Analysis and Synthesis of Mechanisms with Bars and Gears Used in Robots and Manipulators

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Abstract: Bars and gears are used everywhere today, but a wide range of uses is robotics and mechatronics. Since ancient times, automations and mechanization have been used with mechanisms consisting of chains of bars and gears. These were obviously used for the purpose of transmitting the movement and its transformation, that is to say, as a mechanical transmission. Gear and bar automation are used today as modern mechanical transmissions, serial and parallel robots, machine building industry and all industrial areas where automation has penetrated. In fact, robotic gears and gears are the basis for mechanical transmissions to robots and at the same time have other roles such as balancing, support, etc. The most commonly used gears are tapered, conical gears, because they work faster, more dynamically, occupy less space, have fewer toothed gear teeth, low volume, light movement different directions and a multitude of features that make them irreplaceable within mobile mechanical systems. The gears and gears are increasingly used in the construction of manipulators and industrial robots, especially in the MOr. In the kinematic openings of Positioning Mechanisms (MPz) of the robots, also referred to as trajectory generators, a first kinematic chain with bars is attached, to which is attached a kinematic chain with cylindrical, conical and hypoid gears. The mechanical chains that can be made of conical mechanical transmissions and bars are complex, extremely complex and can work on different spaces and axes, with inclines and directional changes as desired. From this point of view, they can’t be replaced by other types of mechanical mechanisms or transmissions. A complex kinematic scheme with bevel gears and conical gears of a manipulator-robot with 6 + 1 mobilities is analyzed, where the positioning mechanism (RRR) is not distinguishable from the RRR orientation mechanism. The two kinematic chains of MPz (RzRxxRx) and MOr (RzRxRxxRz) are staged (in extension). At the end (O6 point) of the articulated chain O0O1O2O3O4O5 the gripping mechanism (MAP), made with two articulated parallelograms, is attached. All 6 + 1 kinematic chains are operated by means of worm gear reducers with electric motors located at the base. The kinematic chain with bars is simplified to the left of Fig. 1 and to the right is an axial projection of the complete kinematic scheme of the gear with gears and gears. The articulated bars (0, 1, 2, 3, 4, 5, 6) with six movable elements are the main kinematic chain to which are attached six kinematic chains with conical gears.

Keywords: Anthropomorphic Robots, Kinematics, Bars and Gear, MPz Structure
Introduction

Today the moving mechanical systems are utilized in almost all vital sectors of humanity (Reddy et al., 2012). The robots are able to process integrated circuits (Aldana et al., 2013) sizes micro and nano, on which the man they can be seen only with electron microscopy (Lee, 2013). Dyeing parts in toxic environments, working in chemical and radioactive environments (Padula and Perdereau, 2013; Perumao and Jawahar, 2013), or at depths and pressures at the deep bottom of huge oceans, or conquest of cosmic space and visiting some new exoplanets, are with robots systems possible (Dong et al., 2013) and were turned into from the dream in reality (Garcia et al., 2007), because of use of mechanical platforms sequential gearbox (Cao et al., 2013; Petrescu et al., 2009). The man will be able to carry out its mission supreme (Tang et al., 2013; Tong et al., 2013), conqueror of new galaxies (de Melo et al., 2012), because of mechanical systems sequential gear-box (robotics systems) (Garcia-Murillo et al., 2013).

Robots were developed and diversified (Lin et al., 2013), different aspects (He et al., 2013), but today, they start to be directed on two major categories: Systems serial (Liu et al., 2013; Petrescu and Petrescu, 2011b) and parallel systems (Petrescu and Petrescu, 2012c). Parallel systems are more solid (Tabaković et al., 2013; Wang et al., 2013) but more difficult to designed and handled and for this reason, the serial systems were those which have developed the most. In medical operations or radioactive environments are preferred mobile systems parallel, because of their high accuracy positioning.

