

A New Weighted Rank Correlation

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Abstract: Problem Statement: There have been many cases in real life where two independent sources have ranked n objects, with the interest focused on agreement in the top rankings. Spearman's rho and Kendall's tau coefficients assigned equal weights to all rankings. As a result, the literature proposed several weighted correlation coefficients with emphasis on the top rankings, including the top-down, weighted Kendall's Tau and Blest's correlation coefficient. **Approach:** This article introduced a new weighted rank correlation coefficient that was sensitive to agreement in the top rankings. It presented the limiting distribution under the null hypothesis of independence and provided a summary of quantiles of the exact null distribution for $n = 3(1)9$. **Results:** The article summarized the power comparison between the new weighted coefficient and other weighted coefficients, and showed that the new weighted rank correlation coefficient provided the locally most powerful rank test. **Conclusions/Recommendations:** The new weighted correlation should be used along with other weighted coefficients when the interest focused on agreement in the top rankings, in order to make an effective inference.

Key words: Weighted correlation, top ranking, top-down correlation coefficient, weighted Kendall's Tau, Blest's correlation coefficient.

INTRODUCTION

Every year many students want to apply for postgraduate courses and research, leading to a large number of applicants to universities. Postgraduate committee can choose only few of them, according to some criteria such as GPA and the average of their grades in the major courses that they have studied before. Since the number of the applicants is large, the aim is to minimize the effort and the cost of interviewing all the candidates while choosing the best among them. In such cases, a measure which gives more weight to those who have higher grades is required. Many other cases in life require more weight for values in the top in order to reach decision. For instance, a couple of panels of judges in one of the Olympic game wants to choose the best participants.

For such cases, correlation measures that give more weighted for the top rankings were presented by^[1,4,5]. To review these measures briefly, let $\{(X_i, Y_i), 1 \leq i \leq n\}$ be an independently and identically distributed (i.i.d.) sample from a bivariate distribution where q_i is the rank of Y whose corresponding X has rank i among $\{X_j\}$. Throughout we assume that no ties occur among the variables being considered. If ties occur, the average of

weighted score can be used. Iman and Conover^[4] introduced the top-down correlation coefficient, R_t , as:

$$R_t = \left(\sum_{i=1}^n S_i S_{q_i} - n \right) / (n - S_1) \quad (1)$$

Where, S_i is Savage score^[4] defined as:

$$S_i = \sum_{j=1}^n 1/j$$

Shieh^[5] proposed the weighted Kendall's Tau, R_k , which is given by:

$$R_k = \frac{2}{m(m-1)} \sum_{1 \leq j < i \leq m} \text{sgn}(i-j) \text{sgn}(q_i - q_j) \quad (2)$$

Where, m is the number of top rankings taken into account and $\text{sgn}(a) = -1, 0$ or 1 , if $a <, =$ or > 0 .

A graphical approach was proposed by^[1], leading to a correlation coefficient R_b , which is given by:

$$R_b = \frac{2n+1}{n-1} - \frac{12}{n(n+1)^2(n-1)} \sum_{i=1}^n (n+1-i)^2 q_i \quad (3)$$

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A new weighted rank correlation, R_w , that depends on weighted scores, will be introduced along with its asymptotic distribution under the null hypothesis of independence. Then some exact and approximated quantiles of R_w are summarized. Power comparisons between R_w and other reviewed coefficients will be presented. Finally, an example is given for illustration.

MATERIALS AND METHODS

A new weighted rank correlation: Let (X_i, Y_i) , $(1 \leq i \leq n)$ be an i.i.d. sample from a bivariate distribution and let (i, q_i) , $i = 1, 2, \dots, n$, be paired rankings of n objects, where q_i is the rank of the Y values whose corresponding X has rank i among all $\{X_j\}$. We define weighted scores as:

$$W_i = w^i \tag{4}$$

Where, i is the rank of the order observations in a sample of size n and $0 < w < 1$.

