Mathematical Derivation of MIMO Based MANET to Improve the Network Performance

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Abstract: The proliferations of research activities in wireless communication stimulated researchers to develop a new technology and a technique to enable efficient and purposeful application in mobile ad hoc network. This new technique was developed by invention of MIMO technology and its tremendous potential in wireless network. The advantage of MIMO technology is it’s high data rate and improved transmission efficiency in ad hoc network, wireless communication, IEEE 802.11n (Wi-Fi) and IEEE 802.16e (WiMAX) standards. Mobile ad hoc network (MANET) has several significant applications in infrastructure-less wireless network. MIMO technology has played a key role to improve the transmission efficiency in MANET. Mobile ad hoc network (MANET) and Multiple Input Multiple Output (MIMO) communication both are very emerging in modern wireless communication system. Main focus of this research work is to improve the spectral efficiency of the MANET. MANET and MIMO communication are studied separately to analyze the performance of these networks. Analysis of this work can further help to implement an integrated network or MIMO based mobile ad hoc network. And finally analysis the performance of the integrated network (MIMO-MANET) to improve the spectral efficiency.

Keyword: Mobile Ad Hoc Networks, Multi Input Multi Output (MIMO), Channel Capacity, Diversity, Space Time Block Code, Spatial Multiplexing

Introduction

Ad hoc wireless networking is an efficient technique to operate in wireless communication system which can serve anywhere and anytime communication by incorporating routing functionalities into a mobile host. This new paradigm of wireless networking is designed by some portable devices to establish infrastructure less network at any time and dismantle at the end. This area of communication and computing is very challenging for computer and communication engineering. This designing method which can represent wireless ad hoc network is called MANET (Paulraj et al., 2004; Basagni et al., 2003; Chlamtac et al., 2003). A serious problem in MANET is to have congestion free transmission in large size networks, which calls for a system with improved spectral efficiency. Several classical efforts were made to improve the spectral efficiency without much impact. Multiple antennas between the nodes to establish a communication links that can improve the transmission quality of wireless communication systems in a significant level without using any extra operational frequency bandwidth (Ali et al., 2010; Akyildiz et al., 2009; Wu et al., 2013a). Because of this quality the Multiple Input Multiple Output (MIMO) is envisaged for the next generation mobile communication systems (Chen, 2006; Papadakis, 2011). Multi Input and Multi Output (MIMO) technology are integrated with ad hoc nodes at the transmitting and receiving ends of the communication system to improve the spectral efficiency (Fakih et al., 2009; Wu et al., 2012a; 2013b). Multi Input Multi Output (MIMO) ad hoc network systems are the promising techniques for improving the data rates particularly in frequency selective fading environments. Utilization of MIMO antenna system can improve the channel capacity (Wu et al., 2012b; Shin and Lee, 2004; Teltar, 1995). In this study we present a mathematical modelling of the MIMO integrated mobile ad hoc network and analyze the performance of this integrated network. Mobile ad hoc networks play an important role in military and civilian application. So, to achieve high channel capacity for next generation wireless technologies MIMO integrated mobile ad hoc network is really a promising approach. MIMO ad hoc networks
present an interesting and important role in future application and expected to be implemented in modern communication system because of its higher spectral efficiency, higher transmission rate and increasing information efficiency. The performance analysis of these networks needs to be addressed properly and become an important area on research nowadays. The nodes of MIMO ad hoc networks are cooperative in nature and communicate to each other through multipath routing and multihop communication rely on implicit trust among themselves. The lack of fixed infrastructure and central concentration (base station) make it difficult to apply the different control mechanisms that are used in mobile communication. Design of efficient and reliable channel mechanism and quality of service provisioning is a challenging issue to improve the spectral efficiency of the integrated network.

The paper has been organized as follows. In section 2, MIMO integrated mobile ad hoc network model is described. Architectural model for a single node is also described to design the node with MIMO channel. Section 3 introduces diversity and multiplexing technique for the designing aspect of the network. The channel capacity or performance of the MIMO integrated network channel is analyzed in section 4. Simulated result and discussion is given in section 5 followed by a conclusion in section 6.

**MIMO Ad Hoc Network Model**

The channel model of MIMO integrated mobile ad hoc network is shown in Fig. 1 considering all the ad hoc nodes consist of \( N_t \) number of transmitting and \( N_r \) number of receiving antennas.

