Approximate Maximum Likelihood Commercial Bank Loan Management Model

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Abstract: Problem statement: Loan management is a very complex and yet, a vitally important aspect of any commercial bank operations. The balance sheet position shows the main sources of funds as deposits and shareholders contributions. Approach: In order to operate profitably, remain solvent and consequently grow, a commercial bank needs to properly manage its excess cash to yield returns in the form of loans. Results: The above are achieved if the bank can honor depositors withdrawals at all times and also grant loans to credible borrowers. This is so because loans are the main portfolios of a commercial bank that yield the highest rate of returns. Commercial banks and the environment in which they operate are dynamic. So, any attempt to model their behavior without including some elements of uncertainty would be less than desirable. The inclusion of uncertainty factor is now possible with the advent of stochastic optimal control theories. Thus, approximate maximum likelihood algorithm with variable forgetting factor was used to model the loan management behavior of a commercial bank in this study. Conclusion: The results showed that uncertainty factor employed in the stochastic modeling, enable us to adaptively control loan demand as well as fluctuating cash balances in the bank. However, this loan model can also visually aid commercial bank managers planning decisions by allowing them to competently determine excess cash and invest this excess cash as loans to earn more assets without jeopardizing public confidence.

Key words: Adaptive loan management, least squares, linear quadratic theory, stochastic modeling, variable forgetting factor

INTRODUCTION

One of the financial institutions in any country is the commercial bank. Its balance sheet position unveils how effectively management has been able to manage the granting of loans to borrowers. It suffices to say that the balance sheet consists of two sides (Assets and Liability). The grand totals of the two sides are always equal. The liability shows sources of funds, while the asset contains application of funds. Also, the bank has control over its internal environment, but it has no such control over its external environment.

Deposits from depositors and contribution from shareholders usually form the primary sources of commercial bank funds\(^{17,23}\).

The other sources are the credit maintained by borrowers\(^{5}\) and external funds, such as federal government funds. Other items in the liability side are deferred incomes, debentures, surpluses and undivided profits\(^{22}\). The sum total of funds placed in a bank such as demand, savings and time deposits is collectively known as deposit. Also, the interest on discount advances made by the bank is known as deferred income. This is the interest due over the entire life of the loan and it is always deducted from the loan before it is given to the borrower. Additionally, there is operating reserve and it is money usually set aside to meet deposit interest payment and other operating expenses that are yet to be met\(^{10,24}\).

However, a bank’s management frowns at any attempt to dilute its equity. Adherence to such philosophy creates a greater leverage for stockholders. This will no doubt limit the bank’s loan portfolio since its capital-to-deposit ratio will be low.

The bank examiner or regulator (Central or Reserve Bank) requires that the bank’s capital-to-deposit ratio should be high enough before it can embark on loan portfolio and other investments. This is to provide ample protecting cushion to depositors, if the bank experiences losses\(^{36}\). Thus, the confidence of the public will still be maintained.

A high capital-to-deposit ratio will, however, lower the earning leverage because it means more shareholders. This shortcoming can be overcome by issuing debentures. Debenture, therefore, acts as subordinated protecting cushion to depositors\(^{25}\).

Undisbursed loan proceeds needed for borrowers, tax owed, cash dividends due, undivided profits and
surpluses are other items of liability. Contribution by stockholders above nominal value of the stock is known as surplus, while undivided profits are funds earned but not allocated elsewhere[22].

The asset side is concerned with the uses of funds, which will eventually lead to the making of profits. The asset comprises buildings and equipment, cash on hand and due from banks, loans and investments. In a commercial bank portfolio management, loan is a “cow”. It has the highest return to the bank than investment in bonds or government securities, such as treasury bills[16].

The banking regulations are normally aimed at stabilizing the banking system[8]. The regulator (Central or Reserve Bank), tends to control bank reserves, equity capital, loans portfolios, deposits, interest rates, entry, branching and mergers, harvesting of non-profitable branches. In addition, an Insurance Scheme cooperates effectively with the Central or Reserve bank to provide insurance coverage for all deposits in banks through emergency loans. In the USA, its functions also include negotiation of mergers of failed banks[8,24].

