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Comparison Between Niemann's and Finite Element Method for the Estimation of Maximum Allowable Stress of Meshing Spur Gear Teeth at Highest Point of Single Tooth Contact

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ABSTRACT

Purpose of this study is the consideration of loading and contact problems encountered at rotating machine elements and especially at toothed gears. The later are some of the most commonly used mechanical components for rotary motion and power transmission. This fact proves the necessity for improved reliability and enhanced service life, which require precise and clear knowledge of the stress field at gear tooth. This study investigates the maximum allowable stresses occurring during spur gear tooth meshing computed using Niemann's formulas at Highest Point of Single Tooth Contact (HPSTC). Gear material, module, power rating and number of teeth are considered as variable parameters. Furthermore, the maximum allowable stresses for maximum power transmission conditions are considered keeping the other parameters constant. After the application of Niemann's formulas to both loading cases, the derived results are compared to the respective estimations of Finite Element Method (FEM) using ANSYS software. Comparison of the results derived from Niemann's formulas and FEM show that deviations between the two methods are kept at low level for both loading cases independently of the applied power (either random or maximum) and the respective tangential load.

Keywords: Highest Point of Single Tooth Contact (HPSTC), Finite Element Method (FEM)

1. INTRODUCTION

First systematic attempt to calculate the position of critically stressed point is attributed to Lewis (1882), who considered that the inscribed isosceles parabola tangent to the dedendum of the tooth flank defines the critically stressed point which is located at the point of tangency at the side which is loaded by tensile stresses.

The "30 degrees tangent" is another method which argues that the critically stressed point is independent of the load location. Instead, it is located at a specific point at the tooth root. Although this method is adopted the ISO standards, (Kawalec *et al*., 2006), it is approximate and applicable only to low stressed gears.

Methods, such as AGMA standard and DIN (Kawalec *et al*., 2006), Heywood's semi-empirical method (Heywood, 1962) and Dolan-Broghamer's (Dolan and Broghamer,1942) empirical formula, can be found at references and are recommended for the determination of the precise stress level caused by the phenomenon of stress concentration at gear tooth root

According to method proposed by DIN 3990 1987 and ISO 6336 1996, standards the bending stresses calculation at gear tooth root is based on the concept of "30 degrees tangent" (Heywood, 1962), which proves to be a disadvantage. Thus, this method is quite approximate and should not be applied to the design of high loaded gearings.

Heywood's method (Heywood, 1962), is applied for the determination of maximum real stress at critically stressed point at the root of a stubby beam with constant width. This method was later modified in order to make more precise prediction of the critical point which is located at a lower position from then on.

2. MATERIALS AND METHODS

The previously mentioned methods of maximum stress calculation at gear tooth root will not be applied at the present study, because the determination of the necessary geometric features, especially at the critical region of gear tooth fillet which requires more precise computations, is a time-consuming procedure. Instead, the applied method assumes that the maximum load during gear tooth meshing is applied to the Highest Point of Single Tooth Contact (HPSTC), (Spitas *et al*., 2005).

2.1. Gear-Tooth Strength at the Critically Stressed Point of Root

According to gearing theory, the total load PN applied to the gear tooth along contact path of the transverse tooth section can be seen at **Fig. 1** and the tangential component of gear tooth load at pitch point C can be found using the following relation Eq. 1:

$$P_u = P_n \cdot \cos\beta_o = P_N \cdot \cos\alpha_{on} \cdot \cos\beta_o \qquad (1)$$

Assuming that gear tooth is a stubby cantilever beam fixed at dedendum circle, it can be easily seen that gear tooth loading is maximum when P_N is applied at the addendum circle, as seen at **Fig. 1.**

When applied at the addendum circle, normal load P_N does not attain its maximum value. If we shift P_N to point M, which is located on the axis of symmetry of the gear tooth, it can be resolved giving a tangential component Eq. 2:

$$P_u = P_N \cdot \cos\alpha' \qquad (2)$$

and a radial component Eq. 3:

$$P_R = P_N \cdot \sin\alpha' \qquad (3)$$

At the critically stressed point, PU causes the following bending moment Eq. 4:

$$M_B = P_N \cdot e_f \cdot \cos\alpha' \qquad (4)$$

Resulting in a bending stress of Eq. 5:

$$\sigma_b = \pm\frac{M_B}{W_B} = \pm\frac{P_N \cdot e_f \cdot \cos\alpha'}{b_{o\delta} \cdot s_f^2 / 6} \qquad (5)$$

and a shear stress τ_Δ (positioned at the critical point boδ ef) Eq. 6:

$$\tau_\Delta = \frac{P_N \cdot \cos\alpha'}{b_{o\delta} \cdot s_f} \qquad (6)$$

while P_R causes a compressive stress Eq. 7:

$$\sigma_d = -\frac{P_N \cdot \sin\alpha'}{b_{o\delta} \cdot s_f} \qquad (7)$$

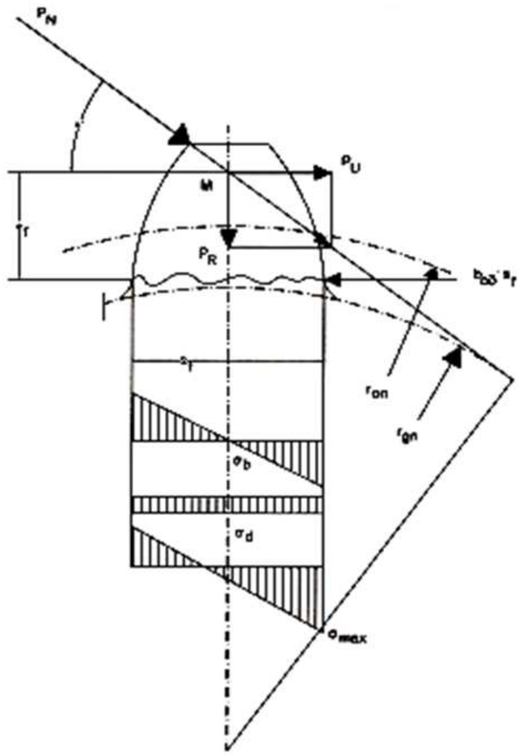


Fig. 1. Gear tooth loading

Shear stress, compared to the bending and compressive stresses which are collinear and normal to the shear stress, is small enough and can be neglected. The maximum normal stress is the compressive one with a value of Eq. 8:

$$\sigma_{max} = \sigma_b + \sigma_d \qquad (8)$$

Or Eq. 9:

$$\sigma_{max} = \frac{6 \cdot P_N \cdot ef \cdot \cos a'}{b_{o\delta} \cdot s_f^2} + \frac{P_N \cdot \sin a'}{b_{o\delta} \cdot s_f} \qquad (9)$$

As gear thickness b is related to gear tooth length $b_{o\delta}$ with the following relation Eq. 10:

$$b = b_{o\delta} \cdot \cos\beta_o \qquad (10)$$

And according to the previous discussion, we can obtain the following result Eq. 11:

$$\sigma_{max} = \frac{P_u \cdot q_k}{b \cdot m} \qquad (11)$$

Where Eq. 12:

$$q_k = \frac{m_n \cdot (6 \cdot e_f \cdot \cos\alpha' / s_f + \sin\alpha')}{s_f \cdot \cos\alpha_{on}} \quad (12)$$

Coefficient q_k depends on the number of pinion teeth and rack cutter shift during pinion generation.

