

Original Research Paper

# Blind Source Separation under Semi-White Gaussian Noise and Uniform Noise: Performance Analysis of ICA, Sobi and JadeR

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**Abstract:** A comparative study is presented to evaluate the performance of three important Blind Source Separation (BSS) techniques under noisy conditions. The ability of FastICA, SOBI and JadeR is tested in separating several kinds of signals under noisy conditions, including human speech and frequency-modulated (quadratic and linear FM) signals. Additionally, different mixing matrices are used to inspect the effect of the mixing process. The influence of two types of noise (semi-white Gaussian and uniform) has been investigated under different Signal to Noise Ratios (SNR). The Pearson correlation coefficient (versus signal to noise ratio) between original and recovered signals is used as a performance metric. Despite the wide use of BSS techniques, there has been no extensive study in these directions. It is found that JadeR out performs other BSS techniques under semi-white Gaussian and uniformly-distributed noise.

**Keywords:** Independent Component Analysis (ICA), Second-Order Blind Identification (SOBI), Joint Approximation Diagonalization Estimation for Real Signals (JadeR), Quadratic FM (QFM), Linear FM (LFM)

## Introduction

Recently, Blind Source Separation (BSS) related to Independent Component Analysis (ICA), has a large attention in an engineering field, is widely used in many applications such as removing additive noise from signals and images, separating crosstalk in telecommunication and preprocessing for multi-probed radar-sonar signals (Murata and Ikeda, 1998).

The problem is how to separate independent sources of a given mixed signal if mixing is done by unknown mixing matrix, hence the name Blind Source Separation (BSS). Thus, some techniques have been employed to extract each source from the mixed signals. These sources should be independent of each other and have non-Gaussian distribution (Hyvärinen *et al.*, 2004).

A problem of this type can be found in many applications such as cocktail party problem, where many persons speak simultaneously, surrounded by loud voices and boisterous music. In this case, ICA techniques will be an efficient solution (Hyvärinen and Oja, 2000; Murata *et al.*, 2001). This problem is fundamental in security applications.

On the other hand, to separate mixed signals, the mixing process should be understood. The emitting of many signals

at the same time is the main reason for the mixing process. As a result, this leads to interfering signals with each other, this process depends on unpredictable parameters such as the distance between the sound and recorder device. In addition, these parameters are represented by matrix  $A$ . To complete separation process successfully, matrix  $A$  must be square and invertible. Matrix  $A$  values represent the mixing weights. This matrix linearly multiplied by the source signals and the result is the mixture  $x(t)$ . Thus,  $A$  is called the mixing matrix as shown in Equation (1) (Hyvärinen and Oja, 2000).

ICA requires achieving of two presumptions. First, the observed mixture must be linear combinations of independent signals (do not give any information about each other), where the second is non-Gaussianity (Hyvärinen and Oja, 2000). ICA techniques easily separate signals, although there are other methods can do that, but ICA can do without prior knowledge about the signals and context (Hyvärinen *et al.*, 2004; Hyvärinen and Oja, 2000).

One of the important algorithms for Independent Component Analysis is called FastICA. It was proposed by Hyvärinen and Oja (2000), as a simple understandable algorithm and other two similar methods which depend on joint diagonalization principle. Second

Order Blind Identification (SOBI) proposed by Belouchrani *et al.* (1997), while Joint Approximation Diagonalization Estimation Real signals (JadeR) was proposed in (Rutledge and Bouveresse, 2013).

The BSS techniques have been tested under the influence of the noise where two types of noise have been added to the signal mixtures. Due to wide use of BSS techniques, their performance under noise will be helpful to a wide range of research directions in communications and signal processing

The remainder of the paper is organized as follows: Section 2 presents the ICA definition, principles and algorithms. Section 3 presents the preprocessing phases before BSS techniques application. Section 4 focuses on different ICA algorithms (JadeR, SOBI and FastICA) simulation under two types of noise (semi-white Gaussian noise and uniform noise). Section 5 states the proposed system. Section 6 shows the simulation and results. Section 7 reviews the conclusions.

## A Brief on BSS Approaches and Algorithms

This section presents an overview of the main BSS approaches and its important algorithms:

### The BSS Problem

Imagine that, there is a room which contains two microphones in different places and in the same room, there are two persons speaking at the same time. The signal resulting from the recording of a speech by each microphone is called mixture, Equation (1) shows the mixing process and Equation (2) shows the un-mixing process:

$$X = AS \quad (1)$$

where,  $X(t) = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$  is the mixture,  $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$  is the mixing matrix and  $S(t) = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix}$  the matrix of source signals.

After estimating the matrix  $A$  using BSS techniques, the inverse can be computed as  $w$ , so the independent components can be simply obtained by:

$$S = wX \quad (2)$$

There is no information about  $A$  and  $S$  hence the name of Blind. Accordingly, BSS techniques can be used to extract  $A^{-1}$ . The next section reviews the ICA which represents the most important BSS techniques.

### Independent Component Analysis (ICA): A Background

Independent Component Analysis (ICA) is a method for finding underlying factors or components from

multivariate (multidimensional) statistical data. The essential characteristic of ICA from other techniques, it looks for components that are both statistically independent and non-Gaussian (Hyvärinen *et al.*, 2004).

ICA is expected to separate the incident signals and detect each signal effectively and conveniently. Each mixture and source signals are considered as a random variable instead of the time signal. This random variable must be with zero mean, if it isn't, it must undergo centering by subtracting sample mean. ICA has some principles to be estimated, the data must be statistically non-Gaussian distributed and should be independent (Hyvarinen and Oja, 2000).

### Meaning of Independence (Hyvarinen and Oja, 2000)

Essentially,  $x_1$  and  $x_2$  are called independent variables if the random variable  $x_1$  does not give any information about the random variable  $x_2$  and vice versa. The joint density of independent variables is given by Equation (3):

$$P(x,y) = P_x(x)P_y(y) \quad (3)$$

where,  $P$  is the probability density function,  $x, y$  are random variables.

Independence implies being uncorrelated but not vice versa, so independence is the stronger requirements. Another approach for ICA estimation, inspired by information theory, is the minimization of mutual information, which leads to the same principle for finding most independence variables:

$$I(x_1, x_2, \dots, x_m) = \sum_{i=1}^m H(x_i) - H(x) \quad (4)$$

where,  $I$  is the mutual information of  $m$  random variables  $\{x_1, x_2, \dots, x_m\}$ ,  $H(x_i)$  is the entropy of  $x_i$ ,  $x_i$  are random variables,  $x = \{x_1, x_2, \dots, x_m\}$ .

The differential entropy can be computed to any random vector  $x$  by Equation (5):

$$H(x) = - \int f(x) \log(f(x)) dx \quad (5)$$

where,  $f(x)$  is the joint density function.

### Non-Gaussianity Measurements

It's impossible to use ICA for Gaussian random variables because Gaussian random variables have symmetric joint density, so mixing matrix  $A$  cannot be estimated (Hyvärinen *et al.*, 2004).

Gaussian distribution (aka "normal distribution") is encountered almost everywhere in nature and has bell-shaped probability distribution function as follows (Hyvärinen *et al.*, 2004):

$$\rho(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu_x)^2}{2\sigma^2}} \quad (6)$$

where,  $\sigma$  is standard deviation and  $\mu_x$  is mean of  $x$ .

Non-Gaussianity must be tested using non-Gaussianity measurements in order to estimate independent components, these measurements are listed below.

#### a. Kurtosis

Kurtosis or the fourth-order cumulant is the classical non-Gaussianity measure. If  $x_1$  and  $x_2$  are two independent random variables, it holds that:

$$kurt(x_1, x_2) = kurt(x_1) + kurt(x_2) \quad (7)$$

where, *Kurt* is kurtosis.

Thus, kurtosis equals to zero if the random variable is Gaussian and non-zero for the non-Gaussian variable (Hyvarinen and Oja, 2000). It is not a powerful measure because it is sensitive to outliers.

#### Negentropy

Another non-Gaussianity measure is called Negentropy. It is defined in information theory as the distance between the entropy of the Gaussian distribution of a random variable and the entropy of the same random variable. For any random variables, Negentropy can be computed using Equation (8):

$$Negentropy(\xi) = H(\xi_{gauss}) - H(\xi) \quad (8)$$

where,  $\xi$  is any random variable,  $\xi_{gauss}$  is Gaussian random variable having the same covariance matrix,  $H$  is the entropy.

The differential entropy can be computed for any random variable according to Equation (5).

#### Approximation of Negentropy

Approximations of Negentropy are very good compromise between the properties of the two classical non-Gaussianity measures given by kurtosis and negentropy.

