Structural Analysis of Spatial Mechanisms

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Abstract: The paper briefly presents how structural analysis is performed on spatial mechanisms, presenting a practical application in the mechatronics of parallel robots, especially in Steward platforms. Structural analysis always helps to better understand the phenomena and especially the way the mechanisms are made. At spatial mechanisms the problems are a little more complex than those raised by plane mechanisms. For this reason, the present paper tries to fill a gap in the field, especially as very few specialists in the theory of mechanisms of robots and machines still work today such structures on the basis of the theoretical knowledge, the modalities and working methods being approached being the most often based on approximate calculations, empirical formulas, or simply on experimental findings, woven with computerized theoretical modeling, but lacking in the essence of the underlying theory that is no longer so well-known. The structure and geometry of the mechanisms represent basically the basic elements that need to be studied primarily when we want to analyze such a mechanism already built and especially when we want to synthesize a new one.

Keywords: Mechanisms, Structure, Spatial Mechanisms Structure, Robots, Mechatronics, Automation, Parallel Robots, Spatial Mechanisms, Steward Platform, Structure and Geometry, Machines Structure

Introduction

The paper briefly presents how structural analysis is performed on spatial mechanisms, presenting a practical application in the mechatronics of parallel robots, especially in Steward platforms.

Structural analysis always helps to better understand the phenomena and especially the way the mechanisms are made.

At spatial mechanisms the problems are a little more complex than those raised by plane mechanisms. For this reason, the present paper tries to fill a gap in the field, especially as very few specialists in the theory of mechanisms of robots and machines still work today such structures on the basis of the theoretical knowledge, the modalities and working methods being approached being the most often based on approximate calculations, empirical formulas, or simply on experimental findings, woven with computerized theoretical modeling, but lacking in the essence of the underlying theory that is no longer so well-known.

The structure and geometry of the mechanisms represent basically the basic elements that need to be studied primarily when we want to analyze such a mechanism already built and especially when we want to synthesize a new one.

Spatial mechanisms are generally more complex than planar ones and therefore both their analysis as well as their optimal design is much more difficult to achieve. Spatial mechanisms are almost as old as planes, but their use has generally been rarer, making it much more difficult to design, build and use.

Engines of all kinds, as well as the various mechanical transmissions, that is, the most widespread mechanisms of all time, are built only by plane mechanisms, which has led to a thorough study of them, to the detriment of the spatial ones, much more difficult to study and still less useful so far.

After the robots appeared, however, things were radically changed, often spatial and often including spatial action mechanisms or elements, so that the necessity of grasping spatial problems into mechanisms became an undeniable objective reality.

Further, the structural analysis of the general, spatial, or plane + spatial mechanisms will be pursued.

The generalized structural formula of the mechanisms (Dobrovolschi) allows to determine the degree of
mobility of the family $f$ mechanisms, taking into account the number $f$ of the common bonding conditions imposed on all the elements of the mechanism before being linked in a single or multi- of the same family).

The kinematic chain is a reunion of kinematic elements of different ranks linked by kinematic couplers of different classes. All elements of the kinematic chain are mobile.

For a kinematic chain to be used, it must first be fastened to one of the component parts.

The classification of the kinematic chains is based on three important criteria: The rank of the component elements, the shape of the chain and the way the elements are moved.

A. By the rank of the elemental elements of the chain, we have:

- Simple kinematics (where each component has at most two kinematic couplings, $j$ being at most 2)
- Complex cinematic lines (at least one element of which has more than two kinematic couplings, or at least one top-level contour of at least 4, belonging to a higher tetrad or higher structural group)

B. According to the kinematic chain, we have:

- Kinematic open lanes (there are also elements with one kinematic couple, e.g., serial robots)
- Closed kinematic lanes (where all elements have at least two kinematic couplers, with the most common mechanisms, including parallel robots)

C. By the way we move elements, we have:

- Kinematic planar lanes (where all elements move in one plane, or in parallel planes)
- Kinematic spatial lanes (at least one of which has a movement in a different plain than the others)

The mechanisms are formed from one or more kinematic chains by fixing an element and the setting of the leading element (or the leading elements).