As examples of such combined mechanisms, several kinematic schemes of gears and gears can be observed, presented by Kojevnikov (1969; AUTORENKOLLEKTIV, 1968; Şaskin, 1963; 1971; Maros, 1958; Rehwald et al., 2000; 2001; Antonescu, 1993; 2003; Antonescu and Mitrache, 1989).

The main problems with plane and spatial gears and gears refer to kinematic analysis and geometric-kinematic synthesis under certain conditions imposed by technological processes, Bruja and Dima (2001; Buda and Mateucă, 1989; Luck and Modler, 1995; Niemeyer, 2000; Tutunaru, 1969; Popescu, 1977; Braune, 2000; Dudita, 1989; Lichtenheldt, 1995; Lederer, 1993; Lin, 1999; Modler et al., 1998; 2001; Modler, 1979; Neumann, 1979; 2001; Stoica, 1977; Petrescu and Petrescu, 2011c-d; Petrescu, 2012d-e; 2016; 2017 a-q; Aversa et al., 2017 a-e; 2016a-o; Mirsayar et al., 2017; Petrescu and Petrescu, 2016a-c, 2013a-d, 2012a-d, 2011a-b; Petrescu, 2012a-c, 2009; Petrescu and Calautit, 2016a-b; Petrescu et al., 2016a-b; Maros, 1958; Modler and Wadewitz, 2001; Manolescu, 1968; Margine, 1999).

Materials and Methods

Bars and gears are used everywhere today, but a wide range of uses is robotics and mechatronics. Since ancient times, automations and mechanization have been used with mechanisms consisting of chains of bars and gears. These were obviously used for the purpose of transmitting the movement and its transformation, that is to say, as a mechanical transmission. Gear and bar automation are used today as modern mechanical transmissions, serial and parallel robots, machine building industry and all industrial areas where automation has penetrated.

In fact, robotic gears and gears are the basis for mechanical transmissions to robots and at the same time have other roles such as balancing, support, etc. The most commonly used gears are tapered, conical gears, because they work faster, more dynamically, occupy less space, have fewer toothed gear teeth, low volume, light movement different directions and a multitude of features that make them irreplaceable within mobile mechanical systems.

The gears and gears are increasingly used in the construction of manipulators and industrial robots, especially in the MO. In the kinematic openings of positioning mechanisms (MPz) of the robots, also referred to as trajectory generators, a first kinematic chain with bars is attached, to which is attached a kinematic chain with cylindrical, conical and hypoid gears.

The mechanical chains that can be made of conical mechanical transmissions and bars are complex, extremely complex and can work on different spaces and axes, with inclines and directional changes as desired. From this point of view, they can not be replaced by other types of mechanical mechanisms or transmissions.

A complex kinematic scheme (Fig. 1) with bevel gears and conical gears of a manipulator-robot with 6 + 1 mobilities is analyzed, where the positioning mechanism (RRR) is not distinguishable from the RRR orientation mechanism. The two kinematic chains of MPz (RzRxxRx) and MO (RzRxRxRz) are staged (in extension). At the end (O6 point) of the articulated chain 0001203O4O5 the gripping mechanism (MAp), made with two articulated parallelograms, is attached.

All 6 + 1 kinematic chains are operated by means of worm gear reducers with electric motors located at the base (Fig. 1). The kinematic chain with bars is simplified to the left of Fig. 1 and to the right is an axial projection of the complete kinematic scheme of the gear with gears and gears. The articulated bars (0, 1, 2, 3, 4, 5, 6) with six movable elements are the main kinematic chain to which are attached six kinematic chains with conical gears (Petrescu and Petrescu, 2011c).