Then the input-output relation of the network can be written as:

\[
Y = HX + W
\]  

(1)

Where:

\[
Y = \begin{bmatrix}
y_1 \\
y_2 \\
\vdots \\
y_{N_r}
\end{bmatrix} \quad \text{and} \quad X = \begin{bmatrix}
x_1 \\
x_2 \\
\vdots \\
x_{N_t}
\end{bmatrix}
\]

and the element \( H \) is a \( N_r \times N_t \) matrix that have a complex Gaussian distribution. \( W \) is the additive white Gaussian noise vector with zero mean and \( C_w \) is covariance matrix covariance. The block diagram of a MIMO integrated mobile ad hoc node is shown in Fig. 2.

In Fig. 2 all the ad hoc nodes in the network consist of multiple inputs multiple output antenna system. Signal transmitted from transmitted nodes reaches to the receiving nodes by MIMO channel. There are two aspect of designing of MIMO ad hoc network one is to improve the reliability of the system to transmit same data across the different propagation (spatial) paths. This is called spatial diversity or simply diversity. And second one is to improve the data rate of the network by placing various parts of the transmitting data on different propagation paths (spatial-multiplexing). Spatial diversity and spatial multiplexing scheme is used to improve reliability and transmission rate of multiple-antenna channels. Multiplexing and diversity technique of MIMO channel is shown in Fig. 3.

The first part of above figure describe how to provide the replicas of the transmitted signal at the receiver consuming minimum power, bandwidth, decoding complexity and other resources. And the second part of the figure is shows how the replica of the transmitted signal is sent to the receiver with minimum probability of error. The main objective is to send two or more copies of the signal through independent fades. Error-correcting codes are used here to reduce the amount of redundancy (Wang and Giannakis, 2004; Kettani and Zaharov, 2010; Blum, 2003). The transmitted signal from mobile ad hoc networks propagate through multiple paths and arrive at the receiver from different directions with different propagation delays in each node, due to use of multiple antennas. The signal received by any receiving node consists of multiple numbers of signals having randomly distributed amplitude, phase and angle of arrival. As a result the signal received by the receiving node is distorted or faded. To mitigate fading effect of the signal different diversity techniques are used. To obtain diversity, the signal is transmitted through multiple (ideally) independent fading paths e.g., in time, frequency or space and combined constructively at the receiver. Multiple-Input-Multiple-Output (MIMO) system accomplishes spatial diversity by having several transmitting and receiving antennas. Diversity is a bandwidth efficient method to mitigate the fading effect (Wam and Dubey, 2000; Merzakreeva et al., 2012; Rihawi and Louet, 2007). This technique uses multiple channels to improve the signal to noise ratio in the presence of fading. Not only using multiple antennas diversity can also be achieved by coding the signal over both space and time (or frequency). STBC and are the technique that combines coding, modulation and signal processing to achieve transmission diversity. STBC scheme supports linear decoding complexity for Maximum Likelihood (ML) decoding. STBC is used for two transmitting antenna where as Orthogonal Space Time Block Code (O-STBC) is a generalized form of arbitrary number of transmitting antennas. It maintains the equity of linear Maximum Likelihood decoding with full transmission diversity. O-STBC, QO-STBC (Quasi Orthogonal Space time Block Code) achieve full transmission diversity due to low decoding complexity and higher code rate.
Fig. 1. MIMO ad hoc network model

Fig. 2. Block Diagram of MIMO based Mobile ad hoc node

Fig. 3. MIMO with diversity and multiplexing
Space Time Block Code (STBC) Diversity

STBC diversity technique for $2\times 2$ MIMO antenna system is consider in Fig. 4. Space time diversity is done 2 period of time and the decoding will be 2-period of time:

\[
Y_1 = \begin{bmatrix}
    y_{11} \\
    y_{12}
\end{bmatrix} = \begin{bmatrix}
    h_{11} & h_{12} \\
    h_{21} & h_{22}
\end{bmatrix} \begin{bmatrix}
    x_1 \\
    x_2
\end{bmatrix} + \begin{bmatrix}
    n_{11} \\
    n_{12}
\end{bmatrix}
\]

(2)

