Kinetostatics of a 2T9R Robot Mechanism

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Abstract: The paper presents in detail a method of calculating the forces acting on a 2T9R type robot. In order to determine the reactions (forces in the kinematic couples), one must first determine the inertial forces in the mechanism to which one or more useful loads of the robot can be added. The torsor of the inertia forces is calculated with the help of the masses of the machine elements and the accelerations from the centers of mass of the mechanism elements, so the positions, velocities and accelerations acting on it will be determined, i.e., its complete kinematics. The calculation method applied by a Math Cad program intelligently uses data entry through the If Log logic function so that the calculations can be automated. So, the effective automation of the calculation program is done exclusively through the If Log functions originally used in the paper.

Keywords: If Log, Robot, 2T9R Robot, Forces, Kinematics, Geometric-Analytical Method, Direct Kinematics, Inverse Kinematics

Introduction

Robots have always fascinated us, but today we use them massively, in almost all industrial areas, especially where they work hard, repetitive and tiring, in toxic, chemical, radioactive environments, underwater, in the cosmos, in dangerous environments, on mined lands, in hard-to-reach areas, etc. It can be said once again that, just as software and microchips have helped us to quickly write various useful programs and implement them directly, so robotics has made our daily work much easier. Thanks to robots, automation is almost perfect today, product quality is very high, the manufacturing price has dropped a lot, you can work in continuous fire, people have escaped hard work, tiring, repetitive, in toxic environments and now can treat other problems more important, such as design, scientific research, to work only 5 days a week with high income and, in the future, due to the massive implementation of increasingly modern robots with increased capabilities, man will reach the work week only 4 days.

An even greater increase is expected in the number of specialized robots implemented in large factories and factories around the world. Due to the massive use of industrial robots, the diversification in this field has gained high levels. For this reason, we want to study in this study a new robot model, 2T9R, extremely complex in movements, useful in any type of work, a versatile robot, which can weld, cut, process different parts, to assemble them, or to manipulate them from one working strip to another and in the same way, he can also paint the different machined components before their assembly. The robot has various advantages due to its complex mode arranged since the design and will be able to easily adapt to any type of automated manufacturing cell. For this reason and because it is an original one and has not been studied before, we want that in this study we review its study completely with the determination of all the forces that act it and that appear within it, the one that it also requires a complete kinematic calculation (Anderson, 1997; CEUP, 2018; García, 2020; Rana, 2020; Garfo et al., 2020; Kumar and Sreenivasulu, 2019; Mishra and Sarawagi, 2020; Welabo and Tesfamariamr, 2020; Antonescu and Petrescu, 1985; 1989; Antonescu et al., 1985a; 1985b; 1986; 1987; 1988; 1994; 1997; 2000a; 2000b; 2001; Aversa et al., 2017a; 2017b; 2017c; 2017d; 2016a; 2016b; 2016c; 2016d; Ayiei, 2020; Brewer, 1991; Chilukuri et al., 2019; Cao et al., 2013; Dong et al., 2013; Saheed et al., 2019; Riman, 2019; Matthews and Yi, 2019; Dwivedi et al., 2019a; 2019b; Eremia, 2020; Franklin, 1930; Hanrahan, 2014; He et al., 2013; Hertel, 2017; Komakula, 2019; Langston, 2015; 2016; Lee, 2013; Lin et al., 2013; Liu et al., 2013; Padula and Perdereau, 2017; 2018; 2019).
2013; Perumaal and Jawahar, 2013; Petrescu, 2011; 2012; 2019a-v; 2020a-g; Petrescu and Petrescu, 2019a-f; 1995a-b; 1997a-c; 2000a-b; 2002a-b; 2003; 2005a-e; 2011a-c; 2012a-b; 2013a-e; 2014a-h; 2016a-c; 2020; Petrescu et al., 2007; 2009; 2016; 2017a-ak; 2018a-w; 2020; Petrescu and Calautit, 2016a-b; Dekkata and Yi, 2019; Fahim et al., 2019; El Hassouni et al., 2019; Riman, 2018; Nacy and Nayif, 2018; Kortam et al., 2018; Welch and Mondal, 2019; Eissa et al., 2019; Younes et al., 2019; Svensson et al., 2004; Rahman, 2018; Richmond, 2013; Kisabo et al., 2019a; Kisabo and Adebimpe, 2019; Kosambe, 2019a-d; Sharma and Kosambe, 2020; Oni and Jha, 2019; Chaudhary and Kumar, 2019; de Lima et al., 2019; Babu et al., 2019; 2020; de Mota Siqueira et al., 2020; Tumino, 2020; Mishra, 2020a; 2020b; Brischetto and Torre, 2020).

Materials and Methods

The present study will start with a description of the 2T9R robot proposed to be analyzed, in terms of the forces acting on it. The 2T9R mechanism (Fig. 1) has a constructive model based on a bimobile kinematic chain having three independent contours (Fig. 2a) obtained from the bicontour chain of the 2T6R mechanism.