In order to take into account the meshing of more than one tooth pairs when the load is applied at the addendum circle, we introduce the overlap coefficient or contact ratio for the transverse tooth section at Eq. 11. Thus Eq. 13:

$$\sigma_{max} = \frac{P_u \cdot q_k}{b \cdot \varepsilon \cdot m} \le \sigma_{\varepsilon\pi} \quad (13)$$

where, $\sigma_{\varepsilon\pi}$ is the allowable stress for the critical point, which depends on gear material, tangential velocity at pitch point and gear meshing mode. In more detail.

For a tangential velocity at pitch point lower than or equal to 5 m sec^{-1}, we have Eq. 14:

$$\sigma_{\varepsilon\pi} = \frac{\sigma_B}{2 \cdots 3} \quad (14)$$

For a tangential velocity at pitch point greater than 5 m/sec and a single trend of rotation, we have Eq. 15:

$$\sigma_{\varepsilon\pi} = \frac{\sigma_{bw}}{1{,}5....2} \quad (15)$$

Where:
σ_B = The tensile strength of gear material
σ_{bw} = The bending strength of gear material

2.2. Highest Point of Single Tooth Contact (HPSTC) During Tooth Meshing

It is proven that the normal load P_N on a gear tooth is not maximum when applied at the addendum circle. As shown in **Fig. 2** during gear tooth meshing, from point A where tooth contact begins to point A´ of tooth contact path and from point B´ to point B, where tooth contact completes, two pairs of teeth are in contact simultaneously. On the other hand, between points A´ and B´ only a single tooth pair is in contact subjected to the total load.

It can, thus, be assumed that the worst loading condition for a tooth of gear 1 does not occur when the load is applied to the highest addendum point (point B), because the total load is distributed at two pairs of gear teeth at this point, but at point B´ of contact path where only a single pair of gear teeth is meshing (Niemann, 1982; Spitas *et al*., 2005).

Point A´ is defined The Lowest Point of Single Tooth Contact (LPSTC) and point B´ is the Highest Point of Single Tooth Contact (HPSTC) for gear 1.

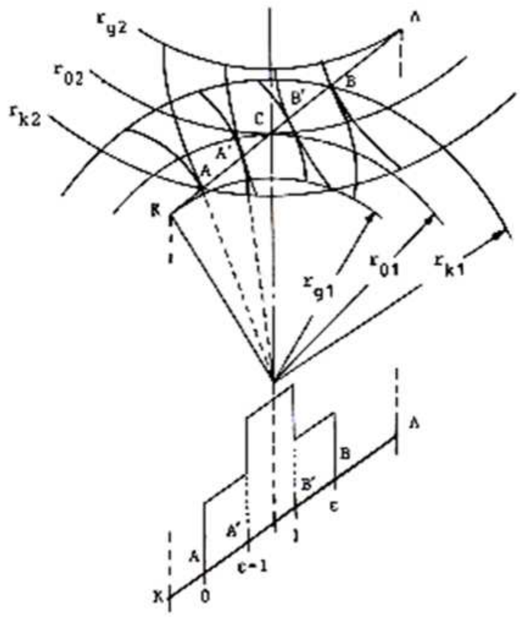


Fig. 2. (a) Meshing teeth profiles of a gear transmission stage (b) Positions of tooth load variation

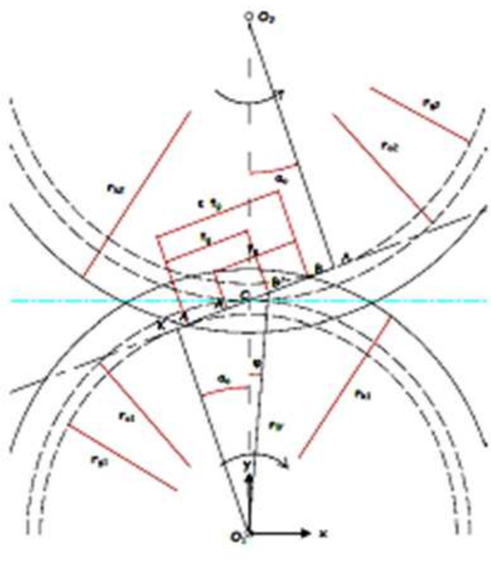


Fig. 3. Geometric determination of HPSTC

That is, during portion A´B´ of the contact path only a single tooth of each gear is loaded, whereas during portions AA´ and BB´ the load is distributed to two teeth of each gear. Thus, we can infer that the maximum gear tooth loading occurs at a point on part A´B´ of the contact path (Spitas *et al*., 2005).

Determination of the point of maximum stresses during gear meshing (**Fig. 3)** is as follows:

$$AB = \varepsilon \cdot t_g = AC + CB \tag{16}$$

$$AC = \sqrt{\left(r_{o2} + m\right)^2 - r_{02}^2 \cdot \cos^2\alpha_o} - r_{02} \cdot \sin\alpha_0 \tag{17}$$

$$BC = \sqrt{\left(r_{01} + m\right)^2 - r_{01}^2 \cdot \cos^2\alpha_0} - r_{01} \cdot \sin\alpha_0 \tag{18}$$

Substituting Eq. (17) and (18) to Eq. (16) results Eq. 19:

$$AB = \sqrt{\left(r_{o2} + m\right)^2 - r_{02}^2 \cdot \cos^2\alpha_o} + \sqrt{\left(r_{01} + m\right)^2 - r_{01}^2 \cdot \cos^2\alpha_0} - \left(r_{01} + r_{02}\right) \cdot \sin\alpha_0 \tag{19}$$

HPSTC is located at point B´. During parts AA´ and B´B of the contact path, load is transmitted through two pairs of gear teeth, while during part A´B´ only a single pair of gear teeth is subjected to the total load. The lengths of parts AB´ and A´B equal the gear circular pitch, tg, at the base circle. Thus, position of HSPTC is determined according to **Fig. 3** as follows Eq. 20 and 21:

$$AC = \sqrt{\left(r_{02} + m\right)^2 - r_{02}^2 \cdot \cos^2\alpha_0} - r_{02} \cdot \sin\alpha_0 \tag{20}$$

$$CB' = AB' - AC = t_g - AC = \pi \cdot m \cdot \cos\alpha_0 - AC \tag{21}$$

Using triangle O1B´C, radius rB´ can be calculated according to the following Eq. 22:

$$r_{B'} = \sqrt{r_{01}^2 + CB'^2 - 2 \cdot r_{01} \cdot CB' \cdot \cos\left(\alpha_0 + 90^0\right)} \tag{22}$$

Cartesian coordinates of point H are: (x, y) = ($r_{B´}$ sinφ, $r_{B´}$ cosφ).