The classical method of approximating Negentropy is using higher-order moments. However, the validity of such approximations may be limited. They suffer from the non-robustness encountered with kurtosis. To avoid the problems encountered with the preceding approximations of Negentropy, other approximations were developed in (Hyvärinen, 1998). The approximation of Negentropy can be computed by Equation (9):

$$J(y) \approx (\mathcal{E}\{G(y)\} - \mathcal{E}\{G(v)\})^2 \quad (9)$$

where,  $v$  is a Gaussian variable of zero mean and unit variance, the variable  $y$  is assumed to be of zero mean

and unit variance and the function  $G$  is a non-quadratic function;  $\mathcal{E}$  is the expectation operator. There are many suggestions for this function as follows:

$$G_1(u) = u^4 \quad (10)$$

$$G_2(u) = \frac{1}{a_1} \log(\cosh\{a_1 u\}) \quad (11)$$

$$G_3(u) = -\exp\left(\frac{-u^2}{2}\right) \quad (12)$$

Because  $g_1(u)$  goes to  $\infty$  very fast it is not robust. In contrast,  $g_2(u)$  and  $g_3(u)$  are more robust, because  $g_2(u)$  slowly goes to  $\infty$  and  $g_3(u)$  is bounded. The derivatives of these functions are used in FastICA to maximize the non-Gaussianity.

Some useful preprocessing steps must be applied to the mixture before applying BSS techniques, to simply extracting source signals from their mixture. The next section will discuss these steps briefly.

## Preprocessing of BSS Techniques

Before each BSS techniques, there are two preprocessing phases that should be applied to the mixture to de-correlate the data and simplify the separating process, these pre-processing phases are described as follows (Zeng *et al.*, 2000):

#### Centering

The essential and necessary preprocess phase is to centre mixture  $x$ , subtract its mean  $m = E\{x\}$  to make  $x$  have zero-mean. This process removes all outliers in the mixture  $x$  and makes the separation process easier.

#### Whitening

Another useful phase which follows the centering phase and proceeds the application of ICA algorithm, it is whitening the observed vector  $x$ . This vector must be transformed linearly after the transformation. The new white vector  $x$  will be obtained. The components of the white vector are uncorrelated and their variances equal unity. The covariance matrix of whitened data will be an identity matrix as Equation (13):

$$\text{cov}(x) = \mathcal{E}\{xx^T\} \quad (13)$$

Whitening algorithm steps will be described as follows:

#### Whitening Algorithm

1. Computing covariance matrix  $C$  for observation signal  $x$
2. Computing eigenvalue decomposition of  $C$  to get diagonal matrix  $D$  and orthogonal matrix  $V$ , where  $V$  is the (orthogonal) matrix of eigenvectors of  $\text{cov}(x)$

3. Computing whitened data  $W_x$  by Equation (14):

$$W_x = VD^{-1/2}V^T x \quad (14)$$

The next section will illustrate the most common BSS techniques and their algorithms.

### BSS Techniques: An Overview

This section displays the most effective BSS techniques in application.

#### FastICA

One of the most crucial BSS techniques is FastICA. It depends on fixed point scheme of Newton Iterations to project the data on the direction that maximizes the non-Gaussianity. The FastICA technique has three versions depending on the non-Gaussianity function used in each algorithm, Kurtosis, Negentropy and Approximation of Negentropy. The algorithm of FastICA which has been discussed in this paper depends on the approximation of Negentropy that maximizes the non-Gaussianity using Equation (9).

Fast ICA algorithm steps can be described as follows:

1.  $C$  = number of the components,  $i = 1$
2. Start  $w_i$  with random values.
3. Select efficient approximation negentropy function  $g$  and compute its derivative  $g'$ .
4. Calculate  $w_i^T x$  where  $x$  is the whitened data.
5. Compute  $g(w_i^T x)$  and  $g'(w_i^T x)$
6. Compute the new vector of weights called  $w_{i+}$  by the following equation:

$$w_{i+} = \mathcal{E}\{x g(w_i^T x)\} - \mathcal{E}\{g'(w_i^T x)\} w_i$$

7. If  $i > 1$  then compute  $w_i = w_i - (w_i^* w_{i-1})^* w_{i-1}$
8. Normalization  $w_i = \frac{w_{i+}}{\|w_{i+}\|}$
9. If the difference between the new weights and previous ones is less than 0.01 (convergence is true) go to step 9, else go to step 4.
10. If  $C = i$  then Exit, else  $i = i + 1$ , go to step 4.

#### Second-Order Blind Identification (SOBI)

Another BSS technique that is widely used to extract source signals from their mixture  $x$ , it exploits the coherence time of the source signals depending on second-order statistics only. Joint diagonalization is an essential step in this technique. This step is applied to the correlation matrices which estimated with a different time lags. Equation (15) shows correlation matrices estimation:

$$R_x(\tau) = \mathcal{E}\{x(t)x(t-\tau)^T\} = AR_x(\tau)A^H \quad (15)$$

For  $\tau \neq 0$ .

SOBI method consists of three primary steps: Whitening, correlation matrices estimation and joint diagonalization. The whitening step involves a linear transformation of the observed data so that the whitened data are uncorrelated data with unit variance as mentioned above.

The algorithm of SOBI can be described as follows (Belouchrani *et al.*, 1997; Gorodnitsky and Belouchrani, 2001):

- 1- Estimating Correlation matrices of the whitened data for each sample using lag  $\tau$  (Matsubara *et al.*, 2015):

$$R(\tau) = \mathcal{E}\{x(t+\tau)x(t)^T\} \quad (16)$$

where,  $x$  is whitened data and  $\tau$  is time-lag.

- 2- Compute joint diagonalization for the resulted correlation matrices
- 3- Estimate the mixing matrix in order to extract the source signals

#### Joint Approximation Diagonalization of Eigen Matrices (JadeR)

Another BSS technique depends on forming a fourth cumulant array. In particular, cumulant can be described as the generalization of the mean (first-order auto-cumulant) and the variance (second-order auto-cumulant) to order higher than 2 (Rutledge and Bouveresse, 2013).

In the JadeR algorithm, the cumulant of the signals with themselves is called auto-cumulant, while the cumulant of all combinations of signals is called cross-cumulant. If the signal vectors are independent, then their fourth-order cross-cumulant will be zero and auto-cumulants maximal (Rutledge and Bouveresse, 2013). JadeR algorithm uses fourth-order statistical cumulants to calculate its cost function, which is a measure of signal independence and repeatedly rotates the set of un-separated signals to minimize the cost and maximize independence.

JadeR algorithm Steps can be described as follows (Hong and Kim, 2015; Sahonero-Alvarez *et al.*, 2017):

- 1) Compute the 4<sup>th</sup> order cumulant matrix of the whitened signals by storing the most significant eigenvectors on a cumulant matrix.
- 2) Apply Joint diagonalization on the output of step 2 by unitary matrix  $U$ .
- 3) Finally, estimate an inverse of  $U$  to recover the original signals.

### Proposed Study

BSS techniques have been studied for separation of speech and bio-signals. However, there are no studies on their performance when the mixture is contaminated by

noise. This study should not contradict the fact that ICA is not working with Gaussian mixtures. The mixture can be a result of several microphones or signal capturing devices. Such devices may introduce noise. In this study, the mixture in Equation (1) is assumed to be contaminated by noise to receive  $X_n$  instead of  $X$  as shown in the following:

$$X_n = X + n = AS + n \quad (17)$$

SNR is defined:

$$SNR = \frac{\text{Power of } (2 * N) \text{ dimensional mixture}}{\text{Power of } (2 * N) \text{ dimensional noise}} = \frac{p_x}{p_n} \quad (18)$$

The mixture power is calculated by:

$$p_x = p_{AS} = \frac{1}{N} \sum_{i=1}^N x(i)^2 \quad (19)$$

The dB-value of SNR can be computed as follows:

$$SNR_{db} = 10 \log_{10}(SNR) \quad (20)$$

Correlation coefficients can be computed by Equation (21) (Sawada *et al.*, 2007).

$$\rho(x_1, s_1) = \frac{E(x_1 s_1) - E(x_1)E(s_1)}{\sigma_{x_1} \sigma_{s_1}} \quad (21)$$

where,  $x_1, s_1$  are two signals,  $\sigma_{x_1}$  standard deviation of  $x_1$ ,  $\sigma_{s_1}$  standard deviation of  $s_1$ .

The next section discusses the effect of such a condition on the performance of BSS algorithms.

## Simulation and Results

MATLAB (R2018b) has been used as an environment to simulate each BSS technique and compute the statistical results. Different signals (speech, QFM and LFM) signals are simulated under the influence of two types of noise. This section discusses the results of the simulation in detail.