The new weighted rank coefficient R_w is obtained by computing the ordinary Pearson correlation coefficient, r , on the weighted scores,

$$R_w = \left(\frac{\sum_{i=1}^n w^{i+q_i} - a_1}{na_2 - a_1} \right) \tag{5}$$

Where, $a_1 = w^2(1 - w^n)^2 / (1 - w)^2$ and $a_2 = w^2(1 - w^{2n}) / (1 - w^2)$.

In another form, we can write R_w as:

$$R_w = \sum_{i=1}^n \left(\frac{w^i - n^{-1}\sqrt{a_1}}{[a_2 - (a_1/n)]^{1/2}} \right) \left(\frac{w^{q_i} - n^{-1}\sqrt{a_1}}{[a_2 - (a_1/n)]^{1/2}} \right) \tag{6}$$

The statistic R_w has a maximum value of 1. However, its minimum possible value is only -1 for $n = 2$, similar as the top-down correlation^[4] and for $n \rightarrow \infty$ it increases from -1 towards approximately a value in the range from -2E -6 to -3E -4, depending on the value of w .

The asymptotic distribution of R_w : Now, the asymptotic distribution of R_w is derived under H_0 , the null hypothesis of independence. The alternative hypothesis of a positive dependence in the rankings can be detected using any of several statistics. The weighted rank correlation R_w is more sensitive to agreement in the top ranks than to agreement in the bottom. For a test of H_0 that is equally sensitive to agreement among ranks at all levels, Spearman's rho or Kendall's tau correlation coefficient can be used. If the marginal distributions are normal and the alternative hypothesis

is bivariate normal with positive correlation, the Pearson correlation coefficient, r , provides the most powerful test of H_0 against the alternative. Under H_0 , the asymptotic distribution of R_w is given by the following theorem:

Theorem 1: Under the null-hypothesis of independence, $E(R_w) = 0$, $V(R_w) = 1/(n-1)$ and the asymptotic distribution of $(n-1)^{1/2} R_w$ is the standard normal distribution.

Proof: The mean and the variance of the R_w , under H_0 , are computed as follows. Since $E(\sum w^{i+q_i}) = nE(w^i)E(w^{q_i}) = (1/n)w^2(1 - w^n)^2 / (1 - w)^2 = a_1/n$, then by substituting in (5) we directly obtain that $E(R_w) = 0$. For the variance,

$$V(R_w) = \text{var}(R_w) = n^2 V(\sum w^{i+q_i}) / (na_2 - a_1)^2$$

Where

$$V(\sum w^{i+q_i}) = nV(\sum w^i)V(\sum w^{q_i}) + n(n-1)\text{cov}(w^i, w^k)\text{cov}(w^{q_i}, w^{q_k})$$

with

$$V(w^{q_i}) = V(w^i) = (1/n)\sum w^{2i} - \left((1/n)\sum w^i \right)^2 = (1/n)\left(a_2 - \frac{a_1}{n} \right)$$

and

$$\begin{aligned} \text{cov}(w^{q_i}, w^{q_k}) &= \text{cov}(w^i, w^k) = E(w^i w^k) - E(w^i)E(w^k) \\ &= \frac{1}{n(n-1)} \sum_{i \neq k} w^i w^k - \left(\frac{\sum w^i}{n} \right)^2 \\ &= \frac{1}{n(n-1)} \left((\sum w^i)^2 - \sum w^{2i} \right) - \left(\frac{\sum w^i}{n} \right)^2 \\ &= \frac{1}{n(n-1)} \left(\frac{a_1}{n} - a_2 \right) \end{aligned}$$

Therefore

$$\begin{aligned} V(\sum w^{i+q_i}) &= n \left(\frac{1}{n} (a_2 - \frac{a_1}{n}) \right)^2 + n(n-1) \left(\frac{1}{n(n-1)} (\frac{a_1}{n} - a_2) \right)^2 \\ &= \frac{1}{n-1} (a_2 - \frac{a_1}{n}) \end{aligned}$$

and

$$V(R_w) = \frac{n^2}{(na_2 - a_1)^2} \frac{1}{n-1} (a_2 - \frac{a_1}{n}) = \frac{1}{n-1}$$