The problem of loan management has been addressed through various approaches, like the rule of thumb. In this method, the decision makers rely on their experience, which is, both unscientific, as it is unreliable. The sedimentary theory approach asserts that certain portions of short-term funds (savings deposits) usually remains in the bank for a long period of time and that, such funds can be granted as loans. Another approach is the successive retention of funds coupled with market share[16]. A more scientific approach uses a distributed lag forecasting model[24].

Some banks embark on credit rationing when demand for loans exceeds the supply at the ruling market price. This is due to lack of accurate forecasting. There is also a linear programming approach[26]. The loan commitment contracts approach specifies the maximum size of loan, the purpose of the loan, the interest rate and the fee to be paid by borrower[14,15]. The problem of loan management also attracted a lot of other approaches, such as the Poisson process and stochastic demand model[30].

Another method adjusts a bank’s portfolio of assets and liabilities using a stochastic approach, which hedges against uncertain shifts in the stochastic state variables affecting the current financial conditions of the bank[38].

However, all these approaches will not give the correct or true estimate of very effective loan management of a commercial bank in a given period. The reason for this is that disturbances and random fluctuations have been left out in all these approaches.

But, the recursive Approximate Maximum Likelihood method (AML) adopted for this study has all the qualities that are deficient in the existing methods.

The aims and objectives of this study are to: (i) identify the important parameters of a bank’s operations, (ii) obtain a stochastic model of a commercial bank’s operations, (iii) adaptively control its loan and (iv) ensure maximum profitability.

The above objectives for this study are achieved by stochastic modeling and subsequent, application of the recursive approximate maximum likelihood method to the identification algorithm. The computer simulation was carried out under many operating conditions (normal and adverse), in order to investigate the stability of the developed algorithm to loan management.

**MATERIALS AND METHODS**

The loan management model employed in this study used the state space equation, identification algorithm and self-organizing controller as the control strategy to model the dynamic behavior of a commercial bank. Least squares, extended least squares and approximate maximum likelihood algorithm with variable forgetting factor to avoid “blow-up” in the covariance matrix of the identifier, coupled with computer simulation and MATLAB were used in this study. The liquidity ratio was obtained in Per Unit (PU) value, which is a fraction of the total deposit. This procedure was adopted to reduce the problems associated with improper choice of parameters for simulation.

**Modeling procedure:** The lending process of a commercial bank usually involves request for loans, subsequent credit evaluations and granting of loans[31]. There could be successful or unsuccessful repayment of loans[9,16]. This is especially so because, the mechanism of using the court to gain defaulter’s asset in the case of international financing is limited[9]. Sometimes, errors can arise in the lending process, although there is always credit analysis to determine the borrowers credit worthiness[2]. As a result of these inescapable flaws, there is a need for optimal approach. Hence, adaptive control is adopted for this study.

Disposable assets can be measured as the sum of deposit liabilities plus networth less reserves (which is mainly primary reserve). This disposable asset is that which the bank can invest. It has been shown that the highest returns, which banks get from investments, are always from loans[10]. It is assumed in this study that the
investment under consideration is loan. There are other investments such as short-term securities (for the sake of protection in time of financial squeeze) and bonds.

The environment and economy in which commercial banks operate are dynamic in nature. A bank’s market share determines to a large extent its ability to diversify its portfolio\(^{11}\). This is due to that fact that, public confidence has to be maintained. Loss of public confidence will arise if the bank is unable to honor depositors withdrawals. To a certain degree, the market share determines the portfolio diversification, which the Central Bank (consequence of reserve requirements) will allow the bank. The demands for loans are always on the increase as the economy expands\(^{29}\). Thus, any attempt to develop a model for credit lending must include uncertainty factor, to account for loan management dynamics.

