

A Reduced Finite Element Model for Analyzing the Transverse Shear Stiffness of Truss-Like Core Sandwich Beam

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Abstract: Problem statement: In structural analysis using finite element software, a number of finite element equations affect the speed of calculation time. A reduced finite element model should be used for analyzing structural responses to reduce the calculation time. **Approach:** This study presents the analysis of transverse shear stiffness of truss-like core sandwich beam using the reduced finite element model based on a unit cell approach. Two kinds of core topologies; an X-truss core and a bi-directional X-truss core, were chosen to be analyzed. The presented reduced finite element model based on the unit cell approach was compared in transverse shear stiffness with the conventional three-point loaded beam approach. **Results:** Results showed that the unit cell approach could be used for analyzing the transverse shear stiffness of truss-like core sandwich beam with a good correlative with the conventional three-point loaded beam approach and a significantly reduced number of finite element equations. **Conclusion:** The transverse shear stiffness of truss-like core sandwich beam can be obtained from the reduced finite element model based on unit cell approach. This reduced finite element model can be used to reduce the number of finite element equations; consequently, the speed of calculation time is increased.

Key words: Finite element analysis, sandwich beam, unit cell, transverse shear stiffness, cell approach, calculation time, finite element equations, factorized transverse, accurate method

INTRODUCTION

In recent structural engineering analysis, a finite element method is a tool for analyzing structural responses, e.g., stiffness of beam structures. This method can be used in corporation with the force and distortion relationship of three-point loaded beam and of unit cell approaches.

The force and distortion relationship of three-point loaded beam approach is an accurate method for analyzing the transverse shear stiffness of sandwich beams (Leekitwattana *et al.*, 2011; Libove and Hubka, 1951). It is, however, an expensive time-consuming method because of the need of repetitive beam tests. In finite element analysis, the repetitive beam tests lead to an expensive calculation time because of a large number of finite element equations.

The force and distortion relationship of unit cell approach is another analytical method for analyzing the transverse shear stiffness of sandwich beams. Libove and Hubka (1951) have used this method in analytical study of simple corrugated core sandwich beams. Lok

and Cheng (2000) have also used it for analyzing the transverse shear stiffness of simple truss core sandwich beams. It was found that the force and distortion relationship of unit cell approach is also an accurate method if applied for the simple truss-like core sandwich beam (Libove and Hubka, 1951; Lok and Cheng, 2000)

This study aims to present an application of the force and distortion relationship of unit cell approach in finite element analysis of complex truss-like core sandwich beams and to outline the advantage of this approach in comparison with the force and distortion relationship of three-point loaded beam approach.

MATERIALS AND METHODS

Three-point loaded beam approach: In practice, the transverse shear stiffness of beams can be deduced from the relationship between the applied vertical load, P and the corresponded vertical deflection at midspan, Δ_z , of a series of three-point loaded beam tests (Nordstrand and Carlsson, 1997; Zangani *et al.*, 2007) as demonstrated in Fig. 1.

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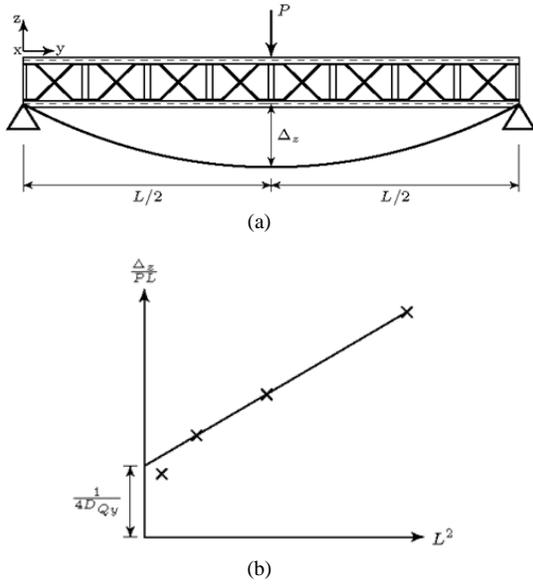


Fig. 1: A method for analyzing the transverse shear stiffness from the force and distortion relationship of three-point loaded beams