Using $a_n(R_{ni}, f) = [a_2 - (a_1/n)]^{-1/2} (w^i - n^{-1}\sqrt{a_1})$ and $a_n(Q_{ni}, g) = [a_2 - (a_1/n)]^{-1/2} (w^{q_i} - n^{-1}\sqrt{a_1})$, we can write $R_w = \sum_{i=1}^n a_n(R_{ni}, f) a_n(Q_{ni}, g)$. That is, R_w is written as a linear rank statistic. Under H_0 , using Theorem V.1.8 in Hájek and Šidák^[3], the distribution of the statistic R_w for $n \rightarrow \infty$ is asymptotically normal with mean 0 and variance $\sigma_{R_w}^2 = V(R_w) = (n-1)^{-1}$.

Exact and approximate quantiles of R_w : When the null hypothesis is true, all permutations of ranks (I, q_i) , $1 \leq i \leq n$, are equally likely where w can take any value between 0 and 1, exclusive. Then, exact and approximate quantiles of R_w can be computed for chosen values of w , say 0.3, 0.6 and 0.9. Exact quantiles for $n = 3(1)9$ are summarized in Table 1 and for large n , approximate quantiles are shown in Table 2.

Table 1: Exact quantiles for the weighted correlation, R_w

w	n	0.90	0.95	0.975	0.99	0.995	0.999	
0.3	3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	
	4	0.9261	0.9934	1.0000	1.0000	1.0000	1.0000	
	5	0.9035	0.9363	0.9920	0.9995	1.0000	1.0000	
	6	0.8930	0.9170	0.9905	0.9947	0.9993	1.0000	
	7	0.8897	0.9143	0.9448	0.9924	0.9950	0.9996	
	8	0.8875	0.9150	0.9246	0.9907	0.9927	0.9993	
	9	0.8884	0.8990	0.9205	0.9904	0.9927	0.9992	
	0.6	3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
		4	0.8366	0.9412	1.0000	1.0000	1.0000	1.0000
5		0.6990	0.8824	0.9170	0.9848	1.0000	1.0000	
6		0.6811	0.7714	0.9015	0.9617	0.9761	0.9956	
7		0.6525	0.7371	0.8595	0.9160	0.9600	0.9883	
8		0.6021	0.7136	0.7989	0.8969	0.9248	0.9710	
9		0.5762	0.6966	0.7704	0.8780	0.9096	0.9678	
0.9		3	1.0000	1.0000	1.000	1.0000	1.0000	1.0000
		4	0.8018	0.8394	1.0000	1.0000	1.0000	1.0000
	5	0.6801	0.7932	0.8906	0.9282	1.0000	1.0000	
	6	0.6063	0.7329	0.8133	0.8945	0.9442	0.9634	
	7	0.5525	0.6809	0.7716	0.8475	0.8900	0.9415	
	8	0.5081	0.6314	0.7205	0.8060	0.8500	0.9141	
	9	0.4731	0.5920	0.6818	0.7704	0.8199	0.8975	

Power comparison: Let $X = X^* + \Delta Z$ and $Y = Y^* + \Delta Z$, where X^* , Y^* and Z are independent random variables and assume that X^* and Y^* have probability density functions $f(x)$ and $g(y)$, respectively, while the distribution of Z is arbitrary. If $f(x)$ and $g(y)$ are continuous almost everywhere and satisfy the conditions $\int_{-\infty}^{\infty} |f'(x)| dx < \infty$ and $\int_{-\infty}^{\infty} |g'(y)| dx < \infty$, then from (6) and Hájek and Šidák^[3], Theorem II.4.11, the locally most powerful rank test of $H_0: \Delta = 0$ versus $H_1: \Delta > 0$ is the test with rejection region $R_w \geq c$ for some suitable chosen constant c .

The power comparisons, from a bivariate normal population, between R_w for chosen values of w and the top-down, weighted Kendall's tau and Blest's correlation, as given in (1), (2) and (3), respectively, are shown in Table 3. From Table 3, we note that R_w has better power than other correlation coefficients, especially for $w = 0.9$ at small sample size (e.g., $n = 8$) and at significant level $\alpha = 0.05$, as shown in Fig. 1.