The direct structural model (Fig. 2b) consists of two initial active modular groups GMAI (A, 1) and GMAI (G, 8) which constitute the linear motors that drive it and two passive modular groups, one of the types of the GMP2 triad (2, 3,4,6) and the other of the GMP1 dyad type (5, 7). The connection of the modular groups for the direct model is shown in Fig. 3.

The direct structural model (Fig. 2b) and the connection of the corresponding modular groups (Fig. 3) are used to determine the reaction torsor in each kinematic coupling using the kinetostatic principle.

To study the main plane mechanism of the 2T9R robot, its kinematic elements, kinematic torques and positioning angles of the elements that also have rotation are initially established (Fig. 4).

For the kinetostatic analysis (determination of the forces in the mechanism) the centers of mass marked with the letter T (Fig. 5) are positioned as follows: O ≡ T5 ≡ T4; B ≡ T2 ≡ T3; E ≡ T6; F ≡ T7. Their placement does not influence the algorithm for calculating the components of the reaction torsion in the kinematic torques.

It is considered a single external force RT acting on the system neglecting other external forces (for example - gravitational forces). This simplification brings some peculiarities in the form of terms from the calculation algorithm without restricting its generality. The forces of weight are not recommended to be introduced in the sizing calculations because their influence is sometimes by addition and sometimes by decrease it being therefore opposite and having negative effects on the sizing of a mechanism. On the other hand, in large (large) robots, if they still work fast (at high speeds), the inertial forces (internal forces, which arise even in the mechanism due to its masses) are considerable and much higher than those weights that automatically become negligible.

Determination of Reactions in the Kinematic Torques of the Triad (2,3,4,6)

The study of forces is always processed inversely to the kinematic one, i.e., not from the motors to the final effector element, but inversely, from the modular group furthest from the motors to them. For this reason, the force calculations start on the triad (2,3,4,6) from Fig. 6.

To determine the unknown forces, the reactions (from the kinematic couplings), the following calculation relations are written (from 2 ROx is made explicit, from 3 RAx, which is introduced in relation 1 and I is obtained and in relation 4 and II is obtained, where I and II represent two linear equations with two unknowns that make up a linear system that can be solved immediately by Kramer III):

\[
\sum M_{Fi}^{0} = 0
\]
\[
M_{Fi} + R_{Fi}^{0} (y_{x} - y_{c}) - R_{Fi}^{0} (y_{x} - x_{0}) + R_{Fi}^{0} (x_{c} - x_{0}) + \ldots = 0
\]

\[
\sum M_{C}^{(0)} = 0
\]
\[
M_{C}^{0} = R_{C}^{0} (y_{0} - y_{c}) - R_{C}^{0} (x_{c} - x_{0}) = 0
\]

\[
\sum M_{B}^{(2)} = 0
\]
\[
M_{B}^{2} - R_{B}^{2} (y_{A} - y_{B}) - R_{B}^{2} (x_{A} - x_{B}) = 0
\]

\[
\sum M_{Tr}^{(3,3,0)} = 0
\]
\[
M_{Tr}^{3} + M_{Tr}^{0} + R_{Tr}^{0} (y_{0} - y_{c}) - R_{Tr}^{0} (x_{0} - x_{c}) - R_{Tr}^{0} (y_{A} - y_{c}) - R_{Tr}^{0} (x_{A} - x_{c}) + \ldots = 0
\]

\[
\sum F_{E}^{(4,3,2,6)} = 0
\]
\[
R_{E}^{4} + R_{E}^{3} + F_{23}^{4} + F_{6}^{4} = 0
\]

\[
\sum F_{T}^{(4,3,2,6)} = 0
\]
\[
R_{T}^{4} + R_{T}^{3} + F_{23}^{4} + F_{6}^{4} + R_{T} = 0
\]
DOI: 10.3844/ajeassp.2022.59.80

\[
R_G = \left[ \begin{array}{c}
(x_0-x_o)
\end{array} \right] + \left[ \begin{array}{c}
(y_0-y_o)
\end{array} \right] + R_E = \left[ \begin{array}{c}
(x_s-x_o)
\end{array} \right] + \left[ \begin{array}{c}
(y_s-y_o)
\end{array} \right] + \left[ \begin{array}{c}
(x_n-x_o)
\end{array} \right] + \left[ \begin{array}{c}
(y_n-y_o)
\end{array} \right]
\]

(II)

\[
M = \left[ \begin{array}{c}
(y_0-y_o)
\end{array} \right] + M_t = F_{T1} + F_{T2} (y_s-y_o) + F_{T3} (x_s-x_o)
\]

Fig. 1: The mechanism 2T9R

Fig. 2: Structural scheme of the mechanism

Fig. 3: Electronic or wiring diagram (block diagram) of the mechanism

Fig. 4: Determining the kinematic elements, the kinematic torques, and the angles that position the elements that also have a rotation