### Two Speech Signals

BSS techniques have been employed to separate mix of two speech signals, these signals are related into two different humans (baby and man), with equal sample rate ( $f_s = 44100$ ) and same length ( $N = 73729$ ). Figures 1 and 2 show the original signals and Figure 3a and 3b shows the mixed signals. Different mixing matrices were used to analyze their effect on the separation process:

$$A_1 = \begin{bmatrix} 0.7 & 0.15 \\ 0.37 & 0.9 \end{bmatrix}; \quad A_2 = \begin{bmatrix} -0.7 & 0.15 \\ 0.37 & 0.9 \end{bmatrix};$$

$$A_3 = \begin{bmatrix} -0.7 & -0.15 \\ 0.37 & 0.9 \end{bmatrix}; \quad A_4 = \begin{bmatrix} -0.7 & -0.15 \\ -0.37 & 0.9 \end{bmatrix};$$

$$A_5 = \begin{bmatrix} -0.7 & -0.15 \\ -0.37 & -0.9 \end{bmatrix};$$

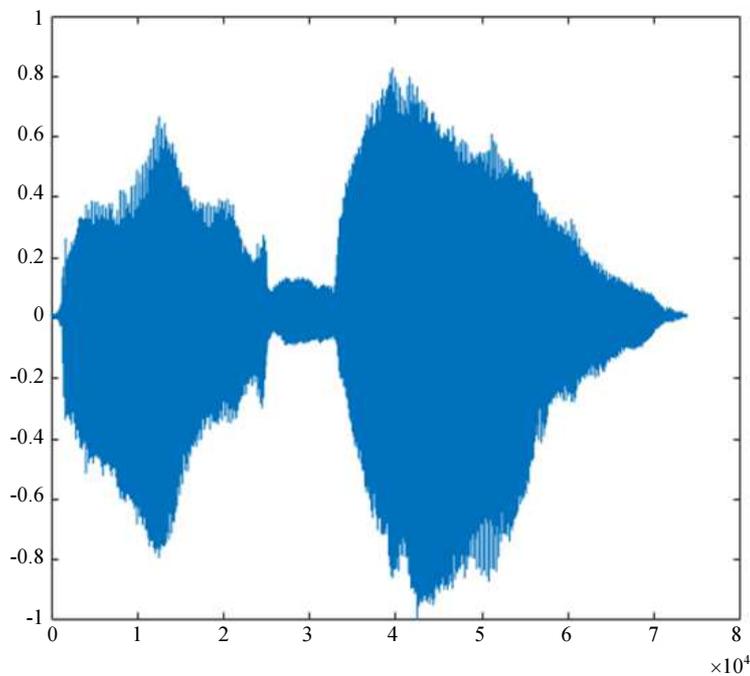


Fig. 1: Source signal 1

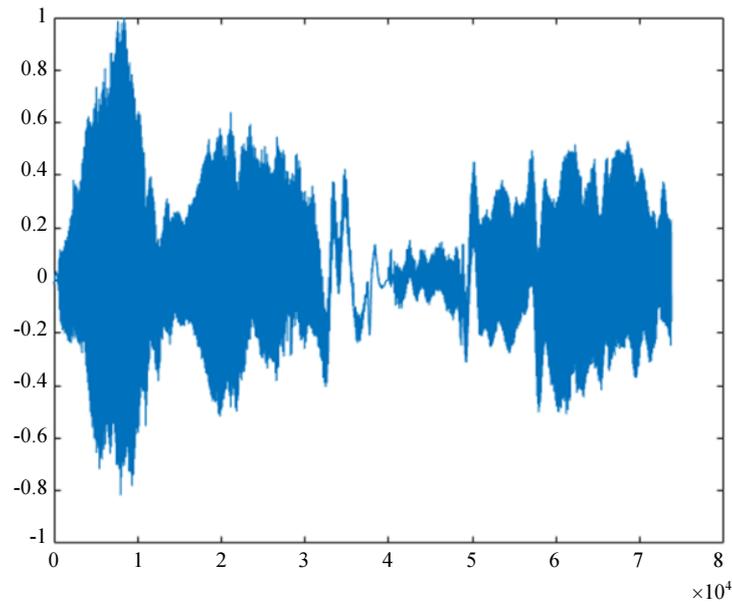


Fig. 2: Source signal 2

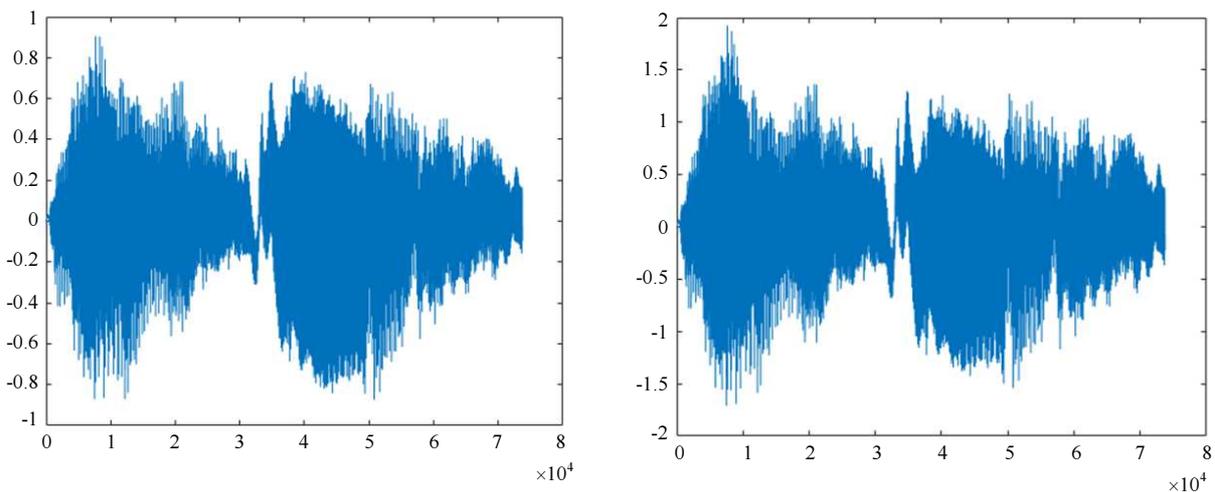


Fig. 3: (a) Mixture1 (b) Mixture2

In case of failure, FastICA algorithm may be re-executed more than once to extract the original signals. Figure 4 shows this state precisely.

Implementation of each algorithm suffers from two ambiguities. First, the extracted signals are not in the same order of the source signals, to solve this ambiguity a permutation step should have occurred in each iteration. Second, the obtained signals do not have the same amplitude as in the original signals, then to solve this ambiguity a normalization process for each amplitude should be done.

BSS algorithms (FastICA, SOBI and JadeR) have been applied to the whitened data. Pearson correlation

coefficient has been used as a performance measure. Figure 5 shows the correlation coefficients between source signals and extracted signals for each technique without using noise. FastICA algorithm was unstable. In contrast, SOBI and JadeR algorithms appear very stable.

#### *Speech Signals with Additive Semi-White Gaussian Noise*

Noisy speech signals have been produced using MATLAB function (*wgn*), this function is used to create 2D noise with different powers corresponding to different signal to noise ratios  $SNR_{dB}$  (ranging from -50

dB to 80 dB). Note that this noise is not exactly white or Gaussian, as it is band limited; also, speech signals themselves behave like non-Gaussian probabilistic signals. Hence, the mixtures are not Gaussian, hence are separable when BSS techniques are applied. BSS

techniques are shown to fail for negative  $SNR_{dB}$  (in the range -50 to -20). From -20dB and above, BSS techniques start to separate the signals.  $SNR_{dB}$  Vs. correlation coefficients have been used as a quality metric for each technique.