**System model:** The starting point is a state space modeling equation, which describes the dynamic behavior of a commercial bank. The state space equation considers the liquidity behavior of the bank\(^{31}\). The following equations can, therefore, be considered:

\[
W = C + I_n \quad (1)
\]

\[
D = D_L + W - R \quad (2)
\]

\[
L = \alpha D \quad (3)
\]

Where:
- \(DL\) = Deposit liabilities
- \(W\) = Networth
- \(C\) = Capital
- \(I_n\) = Retained Reserve
- \(R\) = Reserve as prescribed by the Central Bank and also affected by the frequency of withdrawal by depositors
- \(L\) = Amount of loans
- \(D\) = Disposable assets
- \(\alpha\) = Fraction of the disposable asset, that can be given out \((0 < \alpha < 1)\)

For optimality to be obtained, Eq. 3 must be observable and controllable. Controllability implies that the system can be transferred from an initial state, say \(L(t)\), at time \(t\) to any other state in finite interval of time with the help of an unconstrained control vector. While observability implies that at time \(t\), it is possible to determine its state \(L(t)\) from the observation of the output over a finite time interval. Apart from minimizing the error signal, attempts should be paid to the energy required for the control action.

**Commercial banking identification algorithm:** The two important considerations in the identification algorithm are:

- Possibility of identifying an unstable commercial banking system
- Validity of the algorithm for stochastic cases

The system to be identified is generally assumed to be stable in order to prove the convergence of the identification algorithm\(^{3,7}\). It is only in stochastic approximations that stability of the system as well as bounded input signals are necessary for convergence. As noise is always present, the applicability of a stochastic system is very significant. The stochastic approximation algorithms are acceptable in the stable case.

The state space equation is transferred to a stochastic model, which describes the dynamic behavior of the commercial banking system:

\[
A(Z^{-1})L(t) = B(Z^{-1})D(t - k - 1) + C(Z^{-1})\xi(t) \quad (4)
\]

Where:
- \(L\) and \(D\) = Output and input scalars
- \(Z^{-1}\) = The backward shift operator
- \(\xi(t)\) = A disturbance vector
- \(A(Z^{-1}), B(Z^{-1})\) and \(C(Z^{-1})\) = Polynomials in \(Z^{-1}\)

The Recursive Least Squares (RLS) method of identification is of the form:

\[
\hat{L}(t) = -A(Z^{-1})\hat{L}(t + 1) + B(Z^{-1})D(t + 1) \quad (5)
\]

where, \(A\) and \(B\) are also polynomials in \(Z^{-1}\). Also:

\[
A(Z^{-1}) = \alpha_1 + \alpha_2 Z^{-1} + \ldots + \alpha_p Z^{-p-1} \quad (6)
\]

\[
B(Z^{-1}) = \beta_1 + \beta_2 Z^{-1} + \ldots + \beta_r Z^{-r-1} \quad (7)
\]

The following loan demand vectors can be introduced:

\[
\theta = (\alpha_1, \alpha_2, \ldots, \alpha_p, \beta_1, \beta_2, \ldots, \beta_r)^T \quad (8)
\]

\[
\theta(t) = [-L(t-1), \ldots, -L(t-p), D(t-1), \ldots, D(t-r)]^T \quad (9)
\]

Equation 5 can be rewritten as:

\[
\dot{L}(t) = \theta^T \phi(t) \quad (10)
\]
The modified form of Eq. 5 can be written as:
\[
\hat{L}(t) = -A(Z^{-1})L(t-1) + B(Z^{-1})D(t-1) + \xi(Z^{-1})e(t) \quad (11)
\]
or
\[
\hat{L}(t) = \theta^* (t-1) \phi(t) \tag{12}
\]
where, \(e(t)\) is the prediction error and can be defined as:
\[
\epsilon(t) = L(t) - \hat{L}(t) \tag{13}
\]
and
\[
\xi(Z^{-1}) = \gamma_i + \gamma_i Z^{-1} + \ldots + \gamma_i Z^{-s+1}
\]
which is an equation in polynomial \(Z^{-1}\).