Space mechanisms are today used very often in the robotics and mechatronics industry, in the aerospace industry and in various specialized applications (Frăţilă et al., 2011; Pelecuti, 1967; Antonescu, 2000; Comănescu et al., 2010; Aversa et al., 2016a; 2016b; 2016c; 2016d; 2017a; 2017b; 2017c; 2017d; 2017e; 2017f; 2017g; 2017h; 2017i; 2017j; 2017k; 2017l; 2017m; 2017n; 2017o; 2017p; 2017q; 2017r; 2017s; 2017t; 2017u; 2017v; 2017w; 2017x; 2017y; 2017z; 2017aa; 2017ab; 2017ac; 2017ad; 2017ae; Petrescu and Calautit, 2016a; 2016b; Reddy et al., 2012; Tabaković et al., 2013; Tang et al., 2013; Tong et al., 2013; Wang et al., 2013; Wen et al., 2012; Antonescu and Petrescu, 1985; 1989; Antonescu et al., 1985a; 1985b; 1986; 1987; 1988; 1994; 1997; 2000a; 2000b; 2001; List the first flights, From Wikipedia; Chen and Patton, 1999; Fernandez et al., 2005; Fonod et al., 2015; Lu et al., 2015; 2016; Murray et al., 2010; Palumbo et al., 2012; Patre and Joshi, 2011; Sevil and Dogan, 2015; Sun and Joshi, 2009; Crickmore, 1997; Goodall, 2003; Graham, 2002; Jenkins, 2001; Landis and Dennis, 2005; Clément, Wikipedia; Cayley, Wikipedia; Coandă-1910, Wikipedia; Gunston, 2010; Laming, 2000; Norris, 2010; Goddard, 1916; Kaufman, 1959; Oberth, 1955; Cataldo, 2006; Gruener, 2006; Sherson et al., 2006; Williams, 1995; Venkataraman, 1992; Oppenheim and Volkoff, 1939; Michell, 1784; Droste, 1915; Finkelstein, 1958; Gorder, 2015; Hewish, 1970).

### Materials and Methods

The mechanisms are formed from one or more kinematic chains by fixing an element and the setting of the leading element (or the leading elements).

The $f$ family of a suitable kinematic mechanism or kinetic chain is defined, the space in which elements before being linked by kinematic couplers have 6 degrees of freedom.

In a family space $f$ formed mechanisms can have in their structure only kinematic class couplings $k \geq f + 1$. For example, in a third family space, where $f = 3$ (which may be a plane or a space-spherical one), we can only have fourth and fifth class couplings.

Consequently, in the family space $f$, the isolated elements possess $(6-f)$ degrees of freedom. By linking them through kinematic couples $\sum_{i=1}^{f} c_i$, the degree of freedom of the chain formed will be (I):

$$L_f = (6-f) \cdot e - \sum_{i=1}^{f} (k-f) \cdot c_i \tag{I}$$

Because a kinematic $k$-class coupling suppresses element $(k-f)$ degrees of freedom. Relationship (I) is the structural formula of the single-contour kinematic chain $f$.

If one of the chain elements is attached, the degree of mobility of the family $f$ mechanism (formula
Dobrovolschi, system II) is obtained, where \( m \) is the number of mobile elements:

\[
\begin{align*}
M_f &= L_f - (6 - f) \\
M_f &= (6 - f) \cdot (e - 1) - \sum_{k=1}^{5} (k - f) \cdot c_k \\
M_f &= (6 - f) \cdot m - \sum_{k=1}^{5} (k - f) \cdot c_k
\end{align*}
\] (II)

There are six families of mechanisms, derived from the system (II), according to the system (III):

\[
\begin{align*}
M_f &= (6 - f) \cdot m - \sum_{k=1}^{5} (k - f) \cdot c_k \\
f &= 0 \quad M_0 = 6m - 5c_5 - 4c_4 - 3c_3 - 2c_2 - c_1 \\
f &= 1 \quad M_1 = 5m - 4c_5 - 3c_4 - 2c_3 - c_2 \\
f &= 2 \quad M_2 = 4m - 3c_5 - 2c_4 - c_3 \\
f &= 3 \quad M_3 = 3m - 2c_5 - c_4 \\
f &= 4 \quad M_4 = 2m - c_5 \\
f &= 5 \quad M_5 = m
\end{align*}
\] (III)

The family of the mechanism can be determined using the table method, which is to list in a table all the independent movements of the elements with respect to a convenient coordinate axis system. The number of restrictions common to all elements indicates the family \( f \) of the mechanism. The formula for the obtained family (chosen from system III) is then applied and the mobility of the mechanism is obtained.

Note: The table method cannot be used in any situation to determine the family of a space mechanism.

In order to be used in as many cases as possible, it is sometimes useful to equate smaller upper class couplings with additional elements and fifth-grade inferior couplings. The fixed Cartesian space coordinate system must be judiciously chosen.

The spatial mechanisms of the \( f = 0 \) family are constituted by elements whose movements are not subject to any common restriction.