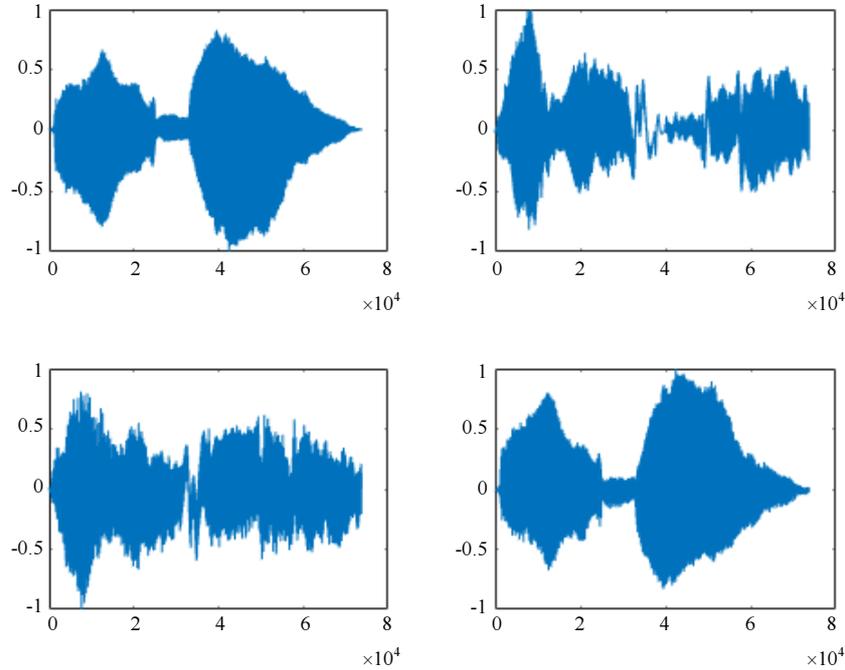


Fig. 4: Extracted signals using ICA

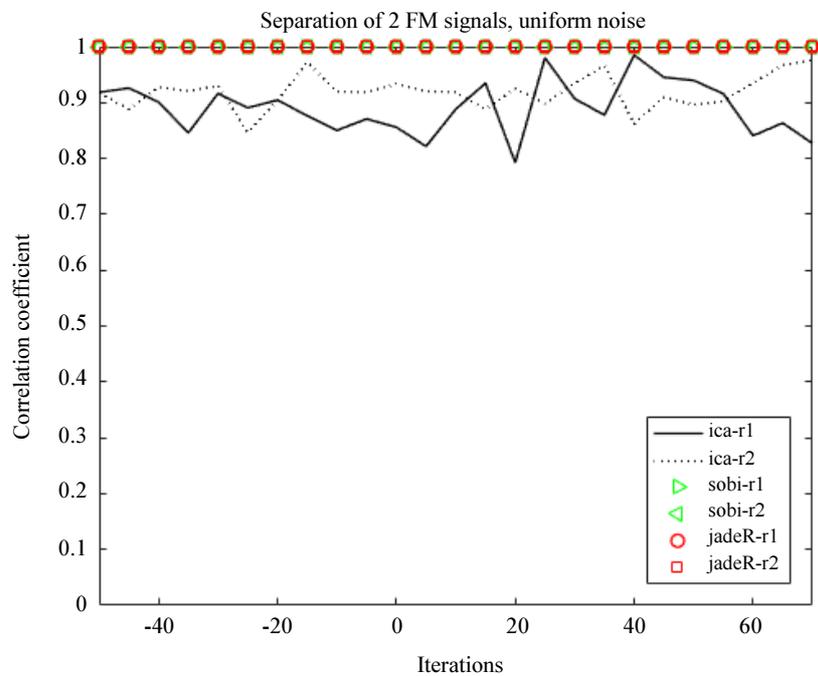
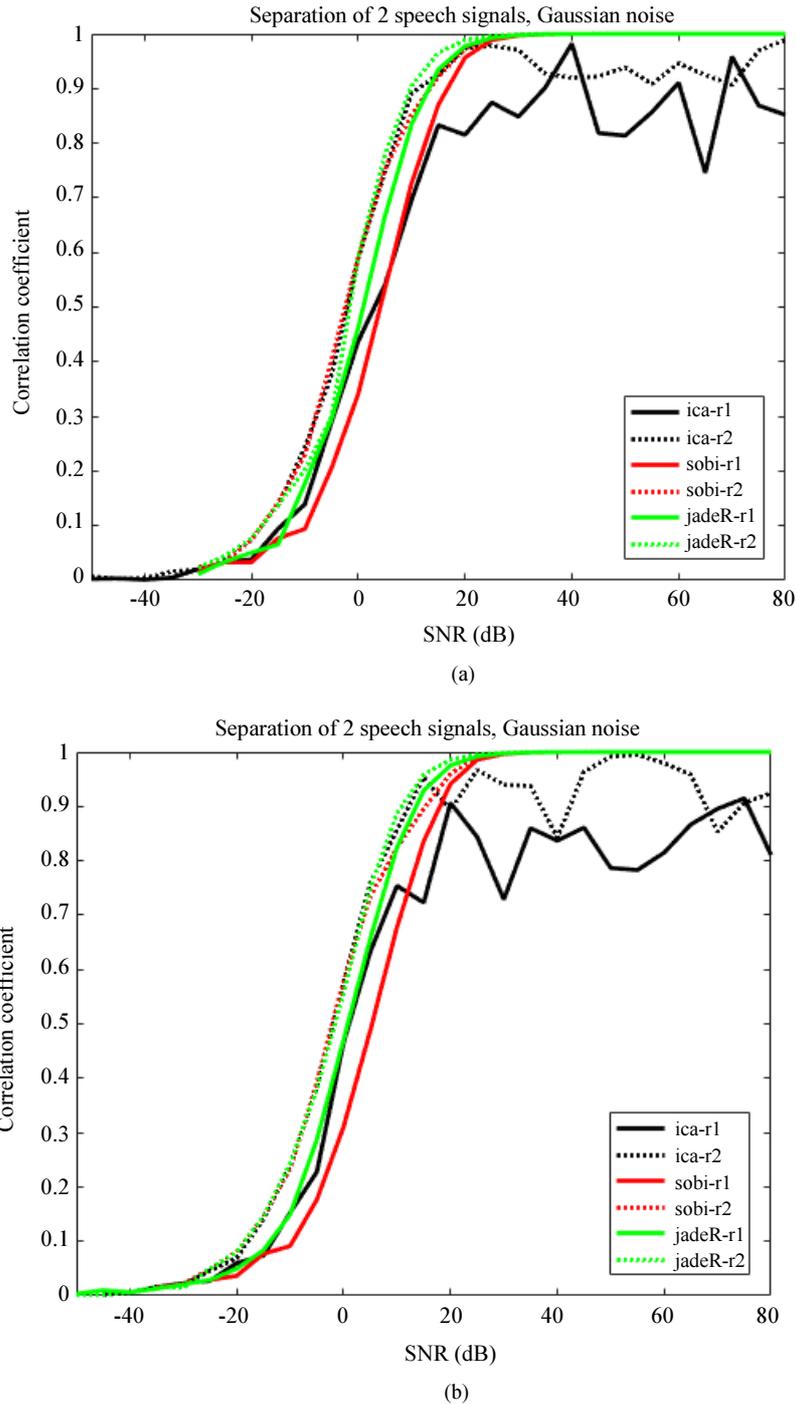


Fig. 5: Separation of speech signals using three methods without noise



**Fig. 6:** (a) Correlation coefficients Vs. SNR for separation of two signals mixed using positive mixing matrix under semi-white Gaussian noise; (b) Correlation coefficients Vs. SNR for separation of two signals mixed using negative mixing matrix under semi-white Gaussian noise

Figure 6a shows the correlation coefficients (between the original signals and the retrieved ones) for each BSS technique under semi-white Gaussian noise, when the signals linearly mixed using positive mixing matrix. Figure 6b shows the correlation coefficients for each

technique under semi-white Gaussian noise when the signals are linearly mixed using negative mixing matrix. The performance of algorithms (FastICA, JadeR) do not affect by the variety of mixing matrix, but the SOBI technique is solely affected.

The analysis of the simulation shows that JadeR is the best algorithm among the compared algorithms, it is very accurate and efficiently more than SOBI and FastICA and it also has more stable results (non-fluctuating). In addition, JadeR requires less execution time than other BSS techniques. Figure 6 shows that JadeR gives better correlation coefficients (between original and recovered signals), hence, better separation accuracy, than SOBI or ICA. A Table 1 of comparison.

### Speech Signal Separation under Uniform Noise

This section explains the effect of uniform distribution noise on speech signals separation. Uniform noise can be computed in Equation (22):

$$r = -a + (2a)\mathcal{U}(0,1) \quad (22)$$

where,  $a = \sqrt{3p_n}, p_n$ , (power of noise),  $\mathcal{U}(0,1)$  is the standard uniform distribution.

Figure 7 shows the mixed speech signals separation under Uniform noise when different mixing matrices are mixing the signals. Under uniform noise, JadeR algorithm remains the best algorithm to separate the speech signals. As a conclusion of BSS techniques implementation, the variety of the mixing matrices do not significantly affect the separation process. Also,

SOBI algorithm affected merely by the diversity of the mixing matrix.

### Two Frequency-Modulated Signals (QFM, LFM)

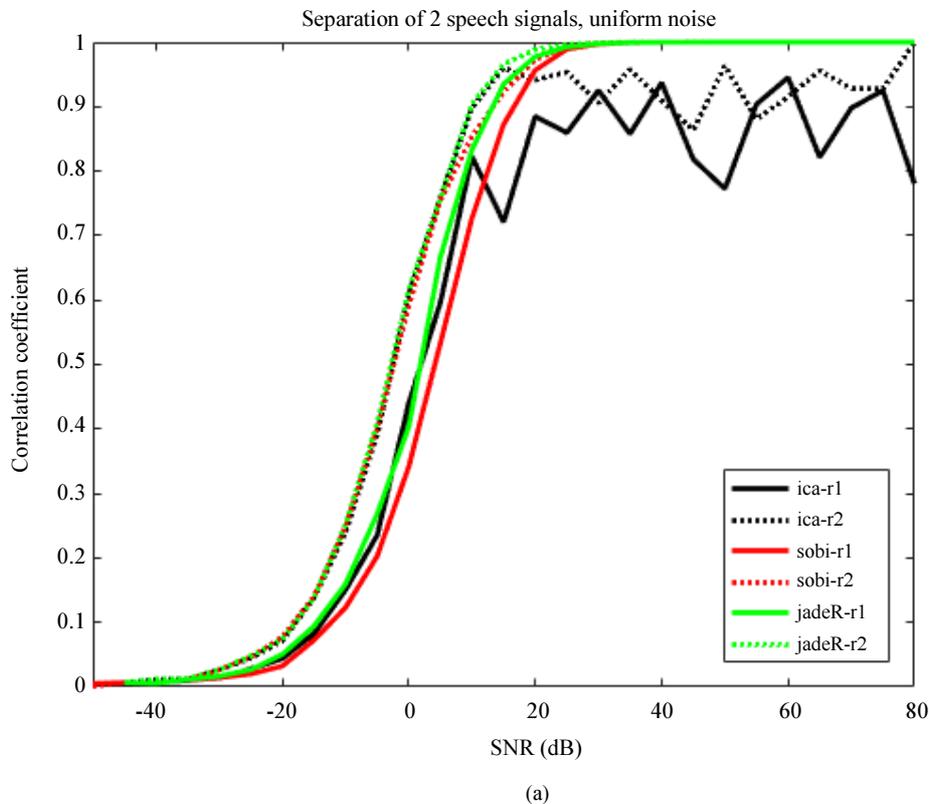
Frequency-Modulated (FM) signals are important in applied signal processing and communications. Linear FM (LFM) has a frequency that changes linearly with time, while non-linear FM has a frequency that follows a non-linear function of time. Here we will only consider quadratic FM (QFM) for non-linear FM. To apply BSS to noisy FM, we consider the original signal  $x(t)$  to be a finite-length LFM or QFM signal of the following forms (Lau *et al.*, 2004):