The mobility of the complex gear with bars and gears is calculated using the general formula (Antonescu, 2003):

\[ M = \sum_{s=1}^{6} (m \cdot C_s) - \sum_{r=2}^{6} (r \cdot N_s) \]  

(1)
Fig. 1: A complex kinematic scheme with bevel gears and conical gears of a manipulator-robot with 6 + 1 mobilities

The structural - geometric parameters of the complex mechanism are:

\[ m = 1, C_1 = 47; m = 2, C_1 = 27; m = 5, \]
\[ C_2 = 7; n = 45, r = 3, N_3 = 29; r = 6, N_4 = 7 \]

The total number of independent closed contours is calculated using the formula:

\[ N_c = \sum_{n=1}^{5} C_n - n = 47 + 27 + 7 - 45 = 36 \]  \hspace{1cm} (2)

Of the 36 contours it identifies \( N_6 = 7 \) şi \( N_3 = 29 \), so from (1) it follows:

\[ M = (1 \cdot 47 + 2 \cdot 27 + 5 \cdot 7) - (3 \cdot 29 + 6 \cdot 7) = 7 \]  \hspace{1cm} (3)

**Results**

**MPz Structure**

It is considered a RRR type RRR variant R || R || R (Fig. 2), which is the kinematic chain with bars, to which are attached two kinematic chains with r.d. conical.

![Fig. 2: The kinematic scheme of an MPz type RRR variant R || R || R](image)

The drive is made by electric motors placed on the base of one side of the open case.

The M1 engine, by means of a cylindrical gear, actuates the arm 1 which rotates about the fixed axis D1 (two coaxial bearings in the fixed housing are provided).

The M2 engine acts arm 2 through the kinematic chain attached to bar 1, consisting of two orthogonal conical gears.

The arm 2 rotates about the mobile axis \( \Delta_2 \), this movement being possible by means of two coaxial bearings mounted in the arm 1.

The M3 engine operates the bar 3 via the kinematic chain formed by four orthogonal conical gears.

The bar 3 rotates around the mobile axis \( \Delta_3 \), which is rotatable in two coaxial bearings mounted at the end of bar 2.

The mobility of the bars (arms) and the conical gears shall be calculated with the formula:

\[ M = C_1 + 2 \cdot C_2 - 3N_3 \]  \hspace{1cm} (4)

Following the kinematic scheme of the complex mechanism with bars and toothed wheels (Fig. 2) the following structural-topological parameters are established:

\[ m = 1, C_1 = 10;r = m = 2, C_2 = 7;r = 3, n = 10, N_3 = 7 \]  \hspace{1cm} (5)

With these numerical values, in formula (4) we obtain:

\[ M = C_1 + 2 \cdot C_2 - 3N_3 = 10 + 2 \cdot 7 - 3 \cdot 7 = 3 \]  \hspace{1cm} (6)

According to the three mobilities, the actual movement of the mechanism is broken down into three
partial movements, so that in the operation of this complex mechanism three distinct phases can be followed, one for each mobility:

I. \( \omega_1 \neq 0, \omega_2 = 0, \omega_3 = 0 \), i.e., the M1 engine is in operation and the other two M2 and M3 are locked. In this case, by actuating bar 1, the two lateral kinematic chains (with conical gears) are partially activated

II. \( \omega_1 = 0, \omega_2 \neq 0, \omega_3 = 0 \), when M2 is in operation and M1 and M3 locked. In this phase, the other kinematic chain is also activated

III. \( \omega_1 = 0, \omega_2 = 0, \omega_3 \neq 0 \), i.e., the M3 engine is in operation respectively M1 and M2 are blocked. In this situation, the movement from M3 does not influence the other two kinematic chains

The transmitting functions performed by the kinetic chains with conical gears shall be determined taking into account the following three unit analysis criteria:

a) In the conical gear, in which the rotational axes have chosen meanings (Fig. 3), the plus or minus sign is associated with the transmission ratio, as the common generator of the rolling cones is in the quadrants with the respective number (II and IV) respectively in odd-numbered quadrants (I and III)

b) In the case of a conical gear with moving axles, when the center wheel is fixed (Fig. 4), the relative rotation of the satellite wheel (relative to the mobile arm) is equal to the angular velocity of the arm, taken with the minus sign, multiplied by the transmission ratio the mobile to the fixed wheel in the "fixed arm" hypothesis

c) When two central wheels are in orthogonal conical gear with a satellite wheel (Fig. 5), if one of the central wheels is stationary, the other central wheel rotates twice with the angular speed of the harness:

**Criterion 1**

It is known that in the cylindrical gear (parallel axes) the transmission ratio is negative (external gearing) or positive (at the inner gearing)

To perform a unitary kinematic analysis, the convex ratio with fixed axes x and y (Fig. 3) is defined as an algebraic size.