Fig. 5: Positioning the centers of mass T of all the elements of the mechanism
From (5) results relation (V) which determines \( R_{Ex} \) and from (6) results the expression (VI) which generates \( R_{Ey} \):

\[
R^i_e = -\left( R^o_e + R^s_e + F^o_{T23} + F^s_{T23} \right) \quad (V)
\]
\[
R^o_e = -\left( R^s_e + F^o_{T23} + F^s_{T23} + R^i_e \right) \quad (VI)
\]

Can now write the next equations (7-15):

\[
\sum F^x_{y^{(0)}} = 0 \Rightarrow R^e_x = X_{34} = -\left( R^o_x \right) \Rightarrow X_{45} = X_{34} = R^o_x \quad (7)
\]
\[
\sum F^y_{y^{(0)}} = 0 \Rightarrow R^e_y = Y_{36} = -\left( R^o_y \right) \Rightarrow Y_{45} = Y_{36} = R^o_y \quad (8)
\]
\[
\sum F^x_{y^{(2)}} = 0 \Rightarrow R^e_x = X_{32} = -\left( R^s_x + F^o_{T23} \right) \Rightarrow X_{23} = -X_{32} \quad (9)
\]
\[
\sum F^y_{y^{(2)}} = 0 \Rightarrow R^e_y = Y_{36} = -\left( R^s_y + F^o_{T23} \right) \Rightarrow Y_{23} = -Y_{32} \quad (10)
\]
\[
\sum F^x_{y^{(6)}} = 0 \Rightarrow R^e_x = X_{36} = -\left( R^s_x + F^o_{T6} \right) \Rightarrow X_{36} = -X_{36} \quad (11)
\]
\[
\sum F^y_{y^{(6)}} = 0 \Rightarrow R^e_y = Y_{36} = -\left( R^s_y + F^o_{T6} + R^i_e \right) \Rightarrow Y_{36} = -Y \quad (12)
\]

With (IV) on determines \( R_{0x} \) si \( R_{3x} \):

\[
\begin{align*}
R^o_x &= M^3_4 + R^{3}_{y} \cdot \left( x_y - x_e \right) \quad (x_y - y_c) \\
R^s_x &= M^3_4 + R^{3}_{y} \cdot \left( x_y - x_b \right) \quad (y_a - y_b) \\
R^i_x &= M^3_4 + R^{3}_{y} \cdot \left( x_y - x_a \right) \quad (y_a - y_b) \\
\end{align*}
\]  

(IV)
In order to perform the triad calculations (2,3,4,6) it is necessary to present briefly the expressions by which the known inertial forces, inside the mechanism, due to the masses of the component elements (16-20) are determined by calculations:

\[ M'_i = -J_{y_2} \cdot \varepsilon_4 = -J^{(5)}_{y_2} \cdot \varepsilon_4 \]  
(16)

\[ F_{t_1}^\alpha = -m_2 \cdot \ddot{x}_b \]
(17)
\[ F_{t_2}^\alpha = -m_2 \cdot \ddot{y}_b \]
\[ M'_i = -J^{(5)}_{y_2} \cdot \varepsilon_4 \]

\[ F_{t_1}^a = F_{t_1}^\alpha + F_{t_1}^\beta = -(m_3 + m_4) \cdot \ddot{x}_b \]
(19)
\[ F_{t_2}^a = F_{t_2}^\alpha + F_{t_2}^\beta = -(m_3 + m_4) \cdot \ddot{y}_b \]
\[ M'_{13} = M_2^a + M_1^a = -J^{(5)}_b \cdot \varepsilon_2 - J^{(5)}_b \cdot \varepsilon_3 \]

\[ F_{t_1}^a = -m_4 \cdot \ddot{x}_b \]
(20)
\[ F_{t_2}^a = -m_4 \cdot \ddot{y}_b \]
\[ M'_i = -J^{(5)}_b \cdot \varepsilon_6 \]

\[ \sum M^{(5)}_i = 0 \]
(21)
\[ M_1^a - R_0^b \cdot (x_0 - y_0) - R_0^b \cdot (x_0 - x_0) + F_{t_1}^\alpha \cdot (x_0 - y_0) - F_{t_2}^\alpha \cdot (x_0 - x_0) = 0 \]
\[ X_{65} = -X_{65} = (x_65 + x_75) = R_x^a + R_y^a \]
(27)
\[ Y_{65} = -Y_{65} = -(R_x^a + R_y^a) \]
(29)
\[ M'_i = -J^{(5)}_b \cdot \varepsilon_6 \]

\[ \sum M^{(5)}_i = 0 \]
(22)
\[ M'_i - R_0^a \cdot (y_0 - y_0) + R_0^a \cdot (x_0 - x_0) = 0 \]
\[ M_1^a - M_2^a \cdot (x_0 - y_0) \]
\[ \sum M^{(5)}_i = 0 \]
(23)
\[ R_0^a = \frac{R_0^a ((y_0 - y_0) - M_2^a) (x_0 - x_0)}{(x_0 - x_0)} \Rightarrow Y_{65} = -Y_{65} = -R_0^a \]
(24)