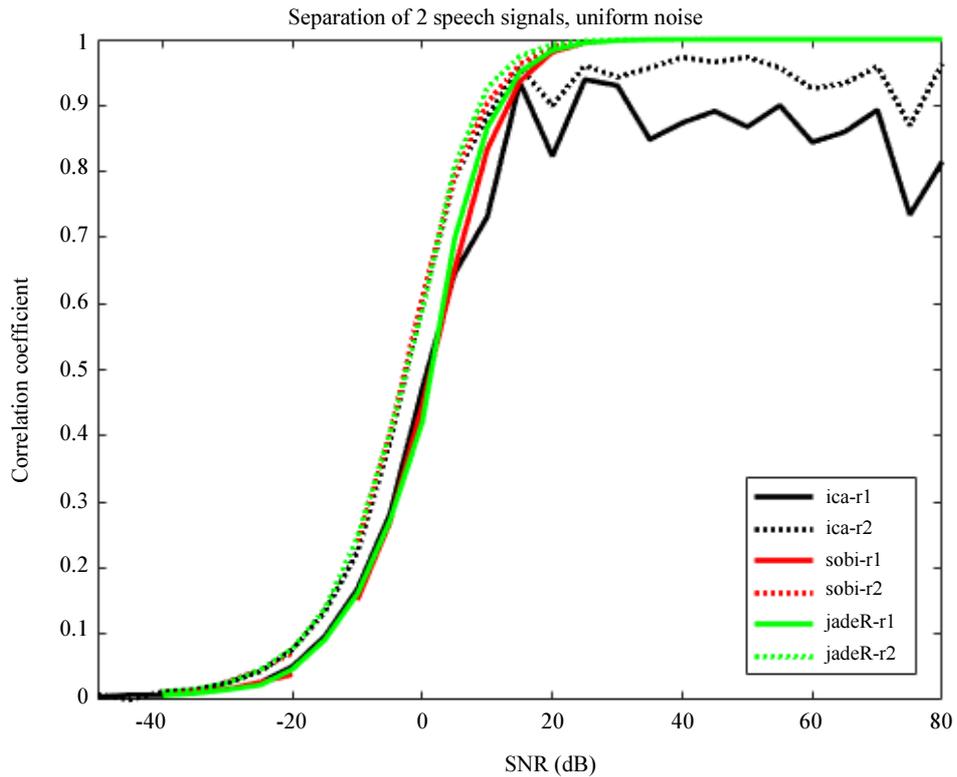
$$\begin{aligned} \text{LFM: } \quad x(t) &= \sin\left[2\pi\left(f_0 t + e t^2 / 2\right)\right] \\ \text{QFM: } \quad x(t) &= \sin\left[2\pi\left(f_0 t + E t^2 / 2 + G t^3 / 3\right)\right] \end{aligned} \quad (23)$$

where,  $e, E$  and  $G$  are the modulation coefficients.

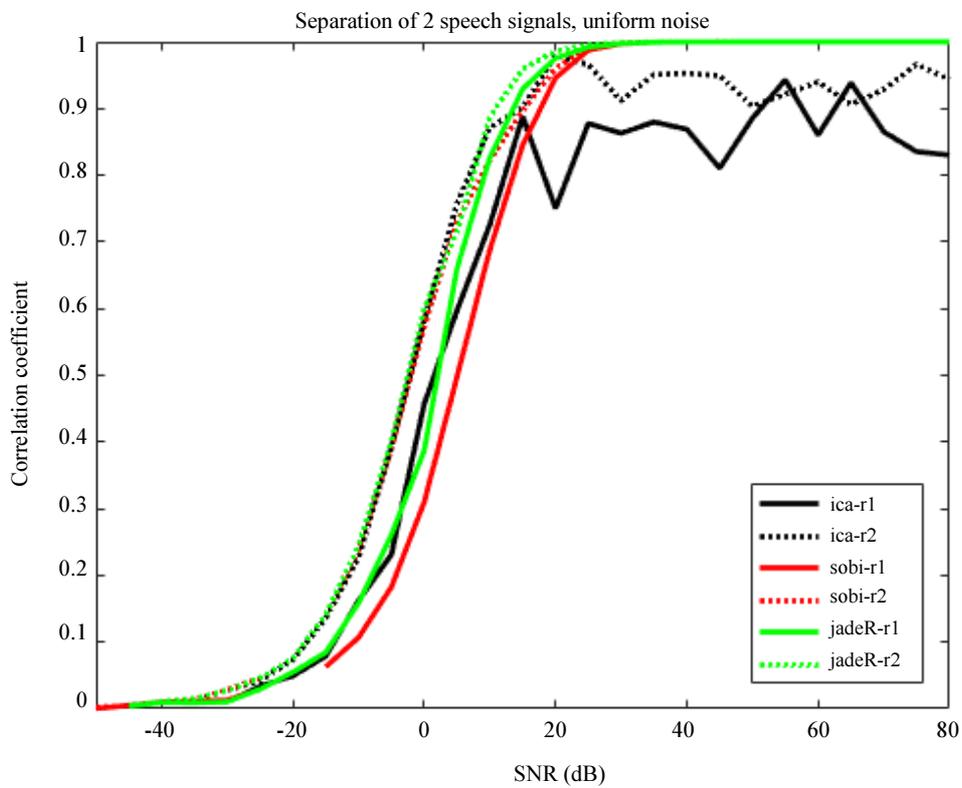
**Table 1:** Correlation Vs. SNR of BSS techniques under semi-white Gaussian noise for mixing matrix  $A = [0.7 \ 0.15; 0.35 \ 0.8]$

| SNR | FastICA | SOBI   | JadeR  |
|-----|---------|--------|--------|
| -10 | 0.1613  | 0.1169 | 0.1943 |
| 0   | 0.3748  | 0.3712 | 0.4944 |
| 20  | 0.9420  | 0.9777 | 0.9798 |

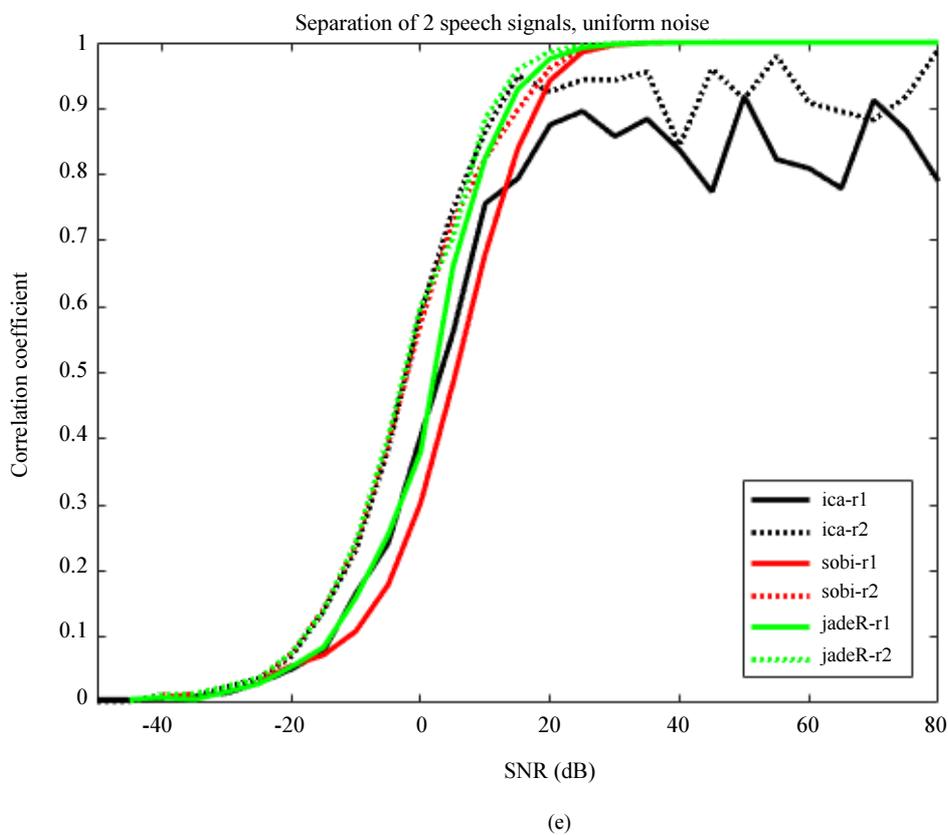
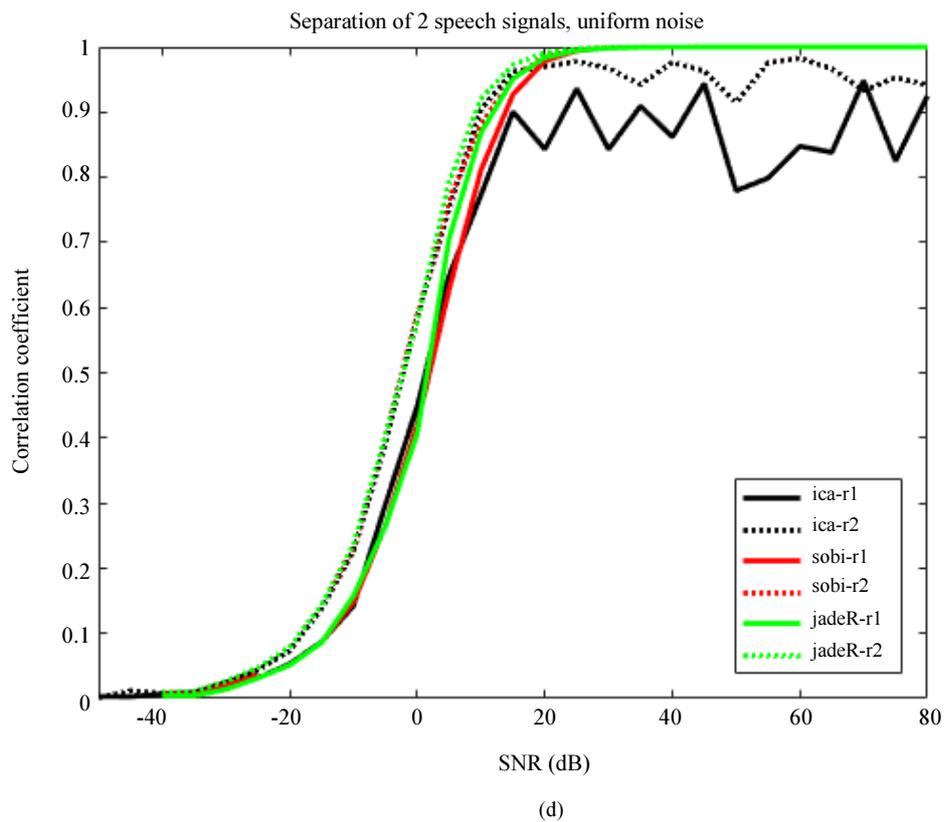