2.2. Application of Niemann's Formulas

Next we will investigate the deviation between results of the referenced maximum stress formulas and computerized results of the finite element method. For this purpose, tangential load P_u is applied to the highest point of single tooth contact during gear tooth meshing.

Ten different gear materials with increasing quality are assumed, starting from GS 52 and resulting to 42 Cr V6. Number of pinion teeth is constant $z_1 = 18$, while ten sets of parameter are considered:

Number of gear teeth: z_2	=	20, 50, 80 and 100
Module: m	=	1, 2, 3, 4, 5 and 6 mm
Tooth length: b_1	=	10, 20, 30, 40 and 50 mm
Input power: P_W	=	10, 18, 27, 34, 42, 53, 58, 63, 65 and 70 kW

The previous sets of parameter values are identical for each type of gear material.

Next, maximum stress for each set is calculated. **Table 1-3** show the detailed results for three types of gear materials, while **Table 4** gathers the results derived for all gear materials.

For the determination of tangential load Pu applied to the highest point of single tooth contact during gear tooth meshing, the above parameters (m, b_1, z_1, z_2, P_w) are set to MITCalc software for each gear material and the value of load are computed automatically. This value is identical for each gear material assuming a common parameter set.

Using Eq. (13), (14) and (15), we determine the allowable and the maximum stress, depending on the ultimate tensile strength of each material. It was found that the maximum stress is constant for a common set of parameters and independent of the value of maximum allowable stress, which generally is different for each material.

The maximum allowable stress increases with material quality. Thus, the values of maximum stresses reach the values of the maximum allowable stress.

Using, then, MITCalc software, we import pinion geometry of each material to Solid Works software. Next, the solid model is imported to ANSYS FEM software and the maximum stress at gear tooth root is computed using the finite element method.

Table 1. Deviation between Niemann's method results and FEM (ANSYS) results assuming GS-52 gear material and random transmitted power

Gear material	Module (mm)	Z_2	Pw (kW)	P_U (Nt)	$\sigma_{\varepsilon\pi}$ (Nt/mm^2)	σ_{max} (Nt/mm^2)	ANSYS (Nt/mm^2)	%
GS 52	1	30	10	10.611,11	200	1.885,79	1.854,00	1.72
	2	50	18	9.500,00	200	432,81	425,454	1.73
	2	80	27	14.325,00	200	412,70	408,265	1.09
	3	100	34	12.025,93	200	199,69	202,509	-1.39
	3	100	42	14.855,56	200	173,05	175,304	-1.28
	4	80	53	14.059,72	200	149,31	147,224	1.41
	5	30	58	12.308,89	200	211,89	209,550	1.12
	5	50	63	13.370,00	200	440,41	433,936	1.49
	6	80	65	11.495,37	173,33	157,28	159,777	-1.56
	6	100	70	12.379,63	173,33	83,37	82,198	1.43

Table 2. Deviation between Niemann's method results and FEM (ANSYS) results assuming Gk-60 gear material and random transmitted power

Gear material	Module (mm)	Z_2	Pw (KW)	P_U (Nt)	$\sigma_{\varepsilon\pi}$ (Nt/mm^2)	σ_{max} (Nt/mm^2)	ANSYS (Nt/mm^2)	%
Ck 60	1	30	10	10.611,11	296,00	1.885,79	1.854,00	1.72
	2	50	18	9.500,00	296,00	432,81	425,454	1.73
	2	80	27	14.325,00	296,00	412,70	408,265	1.09
	3	100	34	12.025,93	296,00	199,69	202,509	-1.39
	3	100	42	14.855,56	296,00	173,05	175,304	-1.28
	4	80	53	14.059,72	296,00	149,31	147,224	1.41
	5	30	58	12.308,89	296,00	211,89	209,550	1.12
	5	50	63	13.370,00	296,00	440,41	433,936	1.49
	6	80	65	11.495,37	293,33	157,28	159,777	-1.56
	6	100	70	12.379,63	293,33	83,37	82,198	1.43

Table 3. Deviation between Niemann's method results and FEM (ANSYS) results assuming 35CrMo4 gear material and random transmitted power

Gear material	Module A(mm)	Z_2	Pw (kW)	P_U (Nt)	$\sigma_{\varepsilon\pi}$ (Nt/mm^2)	σ_{max} (Nt/mm^2)	ANSYS (Nt/mm^2)	%
35 CrMo4	1	30	10	10.611,11	352,00	1.885,79	1.854,00	1.72
	2	50	18	9.500,00	352,00	432,81	425,454	1.73
	2	80	27	14.325,00	352,00	412,70	408,265	1.09
	3	100	34	12.025,93	352,00	199,69	202,509	-1.39
	3	100	42	14.855,56	352,00	173,05	175,304	-1.28
	4	80	53	14.059,72	352,00	149,31	147,224	1.41
	5	30	58	12.308,89	352,00	211,89	209,550	1.12
	5	50	63	13.370,00	352,00	440,41	433,936	1.49
	6	80	65	11.495,37	456,66	157,28	159,777	-1.56
	6	100	70	12.379,63	456,66	83,37	82,198	1.43

Table 4. Review of deviation between Niemann's and FEM results for ten different gear materials and random power rating

Gear material	Module (mm)	Z_2	Pw (kW)	P_U (Nt)	$\sigma_{\varepsilon\pi}$ (Nt/mm^2)	σ_{max} (Nt/mm^2)	ANSYS (Nt/mm^2)	%
GS 52	1	30	10	10.611,11	200	1.885,79	1.854,00	1.72
St 70	2	50	18	9.500,00	274.4	432,81	425,454	1.73
36 Mn5	2	80	27	14.325,00	280,00	412,70	408,265	1.09
Ck 60	3	100	34	12.025,93	296,00	199,69	202,509	-1.39
37 Cr 4	3	100	42	14.855,56	314,00	173,05	175,304	-1.28
42 Mnv7	4	80	53	14.059,72	320,00	149,31	147,224	1.41
35 CrMo4	5	30	58	12.308,89	352,00	211,89	209,550	1.12
31 NiCr14	5	50	63	13.370,00	372,00	440,41	433,936	1.49
34CrNiMo6	6	80	65	11.495,37	500,00	157,28	159,777	-1.56
42CrV6	6	100	70	12.379,63	566,66	83,37	82,198	1.43

Examples of the derived results using FEM are shown in **Fig. 4-7**, regarding GG 52, Ck 60, 36Mn 5 and 35GrMo4 gear materials respectively.

Comparison of the results between the applied methods (Niemann's formulas and FEM) shows a deviation from-1.56 % to+1.73%. This % deviations are common for each material assuming a constant of parameter values, as can be seen at **Table 1-4** in detail.

Figure 8 shows a diagram of maximum stress versus gear tooth length resulting from the concentrated results of **Table 4**.

