lm}c_k,0_k,0_k,0_k)=\infty\end{aligned}$$

And:

$$\begin{aligned}&\|F_k(x_1,...,\lambda_{ak}+\lambda_{bk}+\lambda_{ck})+[d_k e_k f_k],...x_n-\\&\lambda F_k(x_1,...,a_k,...,x_n)\\&-\lambda F_k(x_1,...,b_k,...,x_n)-\lambda F_k(x_1,...,c_k,...x_n)\\&-[g_k(dk)g_k(e_k)F_k(x_1,...,f_k,...x_n)]\\&-[g_k(d_k)F_k(x_1,...,e_k,...,x_n)g_k(f_k)]\\&-[F_k(x_1,...,d_k,...,x_n)g_k(e_k)g_k(f_l)]\\&\|\leq\varphi_k(a_k,b_k,c_k,d_k,e_k,f_k)\end{aligned}\tag{3}$$

For all $a_k, b_k, c_k, d_k, e_k, f_k\in A_k$, $x_i\in A_i$ $(i\neq k)$ and all $\lambda\in T:=\{\mu\in C|\ |\mu| = 1\}$. Then there exists a unique k-th partial derivation $\delta_k$: $A_1\times\cdots\times A_n\to B$ such that:

$$\begin{aligned}&\|F_k(x_1,...,x_n)-\delta_k(x_1,...,x_n)\\&\leq\varphi_k(x_k,x_k,x_k,0_k,0_k,0_k)\end{aligned}\tag{4}$$

For all $x_i \in A_i$ (i = 1, 2,⋯, n).

**Proof:** Let l = 1. In (2.1), putting $a_k = b_k = c_k = x_k$, $d_k = e_k = f_k = 0_k$ and $\lambda=1$, we have:

$$\| F_k(x_1,...,3_{xn},...,x_n) - 3F_k(x_1,...,x_k,...,x_n) \| \le \varphi_k(x_k,x_k,x_k,0_k,0_k,0_k)$$

That is:

$$\left\| F_k(x_1,...,x_k,...,x_n) - \frac{1}{3}F_k(x_1,...,3_{xk},...,x_n) \right\| \le \frac{1}{3}\varphi_k(x_k,x_k,x_k,0_k,0_k,0_k) \quad (5)$$

For all $x_i \in A_i$ (i = 1, 2,⋯, n). In (2.3), dividing the both sides by 3 and replacing $x_k$ with $3_{xk}$, we have:

$$\left\| \frac{1}{3}F_k(x_1,...,3_{xk},...,x_n) - \frac{1}{3^2}F_k(x_1,...,3^2_{xk},...,x_n) \right\| \le \frac{1}{3^2}\varphi_k(3^{xk},3^{xk},3^{xk},0_k,0_k,0_k) \quad (6)$$

It follows from (2.3) and (2.4) that:

$$\left\| F_k(x_1,...,x_k,...,x_n) - \frac{1}{3^2}F_k(x_1,...,3^2_{xk},...,x_n) \right\| \le \frac{1}{3}\varphi_k(x_k,x_k,x_k,0_k,0_k,0_k) + \frac{1}{3^2} \varphi_k(3_{xk},3_{xk},3_{xk},0_k,0_k,0_k)$$

For all $x_i \in A_i$ (i = 1, 2, · · · , n). Continuing this way, we get:

$$\left\| F_k(x_1,...,x_k,...,x_n) - \frac{1}{3^m}F_k(x_1,...,3^m_{xk},...,x_n) \right\| \le \sum_{j=0}^{m-1}\frac{1}{3^{j+1}}\varphi_k(3^j_{xk},3^j_{xk},3^j_{xk},0_k,0_k,0_k) \quad (7)$$

For all positive integers m and all $x_i \in A_i$ (i = 1, 2,⋯, n). For any positive integer p, dividing the both sides by $3^p$ and replacing $x_k$ by $3^p{}_{xk}$ in (2.5), we have:

$$\left\| \frac{1}{3^P}F_k(x_1,...,3^P_{xk},...,x_n) - \frac{1}{3^m+p}F_k(x_1,...,3^{m+p}{}_{xk},...,x_n) \right\| \le \sum_{j=0}^{m-1}\frac{1}{3^{j+p+1}}\varphi_k(3^{j+p}{}_{xk},3^{j+p}{}_{xk},3^{j+p}{}_{xk},0_k,0_k,0_k)$$