\[ \sum F^{(5)}_i = 0 \Rightarrow R_0^b = X_65 = -(R_0^a + F_{t_1}^\beta) \Rightarrow X_{75} = -X_{75} \]
(25)
\[ \sum F^{(5)}_i = 0 \Rightarrow R_0^b = Y_65 = -(R_0^a + F_{t_2}^\beta) \Rightarrow Y_{75} = -Y_{75} \]
(26)
\[ \sum F^{(5)}_i = 0 \Rightarrow Y_{65} = -(Y_{65} + Y_{75}) = R_x^a + R_y^a \]
(28)
\[ X_{65} = -R_x^a = -X_{65} = -(R_x^a + R_y^a) \]
(30)

\[ M'_i = -J^{(5)}_b \cdot \varepsilon_6 \]

**Determination of Reactions in the Kinematic Couplings of the Dyad (5,7)**

Dyad 5.7 has the following charges (Fig. 7), where the already known forces are shown in blue and the unknown ones in green, ie the reactions in the kinematic torques of the dyad, which will be determined.

Can write the relations 21-22:

\[ \sum M^{(5)}_i = 0 \]
(21)
\[ M_1^a + M_2^a - R_0^b \cdot (x_0 - y_0) - R_0^b \cdot (x_0 - x_0) + F_{t_1}^\alpha \cdot (x_0 - y_0) - F_{t_2}^\alpha \cdot (x_0 - x_0) = 0 \]
\[ X_{65} = -X_{65} = -(x_65 + x_75) = R_x^a + R_y^a \]
(27)
\[ Y_{65} = -Y_{65} = -(R_x^a + R_y^a) \]
(29)
\[ M'_i = -J^{(5)}_b \cdot \varepsilon_6 \]

\[ \sum M^{(5)}_i = 0 \]
(22)
\[ M'_i - R_0^a \cdot (y_0 - y_0) + R_0^a \cdot (x_0 - x_0) = 0 \]
\[ M_1^a - M_2^a \cdot (x_0 - y_0) \]
\[ \sum M^{(5)}_i = 0 \]
(23)
\[ R_0^a = \frac{R_0^a ((y_0 - y_0) - M_2^a) (x_0 - x_0)}{(x_0 - x_0)} \Rightarrow Y_{65} = -Y_{65} = -R_0^a \]
(24)

\[ \sum F^{(5)}_i = 0 \Rightarrow R_0^b = X_65 = -(R_0^a + F_{t_1}^\beta) \Rightarrow X_{75} = -X_{75} \]
(25)
\[ \sum F^{(5)}_i = 0 \Rightarrow R_0^b = Y_65 = -(R_0^a + F_{t_2}^\beta) \Rightarrow Y_{75} = -Y_{75} \]
(26)
\[ \sum F^{(5)}_i = 0 \Rightarrow Y_{65} = -(Y_{65} + Y_{75}) = R_x^a + R_y^a \]
(28)
\[ X_{65} = -R_x^a = -X_{65} = -(R_x^a + R_y^a) \]
(29)
\[ Y_{65} = -R_y^a = -Y_{65} = -(R_x^a + R_y^a) \]
(30)

\[ M'_i = -J^{(5)}_b \cdot \varepsilon_6 \]

**Determination of the Reactions in the Kinematic Torques of the Motor Element 8 and Calculation of the Driving Force Fm8**

Figure 8 shows all the forces acting on the linear motor element 8, in the rotation torque G (between elements 8 and 7) and in the translation torque T8 (between elements 8 and 0) materialized by the guideline between the motor piston 8 and its axis of vertical symmetry coinciding with the guide 0, considering as the point of actuation of the forces 08 the center of mass T8. The forces in the torque are the x-axis and y-axis projections of the already known R78 reaction (thus shown in dark blue). Also known the torsion of the inertial forces on element 8, represented here only by an inertial force along the guide axis y (its action being concentrated in the center of mass T8), there is no movement on the x-axis acceleration and automatic acceleration and force inertial on this x-axis is canceled and the inertial moment is also canceled permanently because there is no rotational motion, the angular and automatic acceleration and the inertial moment being canceled.

The driving force that moves the linear motor element 8 also acts in the center of mass. Practically except for the reaction in coupling G all other forces act on the center of mass T8. Relationships can be written (33-36):
\[ F^{(1)}_{y1} = -m_1 \cdot y_1 \]  
\[ \sum F^{(3)}_y = 0 \Rightarrow X_{3_0} + N_{3_0} = 0 \Rightarrow N_{3_0} = -X_{3_0} \Rightarrow N_{3_0} = R^{(3)}_t \]  
\[ \sum F^{(3)}_x = 0 \Rightarrow F_{ax} + Y_{3_0} + F_{ax}^* = 0 \Rightarrow F_{ax} = -Y_{3_0} - F_{ax}^* \Rightarrow F_{ax} = R^{(3)}_t - F_{ax}^* \]  
\[ \sum M^{(3)}_{y_1} = 0 \Rightarrow M_{ax} - X_{3_0} \cdot (y_1 - y_0) = 0 \Rightarrow M_{ax} = R^{(3)}_t \cdot (y_1 - y_0) \]

It is specified here that if the points G and T8 coincide the moment M08 is canceled together with the phase shift \( \left( (y_1 - y_0) = 0 \right) \).