Then, using trial and error method and MITCal software we find for each material the maximum allowable power Pmax, such that the maximum stress equals allowable stress. **Figure 9-12** are some examples of finite element analyses assuming GG 52, St 70, Ck 60 and 5CrMo4 gear materials, respectively.

Application of the former procedure shows that deviation between the results of maximum stress using Niemann's formulas (Niemann, 1982) and FEM (using ANSYS) lie in the range from 1.56% to + 1.73%. This percentages are identical for each material with a common set of parameters as can be seen at **Table 5-8**.

Using the results of **Table 8**, we can create a diagram of maximum allowable stress versus maximum rated power, as shown in **Fig. 13.**

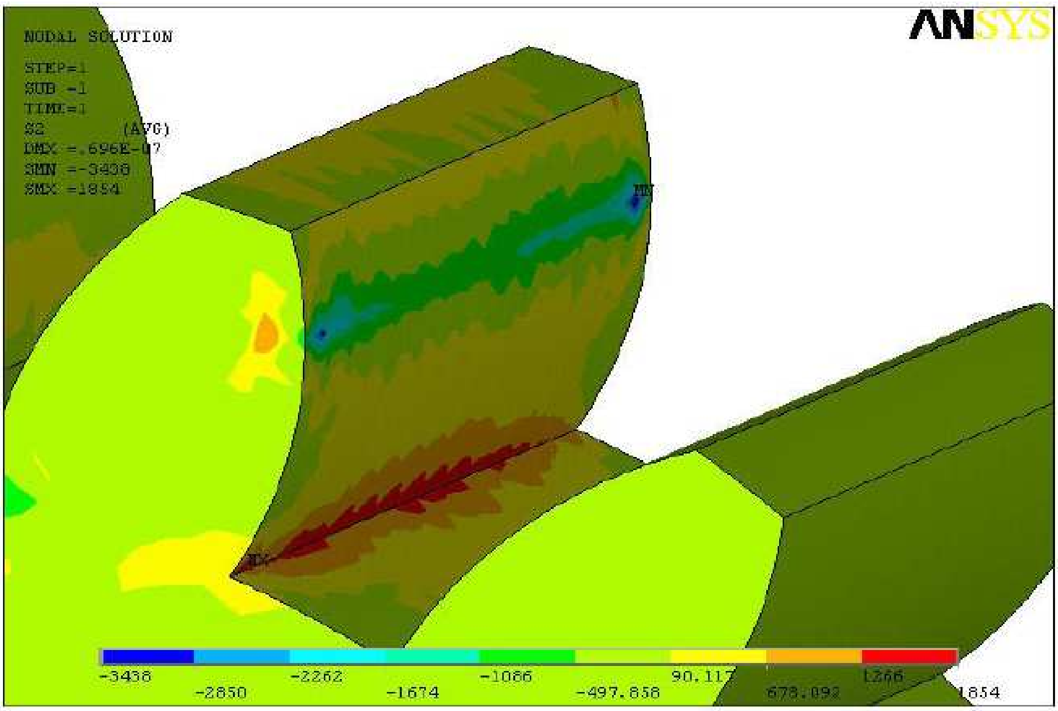


Fig. 4. Gear material: GG52, m = 1, b_1 = 10, z_2 = 30, Pw = 10KW, σ_{max} = 1885,79 N/mm^2

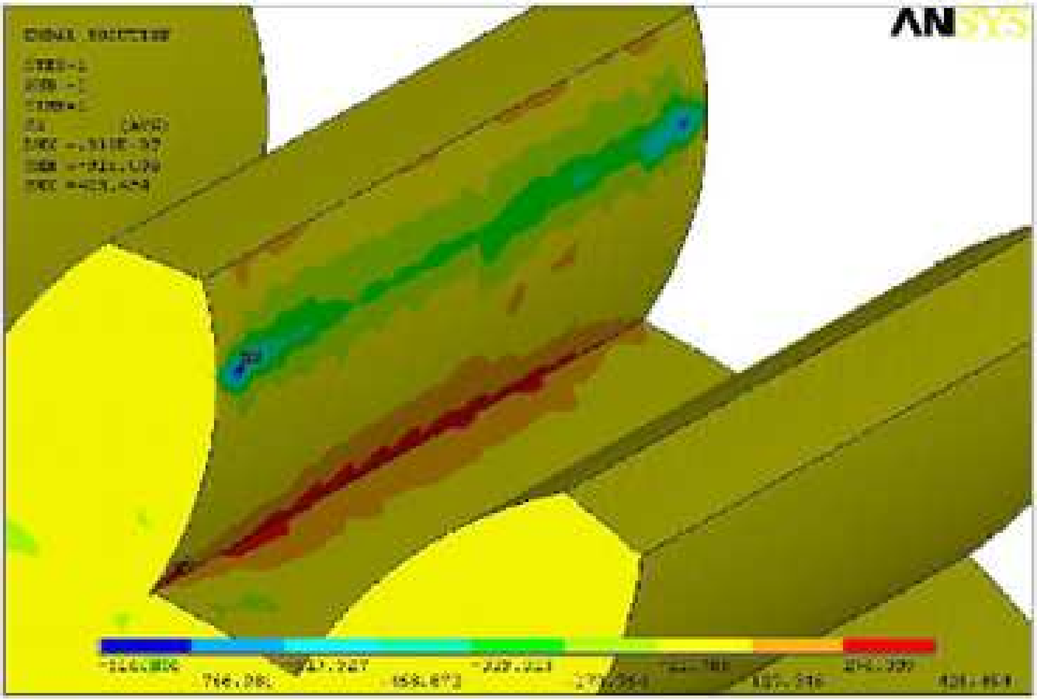


Fig. 5. Gear material: Ck 60, m = 2, b_1=20, z_2 = 50, P_w = 18KW, σ_{max} = 432,81 N/mm^2