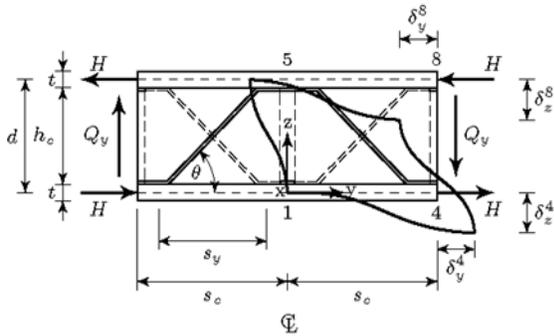


Fig. 2: A method for analyzing the transverse shear stiffness from the force and distortion relationship of unit cell (Leekitwattana *et al.*, 2011)

The relationship between the applied force, P and the corresponded deflection, Δ_z , can be expressed as Eq. 1. This expression provides the basis for determination of the transverse shear stiffness, D_{Qy} , from plotting Δ_z/PL and where L is the length of the beam, from a series of required data P , Δ_z , and L in which D_{Qy} can be yielded from the approximate interception point on the Δ_z/PL axis (Nordstrand and Carlsson, 1997; Souiyah *et al.*, 2009; Urgessa, 2009; Thomas and Dozier, 2010; Mohsen, 2010; Rahman *et al.*, 2009):

$$D_{Qy} = \frac{Q_y}{\frac{\delta_y^4 + \delta_y^8}{d} + \frac{\delta_z^4}{s_c}} \quad (1)$$

Unit cell approach: In practice, the transverse shear stiffness of beams can also be deduced from the relationship between the applied transverse shear force, Q_y and the corresponded deflections Δ_y and Δ_z of a unit cell which is a repetitive unit of a sandwich beam (Leekitwattana *et al.*, 2011), as demonstrated in Fig. 2.

The relationship between the applied transverse shear force Q_y and the corresponded deflections Δ_z and can be expressed as Eq. 2. This expression provides the direct calculation of the transverse shear stiffness, D_{Qy} :

$$D_{Qy} = \frac{Q_y}{\frac{\delta_y^4 + \delta_y^8}{d} + \frac{\delta_z^4}{s_c}} \quad (2)$$

Finite element software: The commercial finite element software ANSYS Release 11 is used in this study. The ANSYS is run under the operating software MS Windows XP Professional Version 2002. The hardware condition is a desktop computer with Intel® Core™ 2 CPU 6600 @ 2.40 GHz and 1.98 GB of RAM.

Finite element models: Three-dimensional finite element models of truss-like core sandwich beam and of its repetitive unit cell, as shown in Fig. 3, are analyzed. The sandwich beam consists of the top and bottom steel faceplates and a series of truss-like core. These parts are modeled using the SOLID45 element type-an eight-node element having three degrees of freedom in nodal translations at each node. In this study, the typical 2 mm finite element mesh size is used. The connections between the faceplates and core elements are defined as fully rigid.

The sandwich beam, as shown in Fig. 3a, has simply supports at the lines 1-1' and 2-2'. An additional constraint boundary condition is set up along the lines 3-3' and 4-4' to reduce the local deformation effect beneath the loading line 5-5' which is subjected to a unit transverse force per unit width of the sandwich beam (Table 1).

The repetitive unit cell, as shown in Fig. 3b, has a fixed support at line 1-1' and a roller support at line 5-5'. Additional constraint boundary conditions are also set up along the lines 4-4' and 8-8' to maintain the displacement equality of both lines in the z -direction.

In this study, the truss-like cores are X-truss core and bi-directional corrugated-strip core, as shown in Fig. 4.

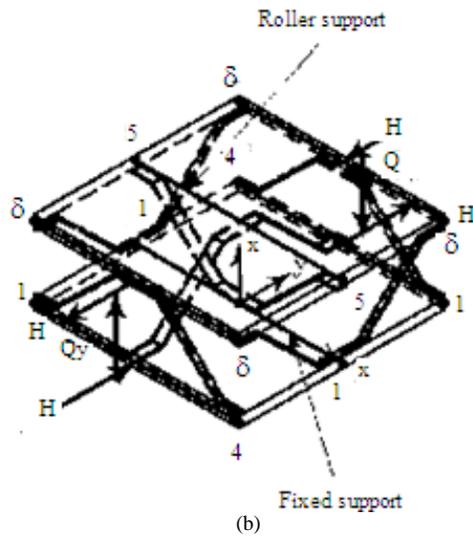
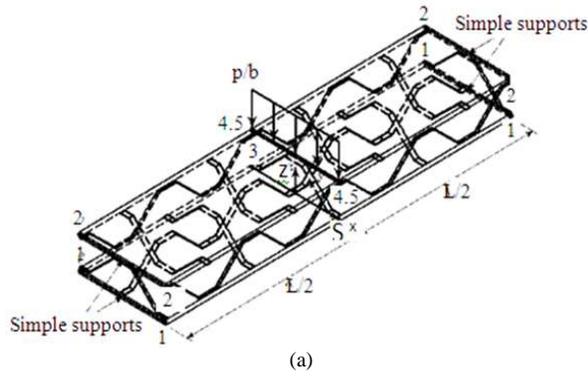