Which tends to zero as $p \to \infty$. So the sequence $\{(\frac{1}{3})^m F_k(x_1,...,3^m xk,...,xn)\}$ is a Cauchy sequence in B. By the completeness of B, $\{(\frac{1}{3})^m F_k(x_1,...,3^m xk,...,xn)\}$ converges and so we can define a mapping $\delta_k$: $A_1 \times \cdots \times A_n \to B$ given by:

$$\delta_k(x_1,...,x_n) = \lim_{n\to\infty}\frac{1}{3^m}F_k(x_1,...,3^m{}_{xk},...,x_n) \quad (8)$$

For all $x_i \in A_i$ (i = 1,⋯, n). In (2.1), letting $d_k = e_k = f_k = 0_k$ and replacing $a_k$, $b_k$, $c_k$ with $3^m{}_{ak}$, $3^m{}_{bk}$, $3^m{}_{ck}$, respectively, we obtain that:

$$\left\| \begin{matrix} \frac{1}{3^m}F_k(x_1,...,3^m(\lambda_{ak}+\lambda_{bk}+\lambda_{ck}),...,x_n) \\ -\lambda\frac{1}{3^m}F_k(x_1,...,3^m{}_{ak},...,x_n) \end{matrix} \right\| -\lambda\frac{1}{3^m}F_k(x_1,...,3^m{}_{bk},...,x_n) -\lambda\frac{1}{3^m}F_k(x_1,...,3^m{}_{ck},...,x_n) \le \frac{1}{3^m}\varphi_k(3^m{}_{ak},3^m{}_{bk},3^m{}_{ck},0_k,0_k,0_k)$$

Which tends to zero as $m \to \infty$. Thus we obtain:

$$\delta_k(x_1,...,\lambda_{ak}+\lambda_{bk}+\lambda_{ck},...,x_n) = \lambda\delta_k(x_1,...,a_k,...,x_n) + \lambda\delta_k(x_1,...,b_k,...,x_n) + \lambda\delta_k(x_1,...,c_k,...,x_n) \quad (9)$$

For all $a_k$, $b_k$, $c_k \in A_k$ and all $\lambda \in T$.

Setting $c_k = 0_k$ and $\lambda = 1$ in (2.7), we have:

$$\delta_k(x_1,...,a_k+b_k,...,x_n) = \delta_k(x_1,...,a_k,...,x_n) + \delta_k(x_1,...,b_k,...,x_n)$$

For all $a_k$, $b_k \in A_k$, all $x_i \in A_i (i \neq k)$.

Setting $b_k = c_k = 0_k$ in (2.7), we have:

$$\delta_k(x_1,...,\lambda_{ak},...,x_n) = \lambda\delta_k(x_1,...,a_k,...,x_n)$$

For all $a_k \in A_k$, all $x_i \in A_i (i \neq k)$ and all $\lambda \in T$.

Let $\gamma = \theta_1 + i\theta_2 \in C$, where $\theta_1$, $\theta_2 \in R$. Let $\gamma_1 = \theta_1 - [\theta_1]$, $\gamma_2 = \theta_2 - [\theta_2]$, where $[\theta_i]$ denotes the greatest integer less than or equal to the number $\theta_i$ (i = 1, 2). Then $0 \le \gamma_i < 1$ (i = 1, 2) and by using Remark 2.2.2 of (Murphy, 1990), one can represent $\gamma_I$ as $\gamma_i = \frac{\lambda_i + \lambda_{i,2}}{2}$ in which $\lambda_{i,j} \in T$ ($1 \le i$, $j \le 2$). Since $\delta_k$ satisfies (2.7), we obtain that:

$$\begin{aligned}
&\delta_k(x_1,\ldots,\gamma_{xk},\ldots,x_n)\\
&= \delta_k(x_1,\ldots,\theta_{1xk},\ldots,x_n)\\
&+ i\delta_k(x_1,\ldots,\theta_{2xk},\ldots,x_n)\\
&= \delta_k\left(x_1,\ldots,\left([\theta_1] + \frac{\lambda_{1,1}+\lambda_{1,2}}{2}\right)x_k,\ldots,x_n\right)\\
&+ i\delta_k\left(x_1,\ldots,\left([\theta_2] + \frac{\lambda_{2,1}+\lambda_{2,2}}{2}\right)x_k,\ldots,x_n\right)\\
&= \delta_k(x_1,\ldots,[\theta_1]x_k,\ldots,x_n) + \frac{1}{2}\delta_k\\
&(x_1,\ldots,(\lambda_{1,1}+\lambda_{1,2})x_k,\ldots,x_n)\\
&+ i(\delta_k(x_1,\ldots,[\theta_2]x_k,\ldots,x_n) + \frac{1}{2}\\
&\delta_k(x_1,\ldots,(\lambda_{2,1}+\lambda_{2,2})x_k,\ldots,x_n))\\
&= \left([\theta_1] + \frac{\lambda_{1,1}+\lambda_{1,2}}{2}\right)\delta_k(x_1,\ldots,x_k,\ldots,x_n)\\
&+ i\left([\theta_2] + \frac{\lambda_{2,1}+\lambda_{2,2}}{2}\right)\delta_k(x_1,\ldots,x_k,\ldots,x_n)\\
&= \theta_1\delta_k(x_1,\ldots,x_k,\ldots,x_n) + i\theta_2\delta_k(x_1,\ldots,x_k,\ldots,x_n)\\
&= \gamma\delta_k(x_1,\ldots,x_k,\ldots,x_n)
\end{aligned}$$