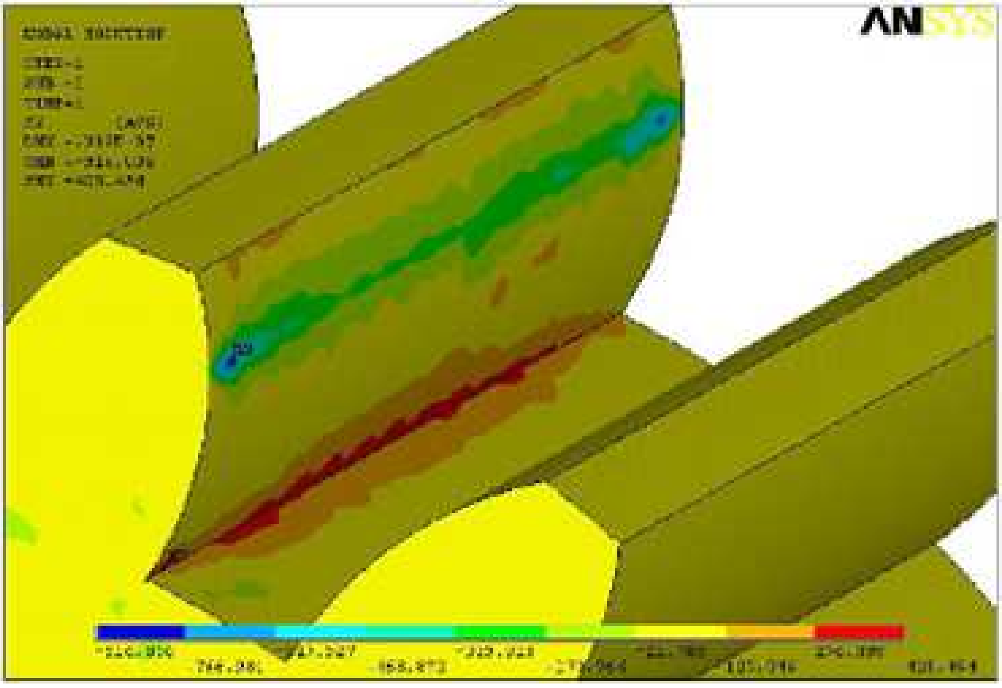


Fig. 6. Gear material: 36Mn 5, m = 2 mm , b_1 = 30 mm, z_2 = 80 και P_w = 27 KW

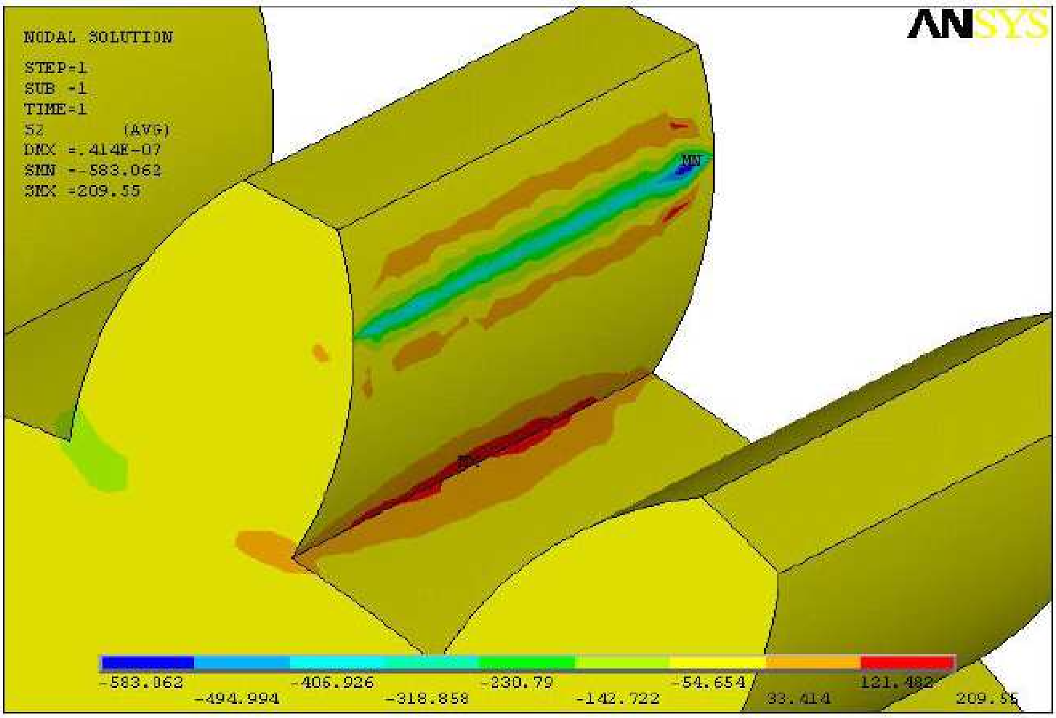
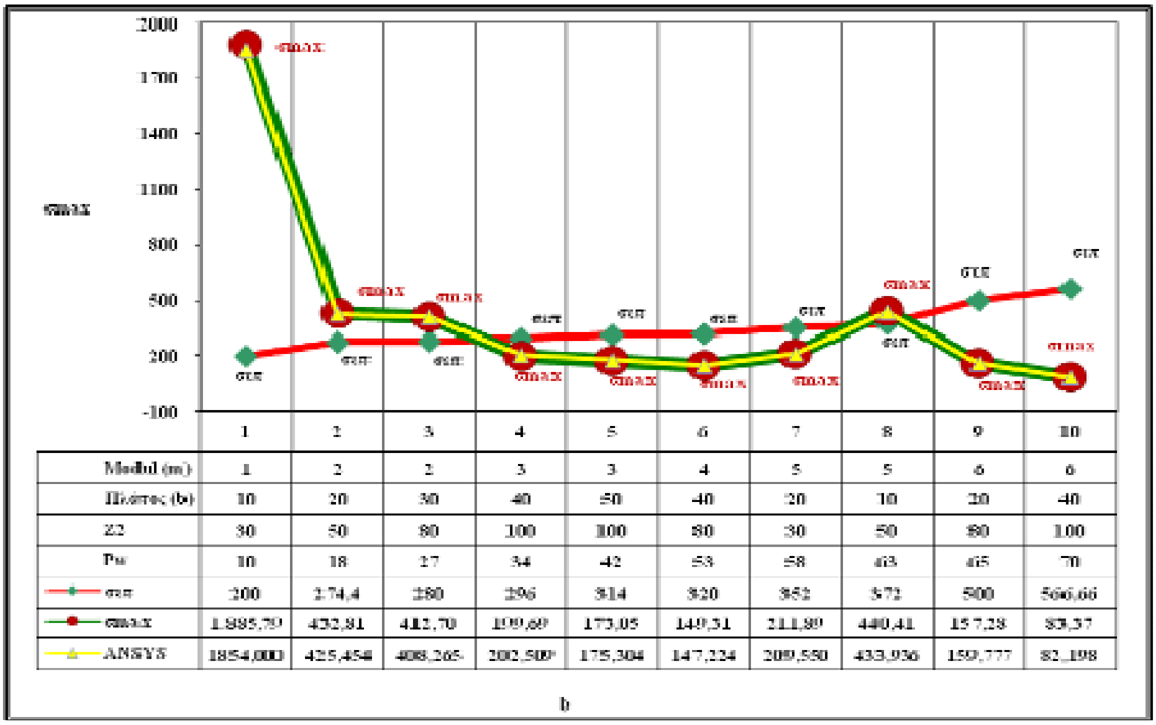


Fig. 7. Gear material: 35CrMo4, m = 5 mm , b1 = 20 mm, z2 = 30 και Pw = 58 KW



	1	2	3	4	5	6	7	8	9	10
Modul (m)	1	2	2	3	3	4	5	5	6	6
Πλάτος (b)	10	20	30	40	50	40	20	10	20	40
z2	30	50	80	100	100	80	30	50	80	100
Pw	10	18	27	34	42	53	58	63	65	70
σεπ	200	274,4	280	296	314	320	352	372	500	566,66
σmax	1.885,79	432,81	412,70	199,69	173,05	149,31	211,89	440,41	157,28	83,37
ANSYS	1854,000	425,454	408,265	202,509	175,304	147,224	209,550	433,936	159,777	82,198