Fig. 3: A finite element model of (a) three-point loaded sandwich beam and of (b) its repetitive unit cell

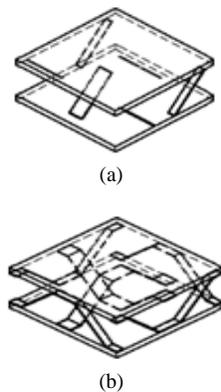


Fig. 2: A model of (a) X-truss core and (b) bi-directional corrugated-strip core

Table 1: Configuration of sandwich beams

Dimensions	Value	Unit
Width of sandwich beam, b	100	mm
Thickness of sandwich faceplate, t	12	mm
Depth of sandwich core, hc	120	mm
Width of core plate, bc	25	mm
Thickness of core plate, tc	2	mm
Length of flat leg of core plate, fc	20	mm

Table 2: Physical properties of steel

Properties	Value	Unit
Yield Stress, f_y	355000	N mm ⁻²
Modulus of Elasticity, E_s	206,000	N mm ⁻²
Poisson's Ratio, ν_s	0.30000	

Material properties of steel: In this finite element study, the steel with perfectly elastic-plastic property is used. In the ANSYS, this material property of steel is defined using the bi-linear model. The tension and compression behaviors of steel are assumed the same. The physical properties of steel are defined in Table 2.

RESULTS

Based on the transverse shear stiffness formulation techniques presented in the materials and methods section, the transverse shear stiffness, D_{Qy} , of the sandwich beam with two core topologies, i.e., the X-truss core topology and the bi-directional corrugated-strip core topology, are obtained and presented in Figs. 5-6.

In both Fig. the transverse shear stiffness, $D_{Qy}/E_b t_b$, is first factorized by F_{st} where E_s is the modulus of elasticity of steel and t is the thickness of sandwich faceplate. Then, it is plotted against s_y/d in the range of $0.25 \leq s_y/d \leq 1$ where s_y is the horizontal projection of the extended local neutral axis of the inclined part of the core (Fig. 2) and d is the effective depth of the sandwich beam, i.e., $d = t$. Here, s_y/d is used to define the angle of the inclined part of the core. It is equal to $(s_c - 2f_c) / (h_c - t)$. Thus, the horizontal length of the unit cell, s_c , can be obtained from this expression.

Table 3-4 present the approximate numbers of finite element equations needed to solve in the analysis module of the ANSYS obtained from the three-point loaded X-truss core sandwich beams and from the three-point loaded bi-directional corrugated-strip core sandwich beams, respectively.

DISCUSSION

From the comparison of the factorized transverse shear stiffness, $D_{Qy}/E_b t_b$, of the X-truss core sandwich beams obtained from the Three-point Loaded Beam approach (TPB) and the Unit Cell approach (UC) as presented in Fig. 5, it can be seen that the unit cell approach agrees very well with the three-point loaded beam approach.

Table 3: Approximate numbers of finite element equations needed to solve in the analysis module of the ANSYS obtained from the three-point loaded X-truss core sandwich beams

Number of unit cells in the sandwich beams									
$s_r/$	1	2	3	4	5	6	7	8	9
0.50	132,167	264,3340	396,50100	528,66800	660,835	793,002	925,169	1,057,336	1,189,503
1.00	201,887	403,7740	605,66100	807,54800	1,009,435	1,211,322	1,413,209	1,615,096	N/A
1.50	271,607	543,2140	814,82100	1,086,428	1,358,035	1,629,642	N/A	N/A	N/A
43,214	814,821	1,086,428	1,358,035	1,629,642	N/A	N/A	N/A	N/A	N/A