For all $\gamma \in C$ and all $x_i \in A_i$ (i = 1, 2, ⋯, n). Hence $\delta_k$ is C-linear with respect to the k-th variable. It follows from (2.5) that:

$$\begin{aligned}
&\| F_k(x_1,\ldots,x_k,\ldots,x_n) - \delta_k(x_1,\ldots,x_k,\ldots,x_n)\\
&\| \le \tilde{\varphi}_k(x_k,x_k,x_k,0_k,0_k,0_k)
\end{aligned}$$

For all $x_i \in A_i$ (i = 1, 2,⋯, n).

To prove the uniqueness of $\delta_k$, let $\delta'_k$: $A_1\times\cdots\times A_n \to B$ be another k-th partial derivation satisfying (2.2). Then we have:

Passing the limit m→∝1, we have $\delta k(x_1,\cdots, x_n) = \delta'_k(x_1,\cdots,x_n)$.

Finally, putting $a_k = b_k = c_k = 0_k$ and replacing $d_k$, $e_k$, $f_k$ with $3^m{}_{dk}$, $3^m{}_{ek}$, $3^m{}_{fk}$, respectively, in (2.1), we obtain:

$$\begin{aligned}
&\| F_k(x_1,\ldots,3^{3m}[d_k e_k f_k),\ldots,x_n)\\
&-[3^m g_k(d_k)F_k(x_1,\ldots,3^m e_k,\ldots,x_n)3^m g_k(f_k)]\\
&-[F_k(x_1,\ldots,3^{3m}d_k,\ldots,x_n)3^m g_k(f_k)]\\
&\le \varphi_k(0_k,0_k,0_k,3^{3m}d_k,3^m e_k,3^m f_k)
\end{aligned}$$

Then we have:

$$\begin{aligned}
&\| \frac{1}{3^{3m}}F_k(x_1,\ldots,[3^{3m}d_k e_k f_k),\ldots,x_n)\\
&-\frac{1}{3^m}[g_k(d_k)F_k(x_1,\ldots,3^m e_k,\ldots,x_n)g_k(f_k)]\\
&-\frac{1}{3^{3m}}[F_k(x_1,\ldots,3^{3m}d_k,\ldots,x_n)g_k(f_k)]\\
&\le \frac{1}{3^{3m}}\varphi_k(0_k,0_k,0_k,3^{3m}d_k,3^m e_k,3^m f_k)
\end{aligned}$$

For all $d_k$, $e_k$, $f_k \in A_k$. Passing the limit m→∝1 in above inequality, we obtain:

$$\begin{aligned}
&\delta_k(x_1,\ldots,[d_k e_k f_k],\ldots,x_n)\\
&= [g_k(d_k)g_k(e_k)\delta_k(x_1,\ldots,f_k,\ldots,x_n)]\\
&-[g_k(d_k)\delta_k(x_1,\ldots,e_k,\ldots,x_n)g_k(f_k)]\\
&+[\delta_k(x_1,\ldots,d_k,\ldots,x_n)g_k(e_k)g_k(f_k)]
\end{aligned}$$

For all $d_k$, $e_k$, $f_k \in A_k$ and all $x_i \in A_i$ ($i \ne k$).

By the same reasoning as above, one can prove the theorem for the case $l = -1$.