The procedure is then repeated for engine 1 (Fig. 9, relations 37-40).

**Determination of the reactions in the kinematical torques of the motor element 1 and calculation of the driving force \( F_{m1} \)**

\[ F^{(0)}_{y_1} = -m_1 \cdot y_1 \]  
\[ \sum F^{(1)}_y = 0 \Rightarrow X_{3_1} + N_{3_1} = 0 \Rightarrow N_{3_1} = -X_{3_1} \Rightarrow N_{3_1} = R^{(1)}_t \]  
\[ \sum F^{(1)}_x = 0 \Rightarrow F_{ax} + Y_{3_1} + F_{ax}^* = 0 \Rightarrow F_{ax} = -Y_{3_1} - F_{ax}^* \Rightarrow F_{ax} = R^{(1)}_t - F_{ax}^* \]  
\[ \sum M^{(1)}_{y_1} = 0 \Rightarrow M_{ax} - X_{3_1} \cdot (y_1 - y_0) = 0 \Rightarrow M_{ax} = R^{(1)}_t \cdot (y_1 - y_0) \]  

It is specified that if points A and T1 coincide the moment M01 is canceled together with the phase shift \( \left( (y_1 - y_0) = 0 \right) \).

**Remarks:** Any torque introduces a reaction that decomposes along the coordinate axes (in the plane) into two components along the x and y axes, while each translation torque introduces a reaction perpendicular to the torque guide axis and a moment.

Any reaction in any pair is easily determined by having the modulus (size) given by the radical in the sum of the squares of the two scalar components of the reaction and its position (the direction of the vector defining it) is given by an alpha angle measured from the horizontal which passes through the origin of the reaction (the respective coupling) and which has the trigonometric scalar and the vector of the respective reaction.

**Determination of Robot Speeds and Accelerations**

The kinematic calculation of the robot's speeds and accelerations is done only by direct kinematics as it is operated in reality, while the positions can be determined in two distinct situations, by direct kinematics when we are interested in the normal operation of the robot, finding the workspace, and the trajectories described by the effector element (or other component kinematic couplings), or by using inverse kinematics when the positions that the final element (effector) must occupy successively are already imposed and the successive positions of the driving elements must be determined, for this robot the linear motors 1 and 8.

**Determination of Robot Speeds and Accelerations to the Dyad 5,7**

As stated, only direct kinematics is used to determine speeds and accelerations, so the calculations from dyad 5.7 are started (Fig. 10).

Write the calculation relationships in the system (41):

The scalar coordinates, velocities and accelerations of points G and O are known, with the help of which, using the equations of the two circles formed, the scalar coordinates of point F are determined. Then easily determine the angles FI5 and FI7 with their derivatives, \( \dot{\varphi}_5, \dot{\varphi}_7, \dot{\varphi}_6, \dot{\varphi}_8 \).

\[ \begin{align*}
(t_x - t_x) \cdot (y_x - y_x) &= t_x \cdot 2(t_x - t_x) \cdot (y_x - y_x) + 2(y_x - y_x) \cdot (y_x - y_x) = 0 \\
(t_x - t_x) \cdot (y_x - y_x) &= f \cdot x \cdot y_x = 0 \Rightarrow 2x \cdot j_x + 2y \cdot j_x \Rightarrow j_x = -\frac{t_x y_x}{j_y} \\
\dot{x}_y &= \left( \frac{t_x y_x}{j_y} \right) \cdot j_y - \frac{t_x y_x}{j_y} \Rightarrow \dot{x}_y = \frac{t_x y_x}{j_y} - \frac{t_x y_x}{j_y} \\
\dot{y}_x &= \frac{-t_x y_x}{j_y} \cdot \dot{y}_x - \frac{t_x y_x}{j_y} \Rightarrow \dot{y}_x = \frac{-t_x y_x}{j_y} - \frac{t_x y_x}{j_y} \\
\ddot{x}_y &= \left( \frac{t_x y_x}{j_y} - \frac{t_x y_x}{j_y} \right) \cdot j_y - \frac{t_x y_x}{j_y} \Rightarrow \ddot{x}_y = \frac{t_x y_x}{j_y} - \frac{t_x y_x}{j_y} \\
\ddot{y}_x &= \frac{-t_x y_x}{j_y} \cdot \ddot{y}_x - \frac{t_x y_x}{j_y} \Rightarrow \ddot{y}_x = \frac{-t_x y_x}{j_y} - \frac{t_x y_x}{j_y}
\end{align*} \]

**Determination of Speeds and Accelerations in the Triad 2,3,4,6**

Figure 11 you can see the positions with the sizes characteristic of triad 2,3,4,6 starting from which the relations of positions, speeds and accelerations are written.