Table 4: Approximate numbers of finite element equations needed to solve in the analysis module of the ANSYS obtained from the three-point loaded X-truss core sandwich beams

Number of unit cells in the sandwich beams									
$s_r/$	1	2	3	4	5	6	7	8	9
0.50	132,167	264,334	396,501	528,668	660,835	793,002	925,169	1,057,336	1,189,503
1.00	201,887	403,774	605,661	807,548	1,009,435	1,211,322	1,413,209	1,615,096	N/A
1.50	271,607	543,214	814,821	1,086,428	1,358,035	1,629,642	N/A	N/A	N/A
43,214	814,821	1,086,428	1,358,035	1,629,642	N/A	N/A	N/A	N/A	N/A

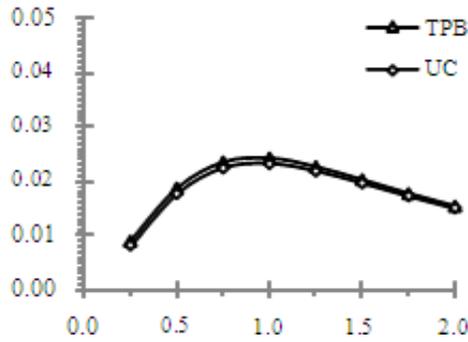


Fig. 3: Factorized transverse shear stiffness, t , of X-truss core sandwich beams obtained at any ratio from the three-point loaded beam approach (TPB) and the unit cell approach (UC)

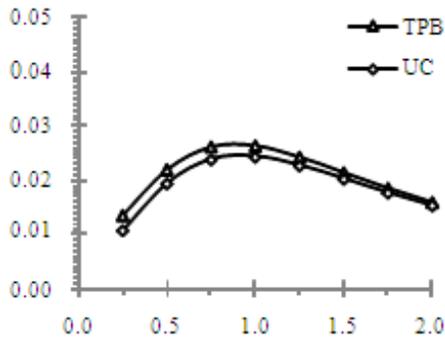


Fig. 4: Factorized transverse shear stiffness, t , of bi-directional corrugated-strip core sandwich beams obtained at any ratio from the Three-Point loaded Beam approach (TPB) and the Unit Cell approach (UC)

From the comparison of the factorized transverse shear stiffness, $D_{Qy}/E_b t_b$, of the bi-directional corrugated-strip core sandwich beams obtained from the Three-Point Loaded ssBeam approach (TPB) and the Unit Cell approach (UC) as presented in Fig. 6, it can also be seen that the unit cell approach agrees very well with the three-point loaded beam approach with a few percentage differences.

According to these comparisons, it can be seen that the transverse shear stiffness, D_{Qy} , of the complex truss-like core sandwich beams obtained from the unit cell approach-a reduced finite element model-is well consistent with the contemporary three-point loaded beam approach. Therefore, it is reasonable to conclude that the reduced finite element model based on the unit cell approach is accurate enough. It can be used for analyzing the transverse shear stiffness, p , of not only the simple truss core sandwich beams but also the complex truss-like core sandwich beams, i.e., the X-truss core and the bi-directional corrugated-strip core sandwich beams presented in this study.

The unit cell approach is considerably more advantageous than the three-point loaded beam approach when applied to the finite element method. This is because the number of finite element equations of the unit cell model is less than the number of finite element equations of the three-point loaded beam model.

It can be seen from Table 3-4 that the numbers of finite element equations obtained from the three-point loaded beams are approximately six times more than those obtained from the single unit cell. In addition, the unit cell approach can provide the direct calculation of the transverse shear stiffness, D_{Qy} , in a

single finite element analysis whereas the three-point loaded beam approach needs a series of repetitive finite element analysis which may be greater than six times in this study.

CONCLUSION

This study presents the application of the force and distortion relationship of unit cell approach to obtain the transverse shear stiffness, D_{Qy} , of the complex truss-like core sandwich beams in finite element analysis. Two core topologies of the X-truss core and the bi-directional corrugated-strip core are presented as examples of complex truss-like core topology. The responses of the transverse shear stiffness, D_{Qy} , obtained from the unit cell approach are presented and compared with those obtained from the three-point loaded beam approach. It is found that the unit cell approach agrees very well with the three-point loaded beam approach. The unit cell approach can be applied to a finite element analysis to reduce the number of finite element equations; consequently, it can be used to increase the speed of calculation time.

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