The transmission ratio of a conical gear (with fixed axes) is uniquely defined by the general expression:

\[
i_{12} = \frac{\omega_1}{\omega_2} = (-1)^n \cdot \frac{z_2}{z_1}
\]

(7)

In quadrants I and III \((n = 1, 3)\) of (7), a negative size results (Fig. 3b).

In quadrants II and IV \((n = 2, 4)\) of (7) a positive value is obtained: (Fig. 3a).

**Criterion 2**

In the case of the conical gears with mobile axles (Fig. 4), this is a gear with gears and gears, in which the kinematic chain with bars (0, 3) is attached to the kinematic chain with two conical wheels forming the conical gear (1, 2).

Depending on the orientation of the rotation axes \((\Delta_1\text{ and } \Delta_2)\) as coordinate axes \((x\text{ and } y)\) of the two conical gears, the transmission ratio of the conical gear to the immobilized bar 3 has positive algebraic expression \(i_{12} > 0\) (Fig. 4a ) or negative \(i_{12} < 0\) (Fig. 4b).

The relative angular velocity of the wheel 2 relative to the bar 3 is calculated with the formula:

\[
\omega_{23} = (\omega_1 - \omega_3) \cdot i_{12}, \text{ where } i_{12} = \frac{1}{i_{12}}
\]

(8)

If the center wheel 1 is immobilized by the lock \((\omega_1 = 0)\), then the satellite wheel 2 rotates relative to the bar 3 at the relative angular velocity:

\[
\omega_{23} = -\omega_3 \cdot i_{12}^3
\]

(9)

![Fig. 3: The convex ratio with fixed axes x and y is defined as an algebraic size](image)
**Criterion 3**

In the kinematic schematics of conical gears and conical gears (Fig. 2), conically-shaped three-wheel kinematic chains are often attached to a kinematic chain with a single articulated bar (Fig. 5).

In the case of orthogonal conical gears (Fig. 5a), the central gears (1, 4) are equal and have the same number of teeth \(z_1 = z_4\).

In the non-orthogonal conic gears (Fig. 5b) axes x and y divide the axial plane of the kinematic diagram into four quadrants and the two central wheels are not equal \(z_1 \neq z_4\).

Write the transmission ratio between wheels 1 and 4 (Fig. 5) relative to bar 3, assuming that the rotation axes \(\Delta_2\) are fixed:

\[
\frac{\omega_4}{\omega_3} = \frac{z_4}{z_1} \quad (10)
\]

For orthogonal gears (Fig. 5a) of (10) we deduce:

\[
\frac{\omega_4}{\omega_3} = \frac{z_4}{z_1} \quad (11)
\]

When one of the central wheels is locked, for example wheel 4 \((\omega_4 = 0)\), it is deduced from (11) that the other central wheel 1 rotates at an angular speed equal to twice the angular speed of the bar 3:

\[
\omega_1 = 2\omega_3 \quad (12)
\]

In the kinematic analysis of the complex spatial mechanism (Fig. 2) the three angular velocities \(\omega_1\), \(\omega_4\) and \(\omega_6\) are known. For a unitary calculation of the angular speeds of bars 1, 2 and 3, the first two criteria (1 and 2) apply.

Start the Phase III kinematic calculation when the M1 and M2 engines are locked, so the M3 engine drives the central kinematic chain 6-7 (7') - 8-9 (9') - 3'(3) without affecting the other.