The above equation is for first time period. Second time period the equation will be:

\[
Y_2 = \begin{bmatrix}
    y_{21} \\
    y_{22}
\end{bmatrix} = \begin{bmatrix}
    h_{11} & h_{12} \\
    h_{21} & h_{22}
\end{bmatrix} \begin{bmatrix}
    -x_1 \\
    -x_2
\end{bmatrix} + \begin{bmatrix}
    n_{21} \\
    n_{22}
\end{bmatrix}
\]

(3)

\[Y_1\] and \(Y_2\) represents received OFDM symbol at first and second time period. Both the equation can be easily combined and arranged to produce result:

\[
Y = \begin{bmatrix}
    y_{11} \\
    y_{12} \\
    y_{21} \\
    y_{22}
\end{bmatrix} = \begin{bmatrix}
    h_{11} & h_{12} & h_{11} & h_{12} \\
    h_{21} & h_{22} & h_{21} & h_{22}
\end{bmatrix} \begin{bmatrix}
    x_1 \\
    x_2 \\
    -x_1 \\
    -x_2
\end{bmatrix} + \begin{bmatrix}
    n_{11} \\
    n_{12} \\
    n_{21} \\
    n_{22}
\end{bmatrix}
\]

(4)

We can isolate \(x_1\) and \(x_2\) by simply multiplying the matrix \(Y\) by inverse of \(H\). Since this matrix is not square, we need to use the Moore-Penrose inverse \(H^+\) to solve our equation:

\[
H^+ = (H^H H)^{-1} H^H
\]

(5)

\[
\begin{bmatrix}
    x_1 \\
    x_2
\end{bmatrix} = (H^H H)^{-1} H^H \begin{bmatrix}
    y_{11} \\
    y_{12} \\
    y_{21} \\
    y_{22}
\end{bmatrix}
\]

(6)

To improve reliability of MIMO channel Space time block coding scheme is implemented. A simplest transmit diversity technique (2*2) is Alamouti STBC coding which is shown in Fig. 5.

For two receive antennas, the received symbols are:

\[
\begin{align*}
    y_{11} &= h_{11}x_1 + h_{12}x_2 + n_{11} \\
    y_{12} &= -h_{11}^*x_1 + h_{12}^*x_2 + n_{12} \\
    y_{21} &= h_{21}x_1 + h_{22}^*x_2 + n_{21} \\
    y_{22} &= -h_{21}^*x_1 + h_{22}x_2 + n_{22}
\end{align*}
\]

(7)

where, \(h_{ij}\) is the path gain between the \(j^{th}\) transmit antenna and \(i^{th}\) receive antenna. If the symbols \(y_{12}\) and \(y_{22}\) from above set of equations are complex conjugated then:

\[
\begin{align*}
    y_{11} &= h_{11}x_1 + h_{12}x_2 + n_{11} \\
    y_{12} &= -h_{11}^*x_1 + h_{12}^*x_2 + n_{12} \\
    y_{21} &= h_{21}x_1 + h_{22}^*x_2 + n_{21} \\
    y_{22} &= -h_{21}^*x_1 + h_{22}x_2 + n_{22}
\end{align*}
\]

(8)

\[
\begin{align*}
    y_{11} &= h_{11}x_1 + h_{12}x_2 + n_{11} \\
    y_{12} &= h_{21}x_1 + h_{22}x_2 + n_{12} \\
    y_{21} &= h_{11}^*x_1 + h_{12}^*x_2 + n_{21} \\
    y_{22} &= h_{21}^*x_1 + h_{22}^*x_2 + n_{22}
\end{align*}
\]

(9)

Which can be rewritten as:

\[
y = H_x x + n
\]

(9i)

At the receiver, Maximal Ratio Combining with Maximal Likelihood decoder technique is used:

\[
\tilde{x} = H^+ y
\]

(10)

Finally, Maximum-Likelihood (ML) decoder received the combined symbols and estimates the transmitted symbols. If in the transmission path is severely faded the ML decoder recovers the transmitted symbols through other propagation paths. Alamouti code is used only for two antennas. For multiple transmit and receive antennas STBC orthogonal transmission matrices can be constructed based on Hurwitz-Radon (HR) theory.