Position relations being considered already solved and all known position values (solved separately by direct or inverse kinematics as required), derived directly twice and thus obtaining triad speeds and accelerations (2,3,4,6), Eq. (42-52).
(42) 
\[ x_e = d \cos \phi_e, \quad y_e = -d \sin \phi_e \]
\[ x_p = x_e - a \cos \phi_e, \quad y_p = y_e - a \sin \phi_e \]
\[ x_2 = y_p + b \cos \phi_e, \quad y_2 = y_p + b \sin \phi_e \]
\[ x_3 = x_2, \quad y_3 = y_2 \]
\[ x_{cs} = y_3 + b \cos \phi_e, \quad y_{cs} = y_3 + b \sin \phi_e \]
\[ x_{ycs} = y_{cs} + b \cos \phi_e, \quad y_{ycs} = y_{cs} + b \sin \phi_e \]
\[ x_5 = x_{ycs} + b \cos \phi_e, \quad y_5 = y_{ycs} + b \sin \phi_e \]
\[ x_6 = x_5 - a \cos \phi_e, \quad y_6 = y_5 - a \sin \phi_e \]
\[ x_7 = x_6 + b \cos \phi_e, \quad y_7 = y_6 + b \sin \phi_e \]
\[ x_8 = x_7 + b \cos \phi_e, \quad y_8 = y_7 + b \sin \phi_e \]
\[ x_9 = x_8 - a \cos \phi_e, \quad y_9 = y_8 - a \sin \phi_e \]

(43) 
\[ -d \sin \phi_e \quad a_1 = x_e + a \sin \phi_e \quad a_2 = b \sin \phi_e \quad \cos \phi_e \]
\[ d \cos \phi_e \quad a_3 = y_e - a \cos \phi_e \quad a_4 = b \cos \phi_e \quad \sin \phi_e \]
\[ x_t = x_e + a \sin \phi_e \quad y_t = y_e - a \cos \phi_e \quad c \quad \cos \phi_e \]
\[ \xi_t = x_t - x_e - a \cos \phi_e \quad \eta_t = y_t - y_e + a \sin \phi_e \quad \cos \phi_e \]
\[ (I): \quad \xi_t \cos \phi_e + \eta_t \sin \phi_e + a_3 \sin \phi_e \quad + \cos \phi_e \quad a_4 \cos \phi_e \]
\[ (II): \quad \xi_t \cos \phi_e + \eta_t \sin \phi_e + a_3 \sin \phi_e \quad + \cos \phi_e \quad a_4 \cos \phi_e \]

(44) 
\[ \xi_t \cos \phi_e + \eta_t \sin \phi_e + a_3 \sin \phi_e \quad + \cos \phi_e \quad a_4 \cos \phi_e \]
\[ (I): \quad \xi_t \cos \phi_e + \eta_t \sin \phi_e + a_3 \sin \phi_e \quad + \cos \phi_e \quad a_4 \cos \phi_e \]
\[ (II): \quad \xi_t \cos \phi_e + \eta_t \sin \phi_e + a_3 \sin \phi_e \quad + \cos \phi_e \quad a_4 \cos \phi_e \]

(45) 
\[ \xi_t \cos \phi_e + \eta_t \sin \phi_e + a_3 \sin \phi_e \quad + \cos \phi_e \quad a_4 \cos \phi_e \]
\[ (I): \quad \xi_t \cos \phi_e + \eta_t \sin \phi_e + a_3 \sin \phi_e \quad + \cos \phi_e \quad a_4 \cos \phi_e \]
\[ (II): \quad \xi_t \cos \phi_e + \eta_t \sin \phi_e + a_3 \sin \phi_e \quad + \cos \phi_e \quad a_4 \cos \phi_e \]

(46) 
\[ \xi_t \cos \phi_e + \eta_t \sin \phi_e + a_3 \sin \phi_e \quad + \cos \phi_e \quad a_4 \cos \phi_e \]
\[ (I): \quad \xi_t \cos \phi_e + \eta_t \sin \phi_e + a_3 \sin \phi_e \quad + \cos \phi_e \quad a_4 \cos \phi_e \]
\[ (II): \quad \xi_t \cos \phi_e + \eta_t \sin \phi_e + a_3 \sin \phi_e \quad + \cos \phi_e \quad a_4 \cos \phi_e \]

(47) 
\[ \xi_t \cos \phi_e + \eta_t \sin \phi_e + a_3 \sin \phi_e \quad + \cos \phi_e \quad a_4 \cos \phi_e \]
\[ (I): \quad \xi_t \cos \phi_e + \eta_t \sin \phi_e + a_3 \sin \phi_e \quad + \cos \phi_e \quad a_4 \cos \phi_e \]
\[ (II): \quad \xi_t \cos \phi_e + \eta_t \sin \phi_e + a_3 \sin \phi_e \quad + \cos \phi_e \quad a_4 \cos \phi_e \]