**Theorem 2.3:** Let $l \in \{1,-1\}$ be fixed and let $F_k$: $A_1\times\cdots\times A_n \to B$ be a mapping with $F_k(x_1,\cdots,0_k,\cdots,x_n) = 0_B$. Assume that there exist a function $\varphi_k : A_k^6 \to [0,\infty)$ and an additive mapping $g_k$: $A_k \to B$ such that:

$$\lim_{m\to\infty} 3^{lm}\varphi(3^{-lm}{}_{ak},3^{-lm}{}_{bk},3^{-lm}{}_{ck},3^{-lm}{}_{ek},3^{-lm}{}_{fk}) = 0$$

$$\tilde{\varphi}_k(a_k,b_k,c_k,0_k,0_k,0_k) := \sum_{m=1}^{\infty} 3^{l(m-1)}\varphi_k\left(\frac{a_k}{3^{lm}},\frac{b_k}{3^{lm}},\frac{c_k}{3^{lm}},0_k,0_k,0_k\right) < \infty$$

And:

$$\begin{aligned}&\Big\| F_k(x1,\ldots,\lambda a_k+\lambda b_k+\lambda c_k\\&+[d_k e_k f_k],\ldots,x_n)-\lambda F_k(x_1,\ldots,a_k,\ldots,x_n)\\&-\lambda F_k(x_1,\ldots,b_k,\ldots,x_n)-\lambda F_k(x_1,\ldots,c_k,\ldots,x_n)\\&-[g_k(d_k)g_k(e_k)F_k(x_1,\ldots,f_k,\ldots,x_n)]\\&-[g_k(d_k)F_k(x_1,\ldots,e_k,\ldots,x_n)g_k(f_k)]\\&-[f_k(x_1,\ldots,d_k,\ldots,x_n)g_k(e_k)g_k(f_k)]\Big\|\\&\le \varphi_k(a_k,b_k,c_k,e_k,f_k)\end{aligned} \quad (10)$$

For all $(a_k, b_k, c_k, e_k, f_k)\in A_k, x_i\in A_i (i\neq k)$ and $\lambda = 1, i$. If for each fixed $x_i\in A_i$ ($i=1, 2,\ldots, n$) the function $t\to F_k(x_1,\cdots, tx_k,\cdots, x_n)$ is continuous on R, then there exists a unique k-th partial derivation $\delta_k$: $A_1\times\cdots\times A_n\to B$ such that:

$$\|F_k(x_1,\ldots,x_n)-\delta_k(x_1,\ldots,x_n)\|\le\tilde{\varphi}_k(x_k,x_k,x_k,0_k,0_k,0_k) \quad (11)$$

For all $x_i \in A_i$ ($i = 1, 2,\cdots, n$).

**Proof:** Let l = 1. In (2.8), putting $d_k = e_k = f_k = 0_k$, $\lambda = 1$ and replacing $a_k$, $b_k$, c = by $\frac{x_k}{3}$ we get:

$$\begin{aligned}&\left\|F_k(x_1,\ldots,x_k,\ldots,x_n)-3F_k(x_1,\ldots,\frac{x_k}{3},\ldots,x_n)\right\|\\&\le\varphi_k(\frac{x_k}{3},\frac{x_k}{3},\frac{x_k}{3},0_k,0_k,0_k)\end{aligned}$$

For all $x_i \in A_i$ ($i = 1, 2,\cdots,n$). Then we have:

$$\left\|F_k(x_1,\ldots,\frac{x_k}{3},\ldots,x_n)\right\|-\frac{1}{3}F_k(x_1,\ldots,x_k,\ldots,x_n) \quad (12)$$

$$\le\frac{1}{3}\varphi_k(\frac{x_k}{3},\frac{x_k}{3},\frac{x_k}{3},0_k,0_k,0_k) \quad (13)$$

For all $x_i\in A_i$ ($i = 1, 2,\cdots,n$). And we obtain that:

$$\begin{aligned}&\left\|3^2F_k(x_1,\ldots,\frac{x_k}{3^2},\ldots,x_n)-3F_k(x_1,\ldots,\frac{x_k}{3},\ldots,x_n)\right\|\\&\le 3\varphi_k(\frac{x_k}{3^2},\frac{x_k}{3^2},\frac{x_k}{3^2},0_k,0_k,0_k)\end{aligned}$$

For all $x_k\in A_k$. By using the induction, we obtain that:

$$\begin{aligned}&\left\|3^mF_k(x_1,\ldots,\frac{x_k}{3^m},\ldots,x_n)-3^pF_k(x_1,\ldots,\frac{x_k}{3^p},\ldots,x_n)\right\|\\&\le\sum_{j=p+1}^{m}3^{j-1}\varphi_k(\frac{x_k}{3^j},\frac{x_k}{3^j},\frac{x_k}{3^j},0_k,0_k,0_k)\end{aligned} \quad (14)$$

For all $m>p\geq 0$ and all $x_i\in A_i$ ($i = 1, 2,\cdots, n$). Thus for $x_i\in A_i$ ($i = 1,\cdots,n$), the sequence $\{3^mF_k(x_1,\ldots,\frac{x_k}{3^m},\ldots,x_n)\}$ is a Cauchy sequence. From the completeness of B, the sequence is convergent. So we can define a mapping $\delta_k$ given by:

$$\delta_k(x_1,\ldots,x_k,\ldots,x_n := \lim_{m\to\infty}3^mF_k(x_1,\ldots,\frac{x_k}{3^m},\ldots,x_n)$$