Orthogonal designs of $2\times 2$ antennas as on:

\[
\begin{bmatrix}
    x_1 \\
    x_2
\end{bmatrix} = \begin{bmatrix}
    h_{11} & h_{12} \\
    h_{21} & h_{22}
\end{bmatrix} \begin{bmatrix}
    y_{11} \\
    y_{12} \\
    y_{21} \\
    y_{22}
\end{bmatrix}
\]

(11)

Orthogonal designs of $4\times 4$ antennas as on:

\[
\begin{bmatrix}
    x_1 & x_2 & x_3 & x_4 \\
    -x_2 & x_1 & -x_4 & x_3 \\
    -x_3 & x_4 & x_1 & -x_2 \\
    -x_4 & -x_3 & -x_2 & x_1
\end{bmatrix}
\]

(12)
Fig. 4. STBC for 2×2 MIMO system

Orthogonal designs of 8×8 antennas as on:

\[
\begin{bmatrix}
  x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 \\
-x_2 & x_1 & x_4 & -x_3 & x_6 & -x_7 & x_5 & -x_8 \\
-x_3 & -x_4 & x_1 & x_2 & x_7 & -x_6 & x_8 & x_5 \\
-x_4 & x_3 & x_2 & -x_1 & x_8 & -x_5 & x_6 & x_7 \\
-x_5 & -x_6 & -x_7 & x_8 & x_1 & x_2 & x_3 & x_4 \\
x_6 & -x_5 & x_8 & x_7 & x_4 & -x_3 & x_2 & -x_1 \\
x_7 & -x_8 & x_5 & x_6 & x_3 & x_4 & -x_2 & x_1 \\
x_8 & -x_7 & -x_6 & x_5 & x_4 & x_3 & x_2 & -x_1 \\
\end{bmatrix}
\]

(14)

The code can be noted as in this scheme \( C(n, k, T) \), where \( m \) is number of transmitting antennas, \( k \) symbols and \( T \) is time slot. The code rate is represented as \( \frac{k}{T} \).


Fig. 5. 2×2 MIMO system with Alamouti coding and ML decoding

The transmission matrix of four numbers of transmit antennas for 1/2 rate code \( C(4,4,8) \) is represented as:

\[
\begin{bmatrix}
  x_1 & x_2 & x_3 & x_4 \\
x_5 & x_6 & x_7 & x_8 \\
-x_4 & x_3 & x_2 & x_1 \\
x_8 & x_7 & x_6 & x_5 \\
\end{bmatrix}
\]

(15)

And for 3/4 rate code \( C(4,3,4) \) represented as:

\[
\begin{bmatrix}
  x_1 & x_2 & x_3 & 0 \\
-x_2 & x_1 & x_3 & 0 \\
x_3 & 0 & -x_1 & x_2 \\
x_2 & -x_3 & 0 & -x_1 \\
\end{bmatrix}
\]

(16)

In this way, transmission matrix of 1/2 and 3/4 rate code for 4,8,16,... numbers of transmit antenna can be generated. To obtain the codes for other number of transmit antennas like 3,5,6,7,..., One or more numbers of column can be eliminated from the transmission matrix of previous codes, keeping the column orthogonality remain same. In this way the transmission matrix of any sets of antennas is generated and the after that the codeword are modulated by different modulation technique. OFDM is a modulation as well as multiplexing scheme. To improve transmission rate multiplexing is necessary in designing of MIMO channel. The transmitted signal is divided into some narrow bands sub stream and modulated by PSK, QAM (for example). A comparative study is done for 16-PSK, 32-PSK and 16-QAM, 32-QAM modulation to estimate Bit Error Rate (BER) shown in Fig. 6a and 6b.
It is found that BER can be minimized by using higher order modulation technique. Further reducing of BER in significance level Trellis Coded Modulation (TCM) combines with STBC which is shown in simulated result (Fig. 7).

After Modulation, Multiplexing (MIMO-OFDM) is done. Transmitted data rate can be improved after the implementing multiplexing. The main objective up to this work is to minimize the BER or increase the signal rate. If the transmitted signal rate is increased then channel capacity of the channel will automatically increase, which effect the improvement of spectral efficiency of the network. Signal generated from STBC encoder is modulated and then modulated data transmit via MIMO channel.