(48) 
\[ \xi_t \cos \phi_e + \eta_t \sin \phi_e + a_3 \sin \phi_e \quad + \cos \phi_e \quad a_4 \cos \phi_e \]
\[ (I): \quad \xi_t \cos \phi_e + \eta_t \sin \phi_e + a_3 \sin \phi_e \quad + \cos \phi_e \quad a_4 \cos \phi_e \]
\[ (II): \quad \xi_t \cos \phi_e + \eta_t \sin \phi_e + a_3 \sin \phi_e \quad + \cos \phi_e \quad a_4 \cos \phi_e \]

(49) 
\[ \vec{x}_e = -d \cos \phi_e \quad \alpha'_e \quad - d \sin \phi_e \quad \alpha_e \]
\[ \vec{y}_e = -d \sin \phi_e \quad \alpha'_e \quad + d \cos \phi_e \quad \alpha_e \]

Fig. 7: Forces of the dyad 5-7

Fig. 8: Forces acting on the engine element 8
Results and Discussion

Table 1 gives the input data, more precisely the known lengths of the mechanism (in the calculation program used these lengths represent the constant geometric parameters).

The point T located on the effector 6 (Fig. 1, 4-5) describes a rectangular trajectory (Fig. 12). Its characteristics are shown in Table 2.

The trajectory of the point T in Fig. 12 is described by the relationships in Table 3.

The coordinates represent the input parameters for the algorithm of the inverse positional model in Table 3.

Going through the connection of the modular groups for the inverse structural model (Fig. 2b, 3) the algorithm presented in Table 2-3 allows the successive calculation of the dependent parameters (Fig. 4), as follows:

- for the dyad RRR (5,6) - Φ5k(XTk, YTk), Φ6k(XTk, YTk) can be seen in Fig. 13 [deg], as Φ50k(XTk, YTk), Φ60k(XTk, YTk)
- for the dyad RRR (3,4) – Φ3k(XTk, YTk), Φ4k(XTk, YTk) can be seen in the Fig. 14 [deg], as Φ30k(XTk, YTk), Φ40k(XTk, YTk)
- for dyad RRT (1,2) – Yαk(XTk, YTk) and Φ2k(XTk, YTk) seen in Fig. 15, where
  - Φ2k(XTk, YTk) in [deg] is Φ20k(XTk, YTk)
- for dyad RRT (8,7) – YGk(XTk, YTk) and Φ7k(XTk, YTk) seen in Fig. 16, where
  - Φ7k(XTk, YTk) in [deg] is Φ70k(XTk, YTk)

It is considered a single external force (technological resistance) RTk that acts on the system neglecting other external forces (for example - gravitational forces) and the system of inertial forces. This simplification brings some peculiarities in the form of terms from the calculation algorithm without restricting its generality.

The external force RTk (Fig. 17) is considered constant on the initial and horizontal portion of the trajectory of the point T (Fig. 12) and is described by the relation (53):

\[ RT_k \text{ if } k \leq 10,20,0 \]

Using the connection of the modular groups for the direct structural model (Fig. 3) the passive module GMP2 (2,3,4,6), a 6R triad (Fig. 5, 6, 18) is analyzed in a first stage, for which elaborated algorithm, relations (1-20).

Applying the calculation algorithm (1-20) for the GMP2 triad (2,3,4,6) is determined reaction torsion components, as follows:

- In the kinematic torque of E → X56k, Y56k from Fig. 19
- In kinematic rotation couple from the point A → X12k, Y12k from Fig. 20
- In the kinematic rotation couple from the point B → X23 k = -X32 k, Y23 k = -Y32 k
- In the kinematic rotation couple from the point C → X43 k = -X34 k, Y43 k = -Y34 k
- In the kinematic rotation couple from the point D → X63 k = -X36 k, Y63 k = -Y36 k
- In the kinematic rotation couple from the point O → X04 k, Y04 k from Fig. 21

Fig. 12: The trajectory of the T-point, the end effector

Fig. 13: Variation of angles FI5 and FI6 considered in [deg] depending on the independent parameter k

Fig. 14: Variation of angles FI3 and FI4 considered in [deg] depending on the independent parameter k

Fig. 15: The variation of the parameter YA and the angle FI2 considered in [deg] depending on the independent parameter k
Fig. 16: Variation of parameter YG and angle FI7 considered in [deg] depending on the independent parameter k

Fig. 17: The external force RTk is considered constant on the initial and horizontal portion of the trajectory of the point T

Fig. 18: Passive module GMP2(2,3,4,6), the triad 6R

Fig. 19: Reaction torque in the kinematic rotation coupling of E → X56 k, Y56 k on the GMP 2 modular group (2,3,4,6), triad type 6 R
DOI: 10.3844/ajeassp.2022.59.80

This bimobile 2T9R mechanism (Fig. 1) can be used by the simultaneous action of active translation torques in A and G point T having a chosen trajectory and law of motion. If one of these active couplings is locked, the mechanism remains with only one degree of mobility. The connections of the modular groups are given in both cases: respectively, for G blocked and for A blocked in Fig. 28 a, b.