Table 2: Approximate quantiles for the weighted correlation, R_w

w	n	0.90	0.95	0.975	0.99	0.995	0.999
0.3	10	0.8895	0.8991	0.9205	0.9902	0.9911	0.9992
	11	0.2536	0.8961	0.9214	0.9903	0.9911	0.9991
	12	0.2377	0.8965	0.9057	0.9278	0.9907	0.9933
	13	0.2474	0.8978	0.9062	0.9263	0.9906	0.9932
	15	0.1983	0.8981	0.9058	0.9256	0.9908	0.9916
	20	0.1994	0.5273	0.9017	0.9097	0.9280	0.9911
	30	0.0475	0.2274	0.9041	0.9064	0.9120	0.9914
	40	0.0376	0.2377	0.9056	0.9059	0.9079	0.9316
	50	0.0465	0.2450	0.2479	0.9066	0.9072	0.9320
	60	-0.0040	0.0779	0.2500	0.9071	0.9072	0.9147
0.6	70	-0.0018	0.0576	0.2532	0.9075	0.9076	0.9151
	80	0.0014	0.0601	0.2557	0.9079	0.9079	0.9108
	90	0.0040	0.0626	0.2577	0.9081	0.9081	0.9104
	100	-0.0107	0.0645	0.2592	0.2615	0.9083	0.9090
	10	0.5457	0.6752	0.7460	0.8576	0.8981	0.9590
	11	0.5291	0.6519	0.7275	0.8439	0.8842	0.9500
	12	0.5108	0.6317	0.7173	0.8162	0.8673	0.9217
	13	0.4795	0.6275	0.7141	0.7907	0.8584	0.9187
	15	0.3898	0.6001	0.6914	0.7685	0.8438	0.9081
	20	0.3156	0.5675	0.6308	0.7311	0.7728	0.8658
0.9	30	0.2875	0.3861	0.5997	0.6582	0.7380	0.8539
	40	0.1901	0.3405	0.6017	0.6358	0.6902	0.7885
	50	0.1683	0.3333	0.4216	0.6212	0.6437	0.7642
	60	0.1623	0.3246	0.3640	0.6174	0.6275	0.7106
	70	0.1161	0.2366	0.3581	0.6193	0.6297	0.7121
	80	0.0943	0.1940	0.3531	0.6211	0.6240	0.7093
	90	0.0983	0.1961	0.3560	0.6234	0.6302	0.7109
	100	0.0764	0.1985	0.3584	0.4384	0.6259	0.6563
	10	0.4441	0.5580	0.6476	0.7360	0.7870	0.8710
	11	0.4243	0.5282	0.6138	0.7022	0.7511	0.8276
12	0.3949	0.5056	0.5888	0.6762	0.7294	0.8206	
13	0.3766	0.4781	0.5664	0.6675	0.7135	0.7970	
15	0.3610	0.4531	0.5263	0.6071	0.6527	0.7455	
20	0.3065	0.3868	0.4523	0.5286	0.5724	0.6699	
30	0.2423	0.3147	0.3838	0.4575	0.5022	0.5643	
40	0.2162	0.2766	0.3345	0.3975	0.4400	0.5327	
50	0.1889	0.2484	0.3002	0.3579	0.4083	0.4907	
60	0.1738	0.2326	0.2798	0.3403	0.3868	0.4768	
70	0.1634	0.2179	0.2657	0.3201	0.3584	0.4337	
80	0.1471	0.1972	0.2465	0.2995	0.3329	0.4037	
90	0.1469	0.1961	0.2408	0.2928	0.3317	0.4178	
100	0.1355	0.1834	0.2235	0.2718	0.3038	0.3736	