### Capacity Analysis

Capacity of the channel is represented by maximum mutual information. The mutual information \( I(X; Y) \) is the amount of uncertainty of difference between the entropies of the channel input and output, where \( X \) and \( Y \) transmitted and receiving signal thus:

\[
I(X;Y) = H(X) - H\left(\frac{X}{Y}\right) = H(Y) - H\left(\frac{Y}{X}\right)
\]

Where:

\[
H(Y/X) = -\sum_{i,j} P(x_i,y_j) \log_2 P\left(\frac{y_j}{x_i}\right)
\]
And:

\[ H(Y) = - \sum_{j=1}^{n} P(y_j) \log_2 P(y_j) \]  \hspace{1cm} (19)

Mean square value of \( X \) is \( S \) and noise is \( N \), then:

\[ Y^2 = S + N \]

\[ H(X,Y) \] is maximum when \( H(Y) \) is maximum:

\[ H_{\text{max}}(Y) = \frac{1}{2} \log_2 \left( 2\pi e (S+N) \right) \]  \hspace{1cm} (20)

Real valued random vector \( X \) consists of \( N \) i.i.D, Gaussian random variable.

Covariance matrix:

\[ C_x = E \left\{ (X - \mu)(X - \mu)^T \right\} \]  \hspace{1cm} (22)

where, \( \mu = 0 \):

\[ H(X) = \frac{1}{2} \left[ N \log_2(2\pi e) + \log \det(C_x) \right] \]  \hspace{1cm} (23)

The mutual information of the channel with Rayleigh fading environment is shown below:

\[ I(X,Y) = H(Y) - H\left( Y \mid X \right) = H(Y) - H\left( H.X + W_X \right) \]  \hspace{1cm} (24)

Putting the value of \( Y \) from equation (1):

Under the condition \( X \) is known \( H_X \) is a constant mean. Hence the conditional entropy:

\[ H\left( Y \mid X \right) = H\left( H.X + W_X \right) = H(W) \]

\[ = H(Y) - \frac{1}{2} \left[ N \log_2(2\pi e) + \log \det(C_x) \right] \]  \hspace{1cm} (25)

Capacity of the channel per vector transmission is:

\[ C = I_{\text{max}}(X,Y) = \frac{1}{2} \left[ N \log_2(2\pi e) + \log \det(C_x) \right] \]

\[ = \frac{1}{2} \left[ \log \det \left( C_x C_x^{-1} \right) \right] \]

\[ = \frac{1}{2} \left[ \log \det \left( H.C_x.H^T + C_x^{-1} \right) \right] \]

\[ = \frac{1}{2} \left[ \log \det \left( I + \frac{X}{N} H.H^T \right) \right] \]

\[ = \frac{1}{2} \log \left[ \text{eye} (N_1, N_1) + \text{SNR} * H^* H^T \right] \]  \hspace{1cm} (27)

where, \( C_x C_x^{-1} = \frac{Y}{N} \); \( \gamma \) is signal to noise ratio.

Simulated Result and Discussion

Channel capacity of the MIMO system with different sets of antennas can be evaluated shown in Fig. 8.

![Fig. 8. Channel capacity at different sets of antennas](image-url)
If we increase the number of antennas it is observed that the channel capacity is increases. MIMO channel capacity can be obtained by spatial multiplexing of transmitted data and space time coding at the transmitter. Finally the spectral efficiency of the network is shown in Fig. 9 by varying the number of antennas.

It is concluded that spectral efficiency cannot be increases indefinitely by increasing the number of antennas. Spectral efficiency is optimized by the number of antennas for a fixed SNR.

**Conclusion**

Design of efficient and reliable channel mechanism and quality of service provisioning is a challenging issue to improve the spectral efficiency of this network. The main aspects of designing a MIMO channel are diversity and multiplexing technique. Space Time Block Code (STBC) diversity scheme with Trellis Coded Modulation (TCM) and MIMO-OFDM multiplexing schemes is implemented to minimize Bit Error Rate (BER). Minimize BER can improve the signal transmission which affect the improvement of channel capacity of the channel. Improvement of channel capacity can improve the spectral efficiency of the integrated network. But the spectral efficiency of the network can’t be increases indefinitely. Optimization of spectral efficiency is done for different number of antennas.

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**Author’s Contributions**

*Swati Chowdhuri*: Done all the research work (simulation results) and contributed to the writing of the manuscript.

*Pranab Banerjee*: Contribute the idea of the research work.

*Sheli Sinha Chaudhury*: Organized the research work and check language of the manuscript.

**Ethics**

There is no conflicts of interest regarding the publication “Mathematical Derivation of MIMO Based MANET to Improve the Network Performance.

**References**