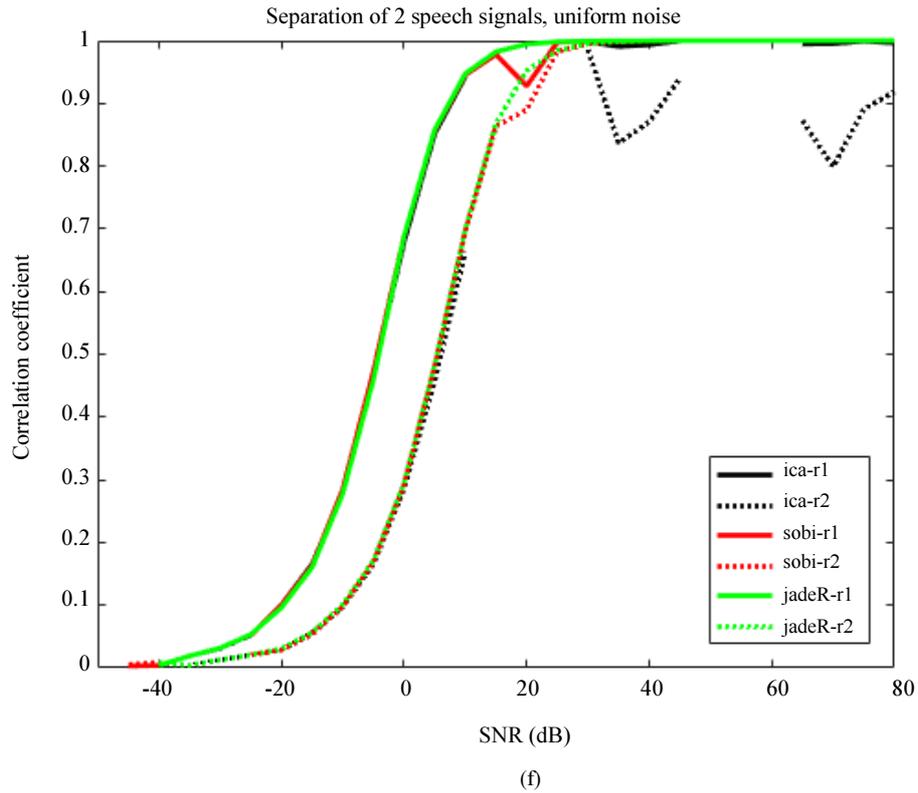


(b)

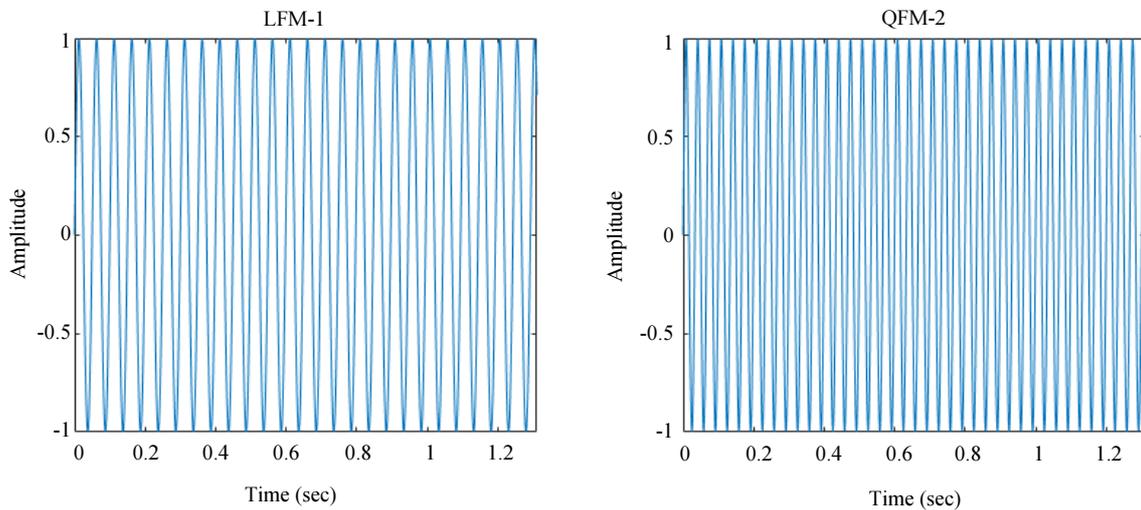


(c)





**Fig. 7:** Correlation coefficients Vs. SNR for separation of two speech signals under uniform noise, the signals are mixed by different types of mixing matrix; (a) all the entries of the mixing matrix are positive,  $A = [0.7 \ 0.15; 0.37 \ 0.9]$ ; (b) mixing matrix has one negative entry only,  $A = [0.7 \ 0.15; -0.37 \ 0.9]$ ; (c) mixing matrix has two negative entries,  $A = [0.7 \ -0.15; -0.45 \ 0.9]$ ; (d) mixing matrix has three negative entries,  $A = [-0.7 \ -0.15; -0.37 \ 0.9]$ ; (e) all the entries of the mixing matrix are negative,  $A = [-0.7 \ -0.15; -0.37 \ -0.9]$ ; (f) entries of the mixing matrix are selected randomly,  $A = \text{random values}$



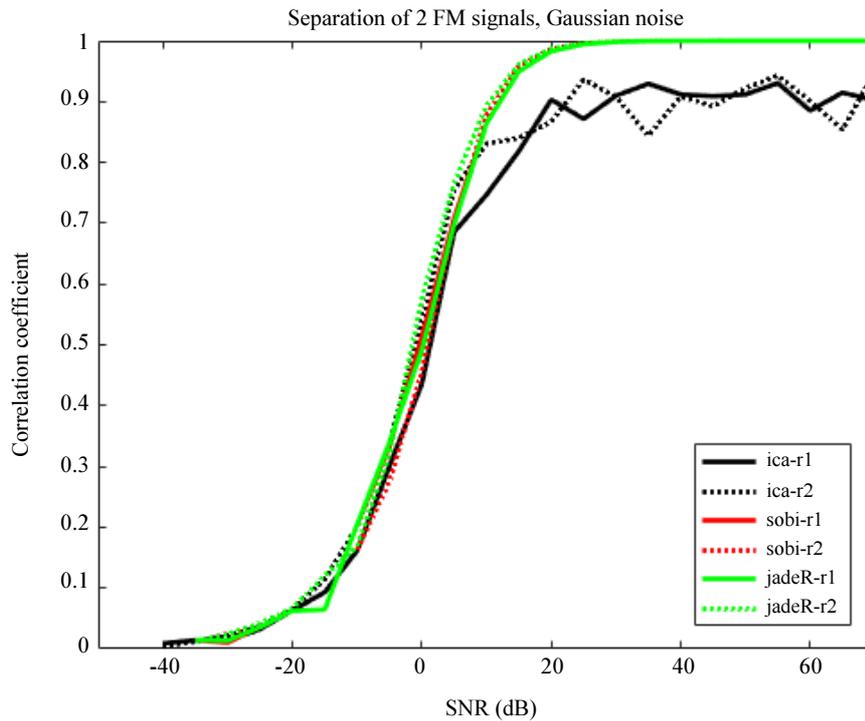
**Fig. 8:** QFM and LFM signals

Figure 8 shows two FM signals (QFM and LFM), these signals are used to inspect the ability of BSS techniques to separate FM signals.