Fig. 8. Diagram of maximum stress (σ_{max}) versus gear tooth length (b)

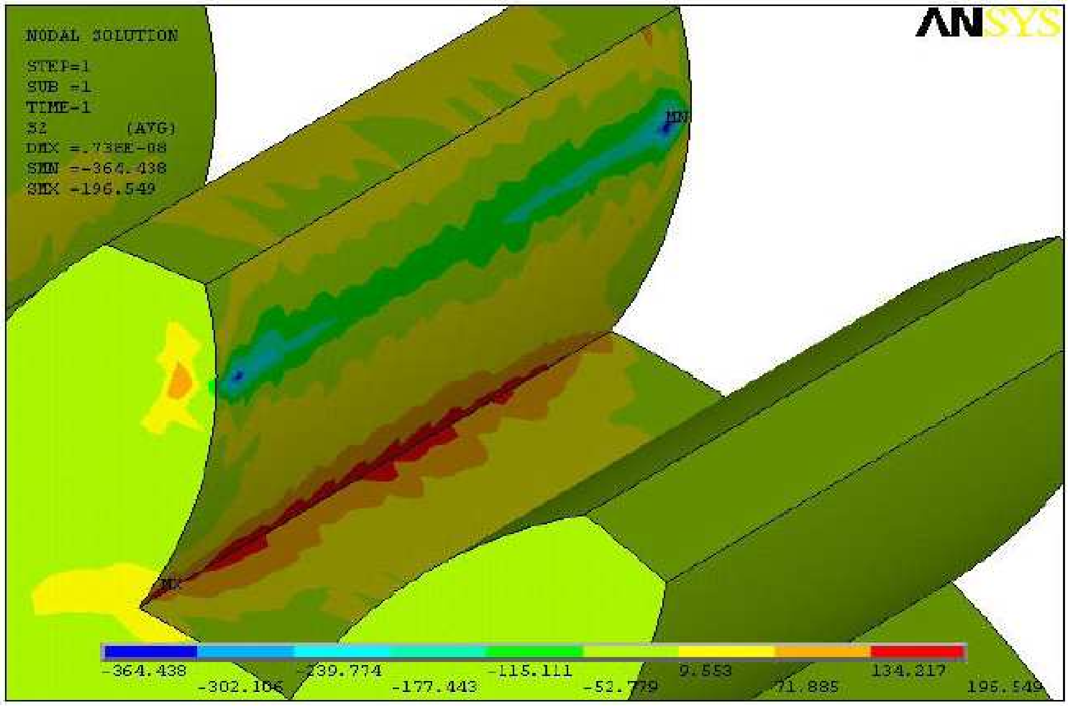


Fig. 9. Gear material: GS 52, m = 1, b_1 = 10, z_2 = 30, P_{wmax}= 0.983kW, σ_{max}= 199,92 N/mm^2

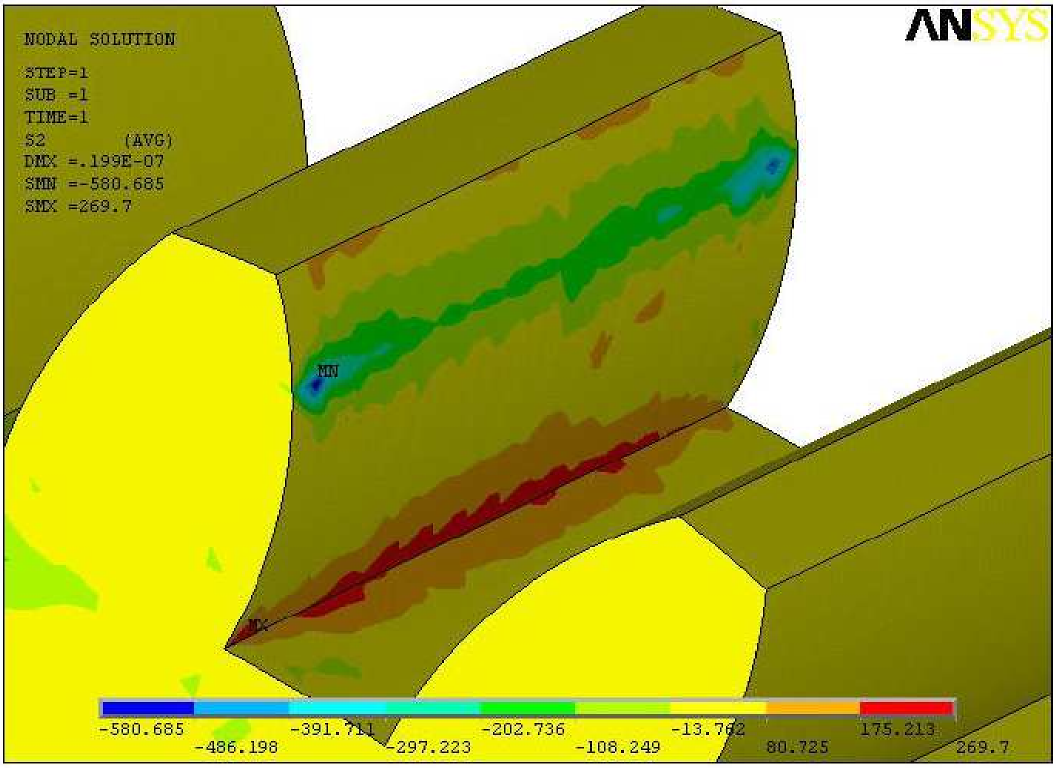


Fig. 10. Gear material: St70, m = 2, $b_1 = 20$, $z_2 = 50$, $P_{wmax} = 11.138$kW, $\sigma_{max} = 274,38$N/mm^2