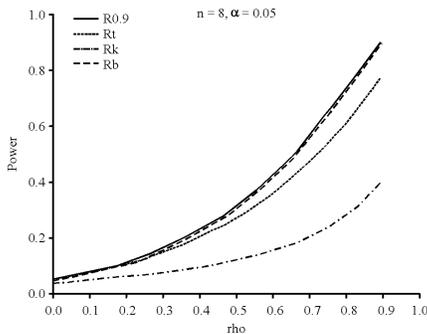


Fig. 1: Power curves of randomized tests of independence for $n=8$

Table 3: Powers of randomized tests of independence

ρ		0	00.1	00.2	00.3	00.4	00.5	00.6	00.7	00.8	00.9
n = 8	R_k	0.0416	0.0510	0.0638	0.0777	0.0976	0.1215	0.1561	0.2075	0.2830	0.4106
	R_t	0.0516	0.0756	0.1057	0.1472	0.2023	0.2705	0.3584	0.4748	0.6138	0.7879
	R_b	0.0503	0.0760	0.1103	0.1593	0.2242	0.3059	0.4171	0.5555	0.7265	0.8970
	$R_{0.1}$	0.0280	0.0410	0.0565	0.0781	0.1072	0.1463	0.2023	0.2783	0.3871	0.5600
	$R_{0.25}$	0.0526	0.0712	0.0966	0.1252	0.1621	0.2097	0.2698	0.3471	0.4519	0.6041
	$R_{0.5}$	0.0536	0.0735	0.1012	0.1321	0.1737	0.2245	0.2904	0.3765	0.4861	0.6385
	$R_{0.6}$	0.0524	0.0745	0.1063	0.1466	0.1974	0.2627	0.3450	0.4601	0.5975	0.7759
	$R_{0.75}$	0.0518	0.0769	0.1117	0.1551	0.2120	0.2878	0.3877	0.5143	0.6690	0.8488
	$R_{0.9}$	0.0528	0.0775	0.1132	0.1645	0.2319	0.3135	0.4258	0.5656	0.7340	0.9055
n = 12	R_k	0.0425	0.0581	0.0758	0.1012	0.1306	0.1677	0.2228	0.3006	0.4177	0.6081
	R_t	0.0508	0.0795	0.1247	0.1858	0.2639	0.3646	0.4911	0.6411	0.7931	0.9340
	R_b	0.0544	0.0933	0.1492	0.2274	0.3309	0.4595	0.6192	0.7760	0.9161	0.9863
	$R_{0.1}$	0.0841	0.1104	0.1421	0.1787	0.2209	0.2713	0.3295	0.4058	0.5001	0.6306
	$R_{0.25}$	0.0407	0.0590	0.0840	0.1141	0.1548	0.2098	0.2726	0.3609	0.4714	0.6206
	$R_{0.5}$	0.0402	0.0591	0.0863	0.1192	0.1612	0.2185	0.2889	0.3844	0.5083	0.6758
	$R_{0.6}$	0.0376	0.0568	0.0864	0.1230	0.1732	0.2400	0.3289	0.4448	0.5969	0.7803
	$R_{0.75}$	0.0381	0.0602	0.0977	0.1499	0.2199	0.3128	0.4392	0.5922	0.7596	0.9218
	$R_{0.9}$	0.0366	0.0647	0.1081	0.1748	0.2627	0.3828	0.5411	0.7090	0.8771	0.9786
n = 20	R_k	0.0302	0.0473	0.0727	0.1089	0.1618	0.2334	0.3325	0.4711	0.6527	0.8641
	R_t	0.0521	0.0978	0.1637	0.2595	0.3821	0.5369	0.7046	0.8554	0.9575	0.9966
	R_b	0.0544	0.1078	0.1914	0.3212	0.4887	0.6704	0.8310	0.9435	0.9924	0.9999
	$R_{0.1}$	0.0496	0.0694	0.0947	0.1285	0.1662	0.2158	0.2717	0.3448	0.4395	0.5856
	$R_{0.25}$	0.0496	0.0694	0.0947	0.1285	0.1662	0.2158	0.2717	0.3448	0.4395	0.5856
	$R_{0.5}$	0.0493	0.0719	0.1003	0.1387	0.1831	0.2440	0.3125	0.4048	0.5251	0.7029
	$R_{0.6}$	0.0501	0.0735	0.1075	0.1525	0.2075	0.2819	0.3714	0.4859	0.6444	0.8340
	$R_{0.75}$	0.0500	0.0903	0.1450	0.2204	0.3252	0.4556	0.6048	0.7606	0.8969	0.9827
	$R_{0.9}$	0.0542	0.1049	0.1847	0.3082	0.4611	0.6396	0.8059	0.9287	0.9862	0.9996
n = 30	R_k	0.0569	0.0987	0.1591	0.2517	0.3640	0.4994	0.6658	0.8155	0.9323	0.9923
	R_t	0.0511	0.1104	0.2068	0.3492	0.5303	0.7226	0.8727	0.9622	0.9945	0.9998
	R_b	0.0501	0.1235	0.2554	0.4493	0.6682	0.8446	0.9504	0.9932	0.9995	1.0000
	$R_{0.1}$	0.0519	0.0780	0.1133	0.1617	0.2242	0.3068	0.4102	0.5312	0.6750	0.8344
	$R_{0.25}$	0.0519	0.0780	0.1133	0.1617	0.2242	0.3068	0.4102	0.5313	0.6751	0.8347
	$R_{0.5}$	0.0568	0.0855	0.1285	0.1831	0.2563	0.3467	0.4650	0.5967	0.7410	0.8954
	$R_{0.6}$	0.0548	0.0847	0.1283	0.1846	0.2580	0.3569	0.4817	0.6269	0.7784	0.9296
	$R_{0.75}$	0.0501	0.0887	0.1452	0.2283	0.3381	0.4767	0.6396	0.8016	0.9236	0.9901
	$R_{0.9}$	0.0513	0.1184	0.2341	0.3985	0.5958	0.7793	0.9105	0.9792	0.9980	1.0000