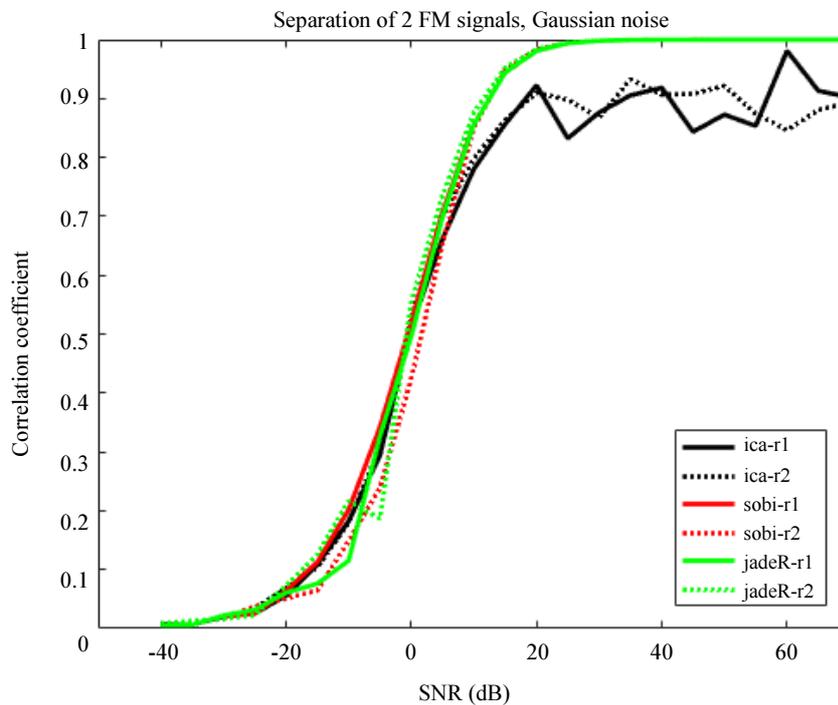
*FM (QFM, LFM) Signals Separation under Noise*

First, FM signals are mixed under semi-white Gaussian noise and separated using BSS techniques which have been

discussed above. Figure 9 shows the correlation coefficient between the extracted signals and the original signals.



(a)



(b)

**Fig. 9:** Correlation coefficients vs. SNR for FM mixed signals under semi-white Gaussian noise (a) QFM mixed with LFM using positive-valued mixing matrix under semi-white Gaussian noise (b) QFM mixed with LFM using negative valued mixing matrix under semi-white Gaussian noise

Second, FM signals are mixed under uniform noise and separated using BSS techniques. Figure 10 shows the correlation coefficients between the extracted signals and the original signals.

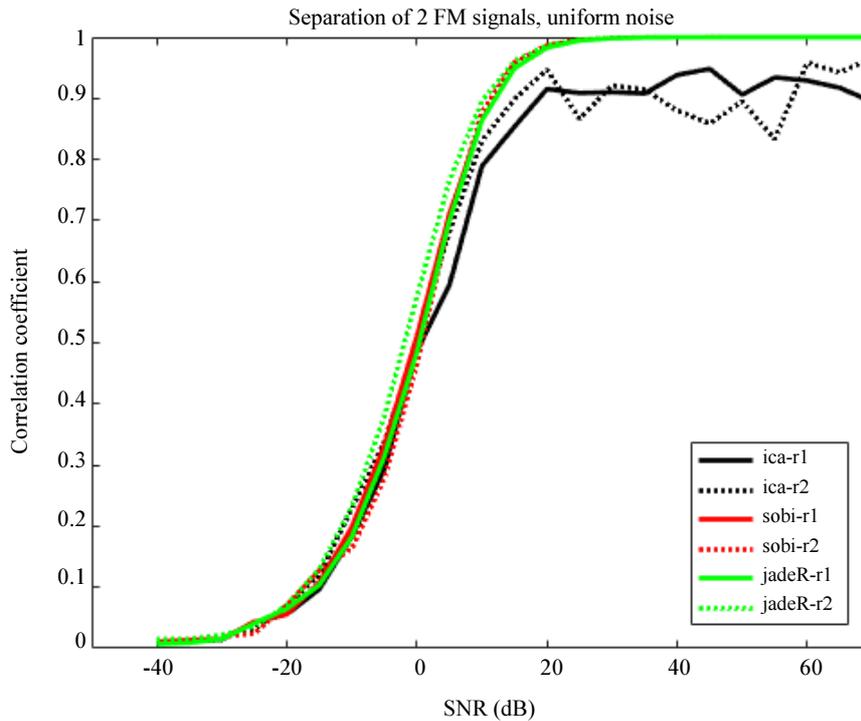
### Discussion

All techniques fail under high-power uniform or semi-white Gaussian noise with  $SNR_{dB}$  (-50 to -20). The separation starts when  $SNR_{dB}$  is (-20) and above. In this

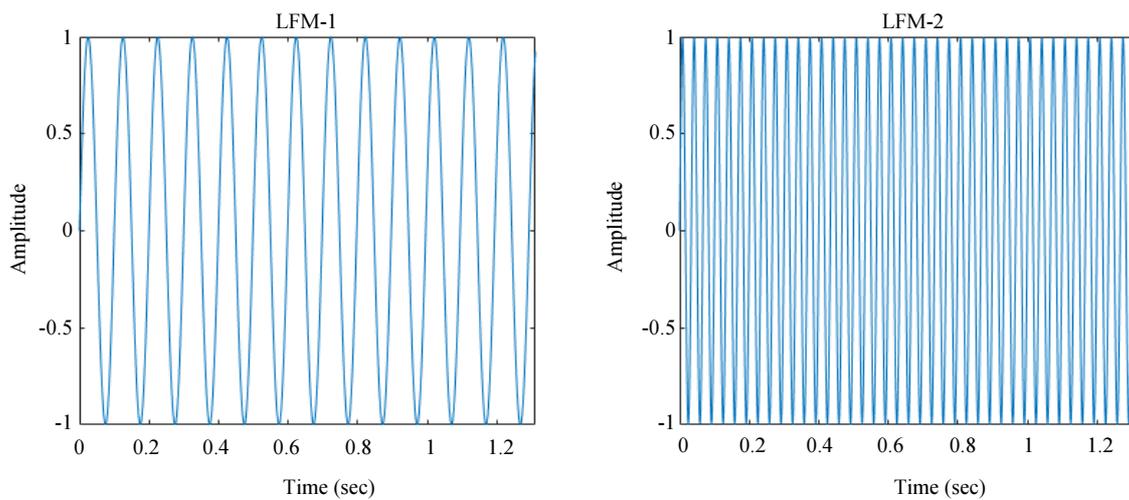
case, JadeR algorithm is better than SOBI and FastICA because it is very stable and has higher correlation values at each SNR.

### Two LFM Signals Separation under Semi-White Gaussian Noise

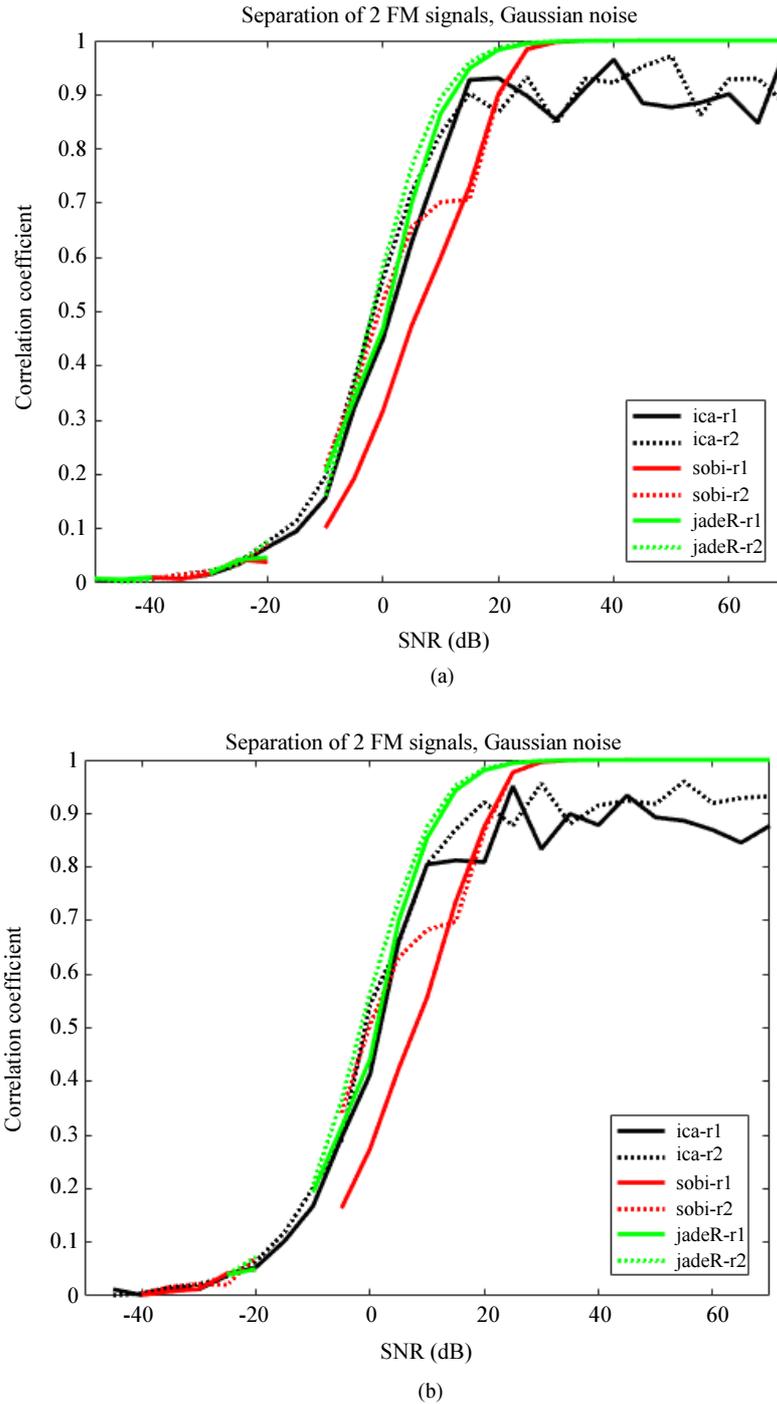
Figure 11 shows two LFM signals with different frequency content, these signals have been mixed using random mixing matrix.