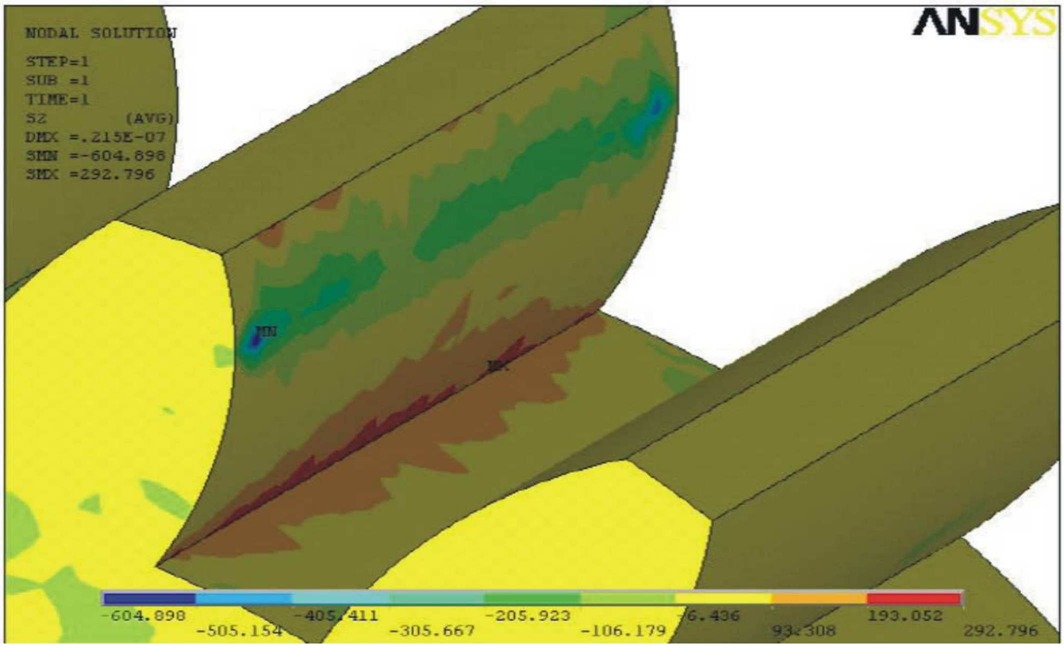


Fig. 11. Gear material: Ck 60, m = 2 mm, $b_1 = 30$ mm, $z_2 = 80$ and $P_{wmax} = 18.414$ kW

3. RESULTS

Consideration of the diagram shown in **Fig. 8** as well as **Table 1-4** derives the following conclusions:

- Maximum stress (σ_{max}) decreases as module (m) increases
- Maximum stress (σ_{max}) decreases as gear thickness (b) increases and vice versa
- Tangential load (P_u) decreases as module (m) increases, whereas as module (m) decreases tangential load (P_u) increases

Consideration of the diagram shown in **Fig. 13** as well as **Table 5-8** derives the following conclusions:

- Increase of both pinion thickness (b) and module (m) causes an increase of maximum rated power (P_{wmax})
- Maximum rated power (P_{wmax}) decreases as pinion thickness (b) decreases, regardless of the module (m) increase
- Maximum rated power (P_{wmax}) is different for each material and increases with enhanced material quality due to an increase in the maximum allowable stress ($\sigma_{\varepsilon\pi}$)

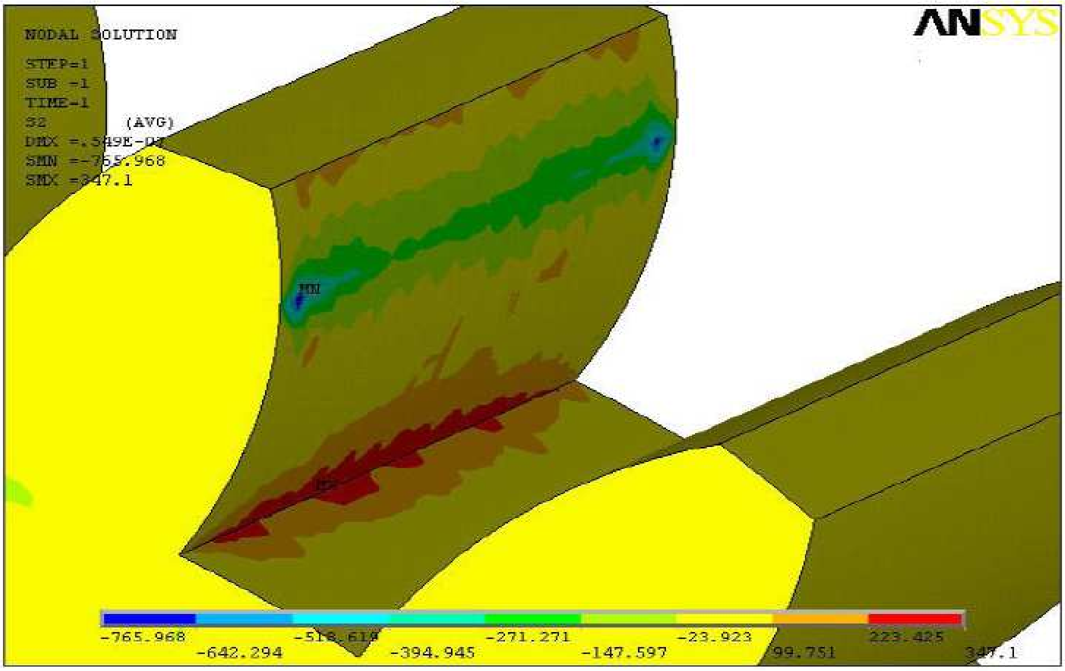


Fig. 12. Gear material: 35CrMo4, m = 4mm, b_1 = 40mm, z_2=80 and P_{wmax} = 116.789 kW

Table 5. Deviation between Niemann's method results and FEM (ANSYS) results assuming GS-52 gear material and maximum rated power

Gear material	Module (mm)	Z_2	$P_{w\,max}$ (kW)	P_U (N)	$\sigma_{\varepsilon\pi}$ (N/mm^2)	σ_{max} (N/mm^2)	ANSYS (N/mm^2)	%
GS 52	1	30	0,983	1.043,28	200	199,92	196,549	1.72
	2	50	8,118	4.307,05	200	199,98	196,592	1.73
	2	80	12,442	6.601,17	200	200,00	197,840	1.09
	3	100	37,621	13.306,69	200	200,00	202,817	-1.39
	3	100	47,027	16.633,62	200	200,00	202,600	-1.28
	4	80	66,357	17.603,04	200	200,00	197,212	1.41
	5	30	49,178	10.436,66	200	200,00	197,775	1.12
	5	50	25,370	5.384,08	200	199,99	197,054	1.49
	6	80	64,697	11.441,78	173,33	173,33	176,083	-1.56
	6	100	130,420	23.065,02	173,33	173,33	170,891	1.43

Table 6. Deviation between Niemann's method results and FEM (ASNYS) results assuming Ck-60 gear material and maximum rated power

Gear material	Module (mm)	Z_2	$P_{w\,max}$ (kW)	P_U (N)	$\sigma_{\varepsilon\pi}$ (N/mm^2)	σ_{max} (N/mm^2)	ANSYS (N/mm^2)	%
Ck 60	1	30	1,455	1.543,92	296,00	295,86	290,860	1.72
	2	50	12,015	6.374,63	296,00	295,98	290,951	1.73
	2	80	18,414	9.769,65	296,00	296,00	292,796	1.09
	3	100	55,681	19.694,58	296,00	296,00	300,175	-1.39
	3	100	69,600	24.617,78	296,00	296,00	299,835	-1.28
	4	80	98,209	26.052,67	296,00	296,00	291,885	1.41
	5	30	72,784	15.446,38	296,00	296,00	292,722	1.12
	5	50	37,549	7.968,73	296,00	296,00	291,655	1.49
	6	80	109,489	19.363,33	293,33	293,33	297,980	-1.56
	6	100	220,713	39.033,50	293,33	293,33	289,195	1.43