RESULTS AND DISCUSSION

Numerical Example: To illustrate our new weighted rank correlation, we use a data set, in Table 4, that was also used by^[5]. The data set considers two techniques, A and B, used to select the most effective variables out of 20 variables for evaluation of some software packages.

We see that the two techniques agree strongly on the top six variables. However, there is large disagreement between these techniques after that. In such circumstances, we may want to place more emphasize on the top rankings rather than equity over all ranking values. Therefore, we calculate some different weighted rank statistics, along with our weighted rank correlation at different weighted values. For each statistic the corresponding p-values are evaluated, these values are given in Table 5.

From Table 5 we can conclude that at different weight values, our weighted rank correlation and the

Table 4: Example: Two techniques A and B for selecting the most effective variables out of 20 variables for evaluation of some software packages.

A	1	2	3	4	5	6	7	8	9	10
	11	12	13	14	15	16	17	18	19	20
B	1	2	4	3	6	5	20	13	15	19
	17	18	14	16	12	9	11	8	10	7

Table 5: Example: Weighted rank correlation statistics and their p-values

	Statistic	p-value
Blest's Rank R_b	0.5602	<0.01
Top-down R_t	0.8206	<0.001
Weighted Kendal R_k (m = 6)	0.7333	<0.01
Weighted rank R_w		
$R_{0.1}$	0.9999	<0.001
$R_{0.25}$	0.9977	<0.001
$R_{0.5}$	0.9851	<0.001
$R_{0.6}$	0.9786	<0.001
$R_{0.75}$	0.9434	<0.001
$R_{0.9}$	0.7135	<0.001

top-down correlation provide strong evidence (p-value <0.001) against the null hypothesis of

independence of A and B. The criteria behind choosing the weight depends on the degree of emphasis the user may wish to apply to top ranks. However, we suggest the weight $w = 0.9$ since as shown in Table 3, our new weighted rank with $w = 0.9$ has higher power than other rank correlation coefficients.

CONCLUSION

This article proposed a new weighted rank correlation coefficient that was sensitive to agreement in the top rankings. Under the null hypothesis of independence, the proposed coefficient's limiting distribution was derived along with the exact and approximated quantiles for different sample sizes. As shown, the test that depended on the new weighted rank correlation coefficient was the locally most powerful rank test. Therefore, when interest focused on the top rankings, we recommended using the new weighted rank correlation coefficient, together with other weighted coefficients, to reach an effective decision.

A generalization of this article, when more than two independent sources rank n objects with focus on top rankings, known as a Concordance measure, will be presented somewhere else.

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