The relative angular velocity of bar 3 relative to bar 2 is calculated in the assumption of fixed rotation axes, so bars 1 and 2 are immobilized:

\[
\omega_{23} = \frac{\omega_4}{\omega_3} \quad (13)
\]

The transmission function specific to this chain is explicitly written by applying criterion 1:
In phase II the M1 and M3 motors are locked and the M2 motor acts on the secondary cinematic chain on the 4-5 (5') - 2' (2) route. The movement of the bar 2 involves the activation of the gears (8,9) and (9', 3') which make the movement towards the immobilized wheel 7'.

The two motion streams in phase II allow the angular velocity to be calculated relative to the axis \( \Delta_2 \) of bar 2 from bar 1 (Fig. 2):

\[
\omega_{24}^{II} = \omega_{12} \cdot i_{24}^{II} \quad (15)
\]

And at the \( \Delta_3 \) axis of bar 3 relative to bar 2, applying criterion 2 above:

\[
\omega_{34}^{II} = -\omega_{32} \cdot i_{34}^{II} \quad (16)
\]

The transmission functions of formulas (15) and (16) are explicitly written, applying criterion 1:

\[
i_{24}^{II} = i_{24}^{II} \cdot i_{24}^{II} = \left( \frac{z_4}{z_2} \right) \left( \frac{z_2}{z_4} \right) = \frac{z_4 \cdot z_2}{z_2 \cdot z_4} \quad (17)
\]

\[
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\]

Phase I is characterized by the blocking of the M2 and M3 engines and the M1 engine through the gear unit with the ratio \( i_c \) acts on the bar 1, the angular speed of which is as follows:

\[
\omega_{12}^{I} = \omega_{ca} \cdot i_c = \omega_c \quad (19)
\]

The rotation of bar 1 determines partial additional movements in each of the other two kinematic chains (Fig. 2), resulting in the relative angular speeds of the bar 2 relative to 1 or the bar 3 relative to 2.

For the calculation of these relative angular speeds, criterion 2 is applied, knowing that central wheels 4 and 6 are stationary:

\[
\omega_{54}^{I} = -\omega_{64} \cdot i_{54}^{I} \quad (20)
\]

Respectively:

\[
\omega_{54}^{I} = -\omega_{64} \cdot i_{54}^{I} \quad (21)
\]

The transmission functions of formulas (20) and (21) are explained in (17) and (14) respectively, the explicit form being according to the teeth numbers:

\[
i_{54}^{I} = \frac{-z_4 \cdot z_2}{z_2 \cdot z_5} \quad (22)
\]

The general case is when all three engines M1, M2 and M3 are started, which corresponds to the overlapping of the three phases analyzed above.

It is of interest in calculating the rotations and angular speeds of the kinematic chain bars 1, 2 and 3 (Fig. 2).

For bar 1, the angular velocity is given by formula (19) and for bar 2 the relative angular velocity relative to the \( \Delta_2 \) axis is calculated by summing the expressions (20) and (15):

\[
\omega_{24}^{I} = \omega_{24}^{II} + \omega_{24}^{II} = -\left( \omega_c - \omega_a \right) \cdot i_{24}^{II} \quad (24)
\]

where, the transmission function \( i_{24}^{II} \) has the expression (22).

The rotation and relative angular velocity of the bar 3 with respect to the \( \Delta_3 \) axis is obtained by summing the angles or angular velocities obtained in the three phases. Thus, the relative angular velocity of the bar 3 relative to the \( \Delta_1 \) axis (Fig. 2) results from the summation of the expressions (21), (16) and (13):

\[
\omega_{34}^{I} = \omega_{34}^{II} + \omega_{34}^{II} + \omega_{34}^{II} = -\left( \omega_c - \omega_a \right) \cdot i_{34}^{II} - \omega_{44} \cdot i_{44}^{II} - i_{24} \quad (25)
\]

**Discussion**

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Conclusions

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Author’s Contributions

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Ethics

Authors should address any ethical issues that may arise after the publication of this manuscript.

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**Source of Figures**

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