Equation 11 is the Extended Least Squares method (ELS). Equations 8 and 9 can be rewritten as:
\[
\theta(t) = (\alpha_1, \ldots, \alpha_p, \beta_1, \ldots, \beta_r, \gamma_1, \ldots, \gamma_s)^T
\]
\[
\phi(t) = [-L(t-1), \ldots, -L(t-p), D(t-1), \ldots, D(t-r), \epsilon(t-1), \theta(t-2), \ldots, \epsilon(t-s), \theta(t-s-1)]^T \tag{16}
\]
\[
\epsilon(t, \theta) = L(t) - \theta^T \phi(t) \tag{17}
\]
where, \(p, r\) and \(s\) are the orders or numbers of parameters in the estimation routines. In this study, \(p, r\) and \(s\) were chosen as 4, respectively.

The sequence of estimates, which are applicable to both the RLS and ELS methods are:
\[
\theta(t) = \theta(t-1) + \lambda(t) \epsilon(t, \theta(t-1))K(t) \tag{18}
\]
\[
K(t) = \frac{R(t)^{-1} \phi(t)}{1 + \lambda(t) R(t)^{-1} \phi(t)} \tag{19}
\]
\[
R(t) = R(t-1) + \lambda(t) [\phi(t) \phi(t)^T - R(t-1)] \tag{20}
\]
\[
\epsilon(t, \theta(t-1)) = L(t) - \theta(t-1)^T \phi(t) \tag{21}
\]

Where:
\(\theta(t)\) = Estimated parameters.
\(K(t)\) = The gain of the identifier
\(\lambda(t)\) = A variable forgetting factor
\(R(t)\) = The covariance of the identifier

The variable forgetting factor has been introduced into the identification algorithm to overcome “blow-up” of the covariance matrix of the identifier. If the system is at a steady state, the error will be negligibly small. This in turn pushes the variable forgetting factor very close to one or unity and so prevents the covariance matrix from “blow-up”.

Occurrence of disturbance can cause the variable forgetting factor to decrease, since the error increases. Fast tracking is achieved for large disturbances and fluctuations as:
\[
\lambda(t) = 1 - [1 - \phi(t-1)^T K(t) \epsilon(t-1)] \lambda_0 \tag{27}
\]
Where:
\[
\lambda_0 = \sigma^2_0 N_0 \tag{28}
\]

It is a pre-selected constant so as to ensure that estimation is based on the same amount of information. \(\sigma^2_0\) is the variance of the noise, \(N_0\) corresponds to the
memory length and it controls the adaptation speed of the identification algorithm. Based on the work reported elsewhere in the literature\(^{[27]}\), the value of \(\Sigma_0\) chosen was 0.8.

**Self-tuning controller strategy:** The self-tuning controller of the commercial banking system is given by Eq. 2. The control strategy is to minimize the cost function\(^{[34]}\):

\[
J = E[(PL(t + k) - NL'(t))^2 + D^T(t)QD(t)]
\]  
(29)

Where:

- \(E\) = Expectation operator,
- \(L^d(t)\) = Set point (desired loan output in system),
- \(N, P, Q\) = Weighting factors, which are properly selected
- \(D\) = Disposable loan

Linear quadratic theory (LQ) is applied to obtain the control law of the controller. LQ is chosen because the control law will be stable and not sensitive to parameter variation if the estimates are accurate unlike the Minimum Variance (MV) strategy\(^{[1, 4, 7, 28]}\). It is assumed that global stability concept exists and also with the convergence of the recursive identification algorithm. The controller is of the form:

\[
D(t) = \frac{C(Z^{-1})L^d(t) - F(Z^{-1})D(t) - d}{B(Z^{-1}) + QC(Z^{-1})}
\]  
(30)

\[
F(Z^{-1}) = \frac{B(Z^{-1})}{b_1} - A(Z^{-1})
\]  
(31)

Where:

- \(b_1 = \frac{L(t + 1)}{D(t)} = \frac{B(Z^{-1})}{C(Z^{-1})} = \frac{B(Z^{-1})}{b_1}\)

\(L(t)\) = System power output at sample instant \(t\)

\(D(t)\) = System input at sample instant \(t\)

\(B, C, F\) = Polynomials in the recursive operator \(Z^{-1}\)

\(d\) = Steady state constant at zero controlled input

**Characteristic polynomial solution:** The characteristic polynomial equation becomes:

\[
B(Z^{-1})[B(Z^{-1}) + QA(Z^{-1})] = 0
\]  
(32)

It is necessary that the roots of both polynomials \(B(Z^{-1})\) and \(B(Z^{-1})+QA(Z^{-1})\) of Eq. 32 lie within the unit circle so as to ensure stability. This is also the requirement for the stability of the Z-transformed transfer function. If \(B(Z^{-1})\) has no roots outside the unit circle, the closed loop system will always remain stable and independent of the values of \(A(Z^{-1})\).

**RESULTS AND DISCUSSION**

The table of results (Table 1) and MATLAB graph (Fig. 1) show the visual display of the polynomial solution for which the granting of loans is no longer very safe in order not to erode and jeopardize public confidence.

Simply because all the results of the simulation for all the weighting factors of 1.0, 1.75, 2.0 and 0.75, showed very slight variations, it was convenient to assume that the weighting factors had little influence on the roots of the polynomials and consequently, use one table to represent all four weighting factors.

**Table 1:** Polynomial roots at 1.0, 1.75, 2.0 and 2.75 weighting factors

<table>
<thead>
<tr>
<th>Per unit</th>
<th>Roots of A (Z^{-1})</th>
<th>Roots of B (Z^{-1})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>-0.10553596 ± 7.339772 ( \times 10^{-3} )</td>
<td>7.230287 ( \times 10^{-4} )</td>
</tr>
<tr>
<td>0.1380660 ± 0.1455781</td>
<td>-7.970028 ( \times 10^{-5} )</td>
<td></td>
</tr>
<tr>
<td>0.25</td>
<td>-0.10108998 ± 0.197992 ( \times 10^{-2} )</td>
<td>5.164292 ( \times 10^{-4} )</td>
</tr>
<tr>
<td>0.35</td>
<td>0.13511654 ± 0.1523604</td>
<td>-5.1622 ( \times 10^{-4} )</td>
</tr>
<tr>
<td>0.45</td>
<td>2.751404 ± 7.147475 ( \times 10^{-2} )</td>
<td>0.1692863</td>
</tr>
<tr>
<td>0.50</td>
<td>2.181172 ± 2.074879 ( \times 10^{-2} )</td>
<td>-1.198381 ( \times 10^{-2} )</td>
</tr>
<tr>
<td>0.56</td>
<td>-2.230876 ± 1.756578</td>
<td>-5.1622 ( \times 10^{-2} )</td>
</tr>
<tr>
<td>0.50</td>
<td>6.113208 ± 6.108464 ( \times 10^{-2} )</td>
<td>1.198381 ( \times 10^{-2} )</td>
</tr>
<tr>
<td>0.50</td>
<td>1.069963 ± 7.07551615</td>
<td>6.251186 ( \times 10^{-2} )</td>
</tr>
</tbody>
</table>
It has also been suggested that the amount of loans control system.

roots are to the origin, the more stable will be the close to the origin. It has been shown that the closer the weighting factors equal to or greater than one, are usually advisable for commercial bank managers to constantly monitor and gauge their loan demand patterns in order to make informed and quality decisions regarding their secondary reserves capacity for enhanced returns on investments, through loans granting and facilitation.

The results show that variation of the weighting factors has very little influence on the roots of the polynomials, since these polynomials roots varied slightly. In order to eliminate the problems of bias and obtain satisfactory performance of the algorithm, weighting factors equal to or greater than one, are usually advisable.