Table 7. Deviation between Niemann's method results and FEM (ASNYS) results assuming 35CrMo4 gear material and maximum rated power

Gear material	Module (mm)	Z_2	$P_{w\,max}$ (kW)	P_U (N)	$\sigma_{\varepsilon\pi}$ (N/mm^2)	σ_{max} (N/mm^2)	ANSYS (N/mm^2)	%
35 CrMo4	1	30	1,731	1.836,78	352,00	351,98	346,032	1.72
	2	50	14,289	7.581,11	352,00	352,00	346,017	1.73
	2	80	21,898	11.618,11	352,00	352,00	348,206	1.09
	3	100	66,215	23.420,49	352,00	352,00	356,964	-1.39
	3	100	82,769	29.275,70	352,00	352,00	356,567	-1.28
	4	80	116,789	30.981,53	352,00	352,00	347,100	1.41
	5	30	86,554	18.368,68	352,00	352,00	348,102	1.12
	5	50	44,653	9.476,36	352,00	352,00	346,834	1.49
	6	80	170,454	30.145,11	456,66	456,66	463,899	-1.56
	6	100	343,609	60.767,89	456,66	456,66	450,222	1.43

Table 8. Review of deviations between Niemann's and FEM results for ten different gear materials and maximum rated power

Gear material	Module (mm)	Z_2	$P_{w\,max}$ (kW)	P_U (N)	$\sigma_{\varepsilon\pi}$ (N/mm^2)	σ_{max} (N/mm^2)	ANSYS (N/mm^2)	%
GS 52	1	30	0,983	1.043,28	200	199,92	196,549	1.72
St 70	2	50	11,138	5.909,33	274.4	274,38	269,700	1.73
36 Mn5	2	80	17,419	9.241,75	280,00	280,00	276,984	1.09
Ck 60	3	100	55,681	19.694,58	296,00	296,00	300,175	-1.39
37 Cr 4	3	100	73,833	26.115,01	314,00	314,00	318,071	-1.28
42 Mnv7	4	80	106,172	28.165,07	320,00	320,00	315,551	1.41
35 CrMo4	5	30	86,554	18.368,68	352,00	352,00	348,102	1.12
31 NiCr14	5	50	47,190	10.014,77	372,00	372,00	366,540	1.49
34CrNiMo6	6	80	186,630	33.005,86	500,00	500,00	507,923	-1.56
42CrV6	6	100	426,377	75.405,56	566,66	566,66	558,671	1.43

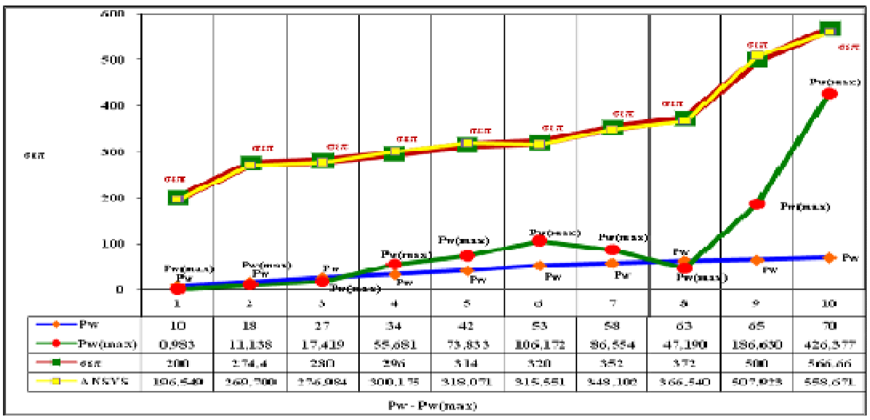


Fig. 13. Diagram of maximum allowable stress ($\sigma_{\varepsilon\pi}$) versus power (P_w) and maximum rated power (P_{wmax})

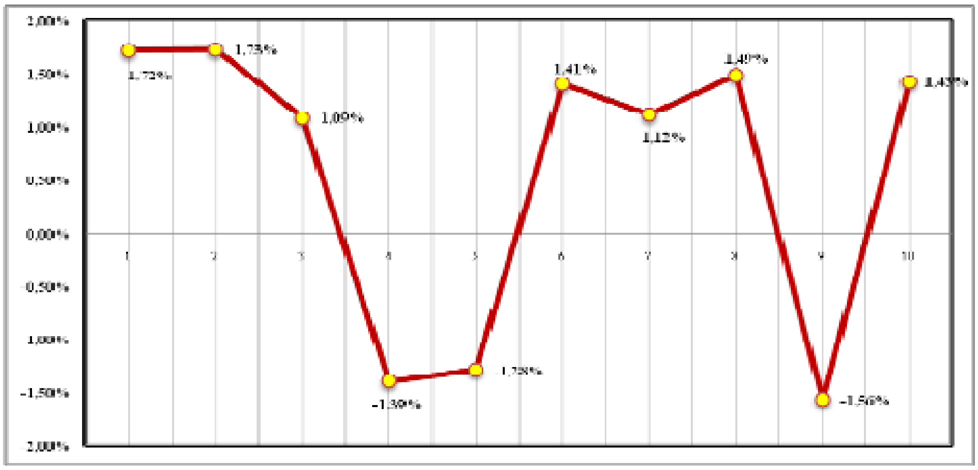


Fig. 14. Concentrating diagram of maximum stress (σ_{max}) deviation between Niemann's estimations and finite element results

From the previous considerations it was derived that deviations between the results of maximum allowable stresses (σ_{max}) calculated using theoretical methods and the Finite Element Method (FEM) with application of ANSYS software are commonly ranged between-1.56% and+.73%, regardless of the transmitted power (random or maximum) and the respective tangential load, as shown in **Table 1-6** as well as in concentrating **Table 4-8 and Fig. 14.**

4. DISCUSSION

Apart from the fatigue failure at the critically stressed point, small cracks at tooth surface have been observed at gear pitch circle (where the gear tooth is subjected to the total load), due to high pressure surface fatigue. Low viscosity lubricants can enter the cracks at high pressures. These initially small cracks can easily grow under the effect of high pressures of penetrating fluids, causing surface fatigue cracks or pitting. Therefore, it is critical to take into account the surface fatigue strength of gear during the design procedure.

5. CONCLUSION

This study investigated the minimum deviation of maximum allowable stress estimations at highest point of single tooth contact of meshing spur gear teeth using Niemann's formulas and finite element method with application of ANSYS software. Number of gear teeth, module and transmitted power were considered as variable parameters. For both loading cases (random and maximum transmitted power), common sets of other parameters (m, z_1, z_2, material) derived identical deviations.

After comparison of the derived results using the discussed methods for both cases, it was concluded that deviations are acceptable, which is reasonable considering the potential errors that can be involved during the procedure.

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