Applying the calculation modules, it is possible to study the behavior of the mechanism with a degree of mobility in the mentioned situations. Thus, if the active coupling G is blocked, the variation of the dependent parameters of the resulting mechanism is studied, with a degree of mobility (Fig. 29) for the extreme blocking positions Φ50 minimum and Φ50 maximum.

Fig. 20: Reaction torque in the kinematic torque of A → X12 k, Y12 k on the GMP 2 modular group (2,3,4,6), 6 R triad type

The next module in the modular group connection of the direct structural model (Fig. 7) is GMP1 (7.5) shown in Fig. 22 a, b, an RRR dyad for which the kinetostatic model is rendered by the relations (21-32).

In this calculation stage it is determined:

- In the kinematic torque from E → X87k, Y87k from Fig. 23
- In the kinematic rotation couple from the point O → X05k, Y05k from Fig. 24

In the following steps, the initial active modular groups GMAI (G, 8) and GMAI (A, 1) shown in Fig. 25 a, b.

The components (NO8k, T08k) of the active translation coupling G are shown in Fig. 26 and for the active coupling of A (NO1k, T01k) in Fig. 27.

Fig. 21: Reaction torsion in the kinematic torque of O → X04 k, Y04 k on the GMP 2 modular group (2,3,4,6), triad type 6R
Fig. 22: Reaction torsor on the GMP1 dyad modular group (7.5)

Fig. 23: Reaction torsor in kinematic coupling E, \( \to \) X87 k, Y87 k, on the GMP 1 dyad modular group (7.5)

Fig. 24: Reaction torsor in kinematic coupling O, \( \to \) X05 k, Y05 k, from the GMP 1 dyad modular group (7.5)

Fig. 25: The reaction torsor of the initial active modular groups GMAI (G, 8) a and GMAI (A, 1) b
Fig. 26: Reaction torsor from the initial active modular group GMAI (G, 8)

Fig. 27: Reaction torsor from the initial active modular group GMAI (A, 1)

Fig. 28: The connections of the modular groups for the two distinct situations when G is blocked and the case when A is blocked, respectively

Fig. 29: The case in which the active coupling G is blocked when studying the variation of the dependent parameters of the resulting mechanism, with a degree of mobility for the extreme locking positions $\Phi_{50}$ minimum and $\Phi_{50}$ maximum

Table 1: Constant geometric parameters

<table>
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<th>XA</th>
<th>ET</th>
<th>XG</th>
<th>OF</th>
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<th>FG</th>
<th>CD</th>
<th>TD</th>
<th>OE</th>
<th>BD</th>
<th>OC</th>
<th>BC</th>
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<td>0.9</td>
<td>0.88</td>
<td>0.7</td>
<td>0.45</td>
<td>0.18</td>
<td>0.45</td>
</tr>
</tbody>
</table>

Table 2: Initial parameters of the T point trajectory

Initial parameters of the T point $T_0 (1.5, -0.9)$
The step of moving the T point horizontally $v = -0.05$
The step of moving the T point vertically $v_1 = 0.05$

Table 3: The input parameters

Points $T$ $X_{Tk} = \begin{cases} \text{if } k \leq 10, X_{T0} + kv, & \text{if } 10 < k \leq 15, X_{T0} + 10v, \\ \text{if } 15 < k \leq 25, X_{T0} + 10v - (k-15)v, & X_{T0} \end{cases}$

$Y_{Tk} = \begin{cases} \text{if } k \leq 10, Y_{T0}, & \text{if } 10 < k \leq 15, Y_{T0} + (k-10)v, \\ \text{if } 15 < k \leq 25, Y_{T0} + 5v, & Y_{T0} + 5v - (k-25)v \end{cases}$
Conclusion

The kinematic and kinetostatic modeling of a 2T9R robotic mechanism is generally quite difficult and lucrative, but it has the advantages of obtaining a well-developed theoretical model that can be used in practice to design or use such robots, extremely interesting and useful, which have increased maneuverability, a large workspace, a correct and fast dynamics of movement, without vibrations or noises, the mechatronic module presented can be designed and built in various ways depending on the requirements and objectives of the workplace in which it will be implemented.

The paper presented the inverse and direct kinematic models, the kinetostatic (forces) model that is always studied inversely, together with the related calculation relations.

In the results and discussions section, the diagrams obtained by calculation using the Math Cad 2000 program were actually presented.

Acknowledgement

This text was acknowledged and appreciated by Dr. Veturia CHIROIU Honorific member of Technical Sciences Academy of Romania (ASTR) Ph.D. supervisor in Mechanical Engineering.

Funding Information

Research contract: Contract number 36-5-4D/1986 from 24IV1985, beneficiary CNST RO (Romanian National Center for Science and Technology) Improving dynamic mechanisms internal combustion engines.

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Author’s Contributions

All the authors contributed equally to prepare, develop and carry out this manuscript.

Ethics

This article is original and contains unpublished material. Author declares that are not ethical issues and no conflict of interest that may arise after the publication of this manuscript.

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