Suffice it to say that the weighting factors were introduced into the model adaptive loan management algorithm to prevent excessive control. The Linear Quadratic Gaussian theory (LQ) was employed to minimize the chosen cost function. The Minimum Variance (MV) approach was not used because the penalty function is zero. This implies that the control could be excessive. Also, the control employing LQ is more stable than that of MV. Additionally, MV is not sensitive to parameter variation. It is therefore, advisable for commercial bank managers to constantly monitor and gauge their loan demand patterns in order to make informed and quality decisions regarding their secondary reserves capacity for enhanced returns on investments, through loans granting and facilitation.

It was also revealed by the Fig. 1 above, which was plotted using MATLAB that as the per unit operating point increases to about 0.45 P.U and beyond, the roots of polynomial A(Z) also increased widely outside the unit circle. This scenario indicates that instability has gradually set into the loan management algorithm and consequently the capacity of the commercial bank to grant loans without jeopardizing public confidence of not failing to honor legitimate depositors withdrawals, begin to become doubtful.

Moreover, an improvement in the financial structure should lead to a more stable situation. This implies that if the bank’s market share improves, then, it can afford to give out more loans to credible borrowers. As a result, the returns to the commercial bank will increase.

CONCLUSION

The availability of cash and loanable funds are important to the successful operations of a commercial bank. However, if there is excess cash, it could lead to a waste of resources unless properly channeled into loans.

If cash is insufficient to meet the demands of customers, especially depositors withdrawals and credible borrowers, it could lead to loss of public confidence and consequent run on the commercial bank leading to bank failure.

As a result, a commercial bank has to hold a certain amount of cash that will meet with depositors withdrawal requirements and other liquidity needs of the commercial bank, using stochastic methods as proposed in this study.

In the past, the problems of holding appropriate levels of reserve to meet loan demand requirements have been addressed by a variety of methods. Amongst them is the rule of thumb in which the decision makers always rely on their experience to guide their loan decisions. Such decisions are more or less guesswork, unreliable and arbitrary.

Another approach used by the commercial bank as loan management methodology is the “sediment” theory, in which it is assumed that only some portions of the short-term funds remains unutilized for a long time and that the remaining portion is volatile. One of the shortcomings of this approach is that it cannot be applied into the interbank money market nor can it be
applied to the proceeds from foreign exchange market. Also, the method of estimation is subject to the risk preference of the decision makers.

Additionally, all the earlier methods fail to take into account uncertainties, fluctuations and random variations in the environment of the commercial banking system. Realizing that there are a lot of uncertainties inherent in the operations of a commercial bank, any attempt to model its behavior without including some elements of uncertainty will not give a true picture of its dynamical properties.

Consequently, these earlier approaches fail to give effective and efficient loan management methodology as explained in this study. This study, therefore, addressed the issue of loan management by employing a self-tuning device, which adjusts itself automatically in accordance with the prevailing circumstances for proper control. It also takes cognizance of uncertainties and random variations that are associated with the unknown future.

Since the socio-economic conditions surrounding the operations of a commercial bank are dynamic, this method should provide a better and more efficient loan management methodology. It is sufficient to say also, that the market shares of commercial banks could influence their investment structure and profile, especially as a consequence of deregulation. These will influence and affect the amount of loans commercial banks can offer and also the amount of cash balances at their disposal.

The stochastic loan management model developed in this research should be more useful to commercial banks operating in large cities and heavily industrialized areas, because they are more susceptible to disturbances and random fluctuations than are rural or semi-urban banks. This assertion is essentially so, because it would be a credible cash deposit base measuring instrument for assessing both the assets and liability positions of the commercial bank for timely and quality decision making capabilities.

Additionally, it will be a valuable tool in the planning and policy decisions of bank managers since it will enable them determine excess cash, which could be invested as loans to earn more assets without jeopardizing public confidence.

However, the importance of this control technique in the policy decision-making processes of loan management problems is yet to be realized. As a result, further research and simulation studies are needed to investigate the full spectrum of its potential when it is used to manage loan disbursement in an existing commercial bank setting.

REFERENCES