For all $x_i\in A$= ($i = 1,\cdots,n$). Letting $\lambda = 1$, $d_k = e_k = f_k = 0_k$ and replacing $a_k$, $b_k$, $c_k$ by $\frac{a_k}{3^m},\frac{b_k}{3^m},\frac{c_k}{3^m}$ respectively, in (2.8), we have that:

$$\begin{aligned}&\Bigg\|3^mF_k(x_1,\ldots,\frac{a_k+b_k+c_k}{3^m},\ldots,x_n)\\&-3^mF_k(x_1,\ldots,\frac{a_k}{3^m},\ldots,x_n)\\&-3^mF_k(x_1,\ldots,\frac{b_k}{3^m},\ldots,x_n)\\&-3^mF_k(x_1,\ldots,\frac{c_k}{3^m},\ldots,x_n)\Bigg\|\\&\le 3^m\varphi_k(\frac{a_k}{3^m},\frac{b_k}{3^m},\frac{c_k}{3^m},0_k,0_k,0_k)\end{aligned} \quad (15)$$

Passing the limit $m\to\infty$, we obtain:

$$\begin{aligned}&\delta_k(x_1,\ldots,a_k+b_k+c_k,\ldots,x_n)=\delta_k(x_1,\ldots,a_k,\ldots,x_n)\\&+\delta_k(x_1,\ldots,b_k,\ldots,x_n)+\delta_k(x_1,\ldots,c_k,\ldots,x_n)\end{aligned} \quad (16)$$

## CONCLUSION

For all $a_k, b_k, c_k \in A_k$ and all $x_i \in A_i$ ($i \neq k$). Since $F_k(x_1, \cdots, tx_k, \cdots, x_n)$ is continuous at $t\in R$ for each fixed $x_i\in A_i$ ($i = 1,\cdots, n$), the mapping $\delta_k$ is R-linear with respect to the k-th variable by the same reasoning as the proof of the main theorem of (Rassias, 1978). Putting $b_k = c_k = d_k = e_k = f_k = 0_k$, $\delta = i$ and replacing $a_k$ with $\frac{a_k}{3^m}$ in (2.8), we can easily obtain the inequality:

$$\begin{aligned}&\left\|3^mF_k(x_1,\ldots,\frac{ia_k}{3^m},\ldots,x_n)-i3^mF_k(x_1,\ldots,\frac{a_k}{3^m},\ldots,x_n)\right\|\\&\le 3^m\varphi_k(\frac{a_k}{3^m},0_k,0_k,0_k,0_k,0_k)\end{aligned} \quad (17)$$

For all $m\in N$ and $a_k\in A_k$. Since the right-hand side in (2.14) tends to zero as $m\to\infty$, we have:

$$\begin{aligned}&\delta_k(x_1,....,ix_k,....,x_n)\\&=\lim_{m\to\infty}3^m F_k(x_1,...,\frac{ix_k}{3^m},....,x_n)\\&=\lim_{m\to\infty}i3^m F_k(x_1,...,\frac{x_k}{3^m},....,x_n)\\&=i\delta_k(x_1,...,x_k,...,x_n)\end{aligned}$$

For all $x_i \in A_i$ (i = 1, ..., n). Thus $\delta_k$ is C-linear with respect to the k-th variable. Now, let p = 0 in (2.11), we obtain the following:

$$\begin{aligned}&\left\|F_k(x_1,...,x_k,...,x_n)-3^m F_k(x_1,...,\frac{x_k}{3^m},....,x_n)\right\|\\&\le\sum_{j=1}^{m}3^{j-1}\varphi_k(\frac{x_k}{3^j},\frac{x_k}{3^j},\frac{x_k}{3^j},0_k,0_k,0_k)\end{aligned}$$

Passing the limit $m \to \infty 1$, we have:

$$\begin{aligned}&\left\|F_k(x_1,....,x_k,....,x_n)-\delta(x_1,.....,x_k,...,x_n)\right\|\\&\qquad\le\tilde{\varphi}_k(x_k,x_k,x_k,0_k,0_k,0_k)\end{aligned}$$

For all $x_i \in A_i$ (i = 1,⋯, n). By a similar method to the proof of Theorem 2.2, one can prove that $\delta_k$ is a unique k-th partial derivation which satisfies (2.9).

By the same reasoning as above, one can prove the theorem for the case $l = -1$.

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