**Fig. 10:** Correlation coefficients Vs. SNR for FM (QFM and LFM) mixed signals under uniform noise



**Fig. 11:** Two LFM signals



**Fig. 12:** (a) Correlation coefficients vs. SNR for two LFM mixed signals mixed by positive mixing matrix under semi-white Gaussian noise; (b) Correlation coefficients vs. SNR for two LFM mixed signals mixed by negative mixing matrix under semi-white Gaussian noise

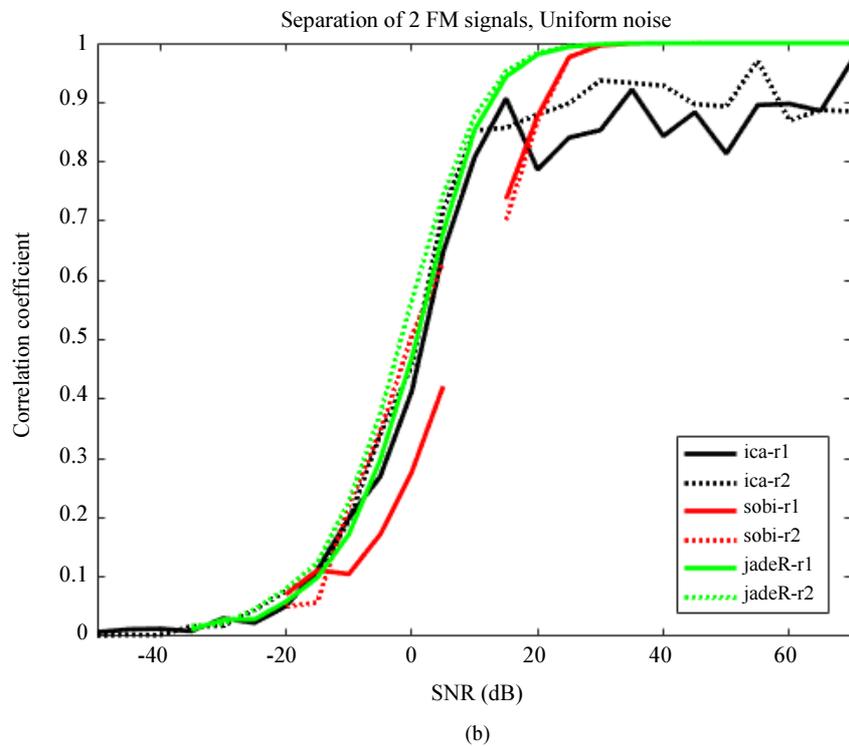
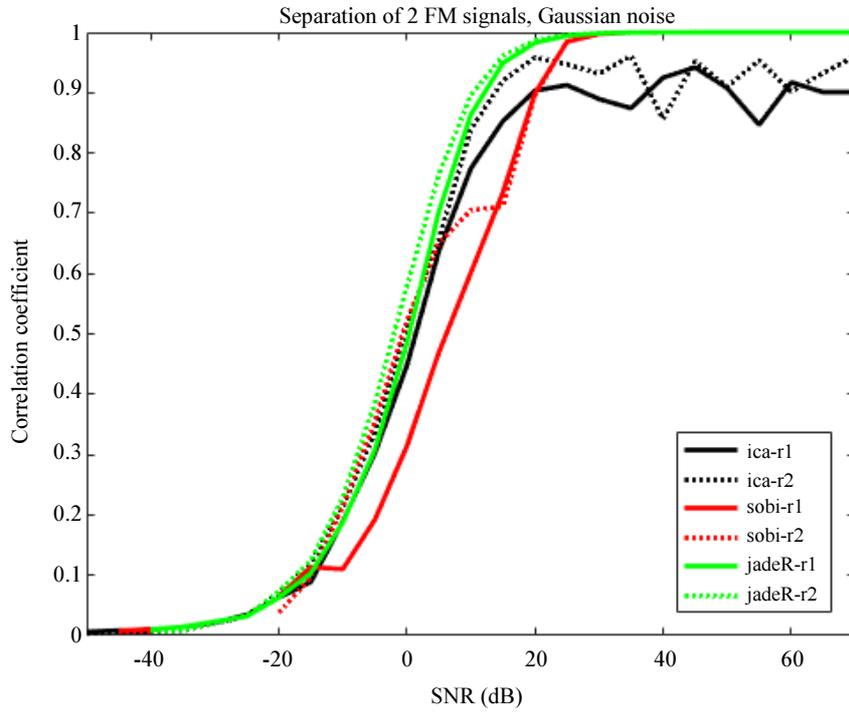
Mixed FM signals are separated using BSS techniques. Figure 12a shows correlation coefficients (between extracted signals and original signals) vs. SNR when original signals are mixed using positive mixing matrix, Figure 12b consider negative mixing matrix.

### Discussion

JadeR algorithm is the most efficient algorithm and SOBI gives lower correlation coefficients as compared with JadeR and FastICA when fixed SNR is considered. Hence,

all previous experiments show that JadeR is the best algorithm as compared with SOBI and FastICA algorithms

and FastICA algorithm is better than SOBI algorithm where SOBI algorithm is unstable in each of the discussed cases.



**Fig. 13:** (a) Correlation coefficients Vs. SNR for two LFM mixed with positive mixing matrix under uniform noise; (b) Correlation coefficients Vs. SNR for two LFM mixed with negative mixing matrix under uniform noise

## Two LFM Signals Separation under Uniform Noise

Figure 13 shows the performance when the mixture is two LFM signals under uniformly-distributed noise.

Figure 13 shows that performance of BSS algorithms under uniform noise is similar to that given by the test results under semi-white Gaussian noise, where JadeR outperforms other techniques.

Future directions will involve testing these algorithms over modern engineering systems, especially wireless channels as in (Mahmoud *et al.*, 2002; 2006) and chaotic communication systems as in (Lau and Hussain, 2005).

## Conclusion

Three major BSS techniques (JadeR, SOBI and FastICA) have been extensively studied and applied to mixed speech and FM signals for the purpose of recovering original signals. Tests included noisy conditions, where two kinds of band-limited noise have been used.

For a noise-free observed data, JadeR and SOBI techniques are more stable than FastICA where their correlation coefficients (between the original and the separated signals) range over (0.99 to 1), whereas FastICA is volatile and requires re-execution more than once to extract the original signals accurately.

For noisy observed data under semi-white Gaussian or uniform noise, the BSS techniques fail at high noise powers (SNR less than -20 dB). Hence, the correlation coefficients are low. JadeR algorithm is the best algorithm among BSS techniques because it is the most resistant to noise.

For noisy and noise-free observed data, JadeR algorithm requires less running time among the compared techniques. This is because it applies the joint diagonalization steps to the cumulant array which has smaller dimensions.

Future directions will involve testing these algorithms over modern engineering systems, especially wireless channels as in (Mahmoud *et al.*, 2002; 2006) and chaotic communication systems as in (Lau and Hussain, 2005).

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## Author's Contributions

**Muna H. Fatnan:** Has contributed to the design and simulation of the proposed system. She did most of the paper write-up.

**Zahir M. Hussain:** Has contributed to the design, analysis, and simulation of the proposed system. He has also contributed to the write-up and language revision.

**Hind R. Mohammed:** Has contributed to the simulation and critical analysis of the proposed system.

## Ethics

The Authors declare that there are no ethical issues associated with this work.

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### Appendix: Flowchart of the Proposed Testing Approach