This category includes the spatial mechanisms whose elements can perform the most general movements (e.g., the steering mechanism of the road vehicles, the braking mechanism of the railway vehicles, the suspension mechanism, the motorbike, the automatic pilot steering mechanism, the modern parallel systems, etc.).

Such a family mechanism \( f = 0 \) is the spatial quadrilateral mechanism of Fig. 1, generally used as a steering mechanism on various road vehicles. To the right, you can see the table of elements moves towards the \( xOyz \) cartesian axis system chosen.

There is no common restriction, so the mechanism has the family 0 and the mobility is determined by the zero family formula (see relationship IV):

\[
\begin{align*}
M_f &= (6 - f) \cdot m - \sum_{k=1}^{5} (k - f) \cdot c_k \\
f &= 0 \quad M_0 = 6m - 5c_5 - 4c_4 - 3c_3 - 2c_2 - c_1 \\
&= 6 \cdot 3 - 5 \cdot 2 - 4 \cdot 0 - 3 \cdot 2 - 2 \cdot 0 - 0 = 18 - 10 - 6 = 2
\end{align*}
\] (IV)

The degree of mobility of the mechanism resulted in two, but the kinematic mechanism is desmodrome with a single actuation, so its real mobility degree is 1.

The second mobility is the possibility of the space bar 2 to rotate randomly around its own longitudinal axis due to the permittivity of the two couplings Spherical third-class spheres at its ends.

Another family zero mechanism is represented in Fig. 2.

Figure 3 shows a RCCR space family mechanism 1.

The inlet and outlet axes of the cranks 1 and 3 are constructed with 5th-class rotary couplings, but the bell 2 is connected to the cranks by fourth-class cylindrical couplings.

There is a common restriction (none of the three movable elements can rotate around the x-axis).

The degree of mobility of the mechanism is obtained with the relation (V) related to the spatial mechanisms of the family 1:

\[
\begin{align*}
M_f &= (6 - f) \cdot m - \sum_{k=1}^{4} (k - f) \cdot c_k \\
f &= 1 \quad M_1 = 5m - 4c_5 - 3c_4 - 2c_3 - c_2 \\
&= 5 \cdot 3 - 4 \cdot 2 - 3 \cdot 2 - 2 \cdot 0 - 0 = 15 - 8 - 6 = 1
\end{align*}
\] (V)

Figure 4 is a family 2 mechanism.

The degree of mobility of the mechanism with screw 1 and rod 2, family 2, is obtained with the corresponding formula (VI):

\[
\begin{align*}
M_f &= (6 - f) \cdot m - \sum_{k=1}^{3} (k - f) \cdot c_k \\
f &= 2 \quad M_2 = 4m - 3c_5 - 2c_4 - c_3 \\
&= 4 \cdot 3 - 5 \cdot 2 - 4 \cdot 0 - 3 \cdot 2 - 2 \cdot 0 - 0 = 16 - 15 = 1
\end{align*}
\] (VI)

The family mechanisms \( f = 3 \) consist of elements whose movements have three common restrictions. There are three main categories in this family:
A. Spherical mechanisms (the elements of these mechanisms are forbidden by all three translations, the elements are located on a sphere, they have the possibility to perform only the three rotations). Example (fourth-class couplings). Refer to the universal or universal coupling in Fig. 5.

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Fig. 1: Spatial quadrilateral mechanism used as steering mechanism on road vehicles

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Fig. 2: Space mechanism used as a mobile coupling to electric locomotives
Fig. 3: Space RCCR Family 1 Mechanism

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Fig. 4: Space Family 2 Mechanism

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The Cardan Cross (Universal Fourth Coupling) is a spherical spatial mechanism of family 3.

Double cardan articulation is a spherical spatial mechanism of the family 1.

The mobility of such a mechanism \((m = 2, C5 = 2, C4 = 1)\) is determined with the related relationship (VII):

\[
\begin{align*}
M_j &= (6 - f) \cdot m - \sum_{k=1}^{5} (k - f) \cdot c_k f = 3 \\
M_5 &= 3m - 2c_3 - c_4 = 3 \cdot 2 - 2 \cdot 2 - 1 = 6 - 4 - 1 = 1
\end{align*}
\]

It should be noted here that the mechanism with two cardanic crossings and a cardan shaft between them (as used in vehicles, Fig. 6) turns into a family mechanism 1 \((f = 1)\) because if we take a system spatial Cartesian axes having a common axis with the longitudinal axis of the shaft, we can see that the spindle has the three space rotation imposed by the cardan couplings at its ends plus two spatial translations along the radial directions but does not translate along its own longitudinal axis the only common restriction to the entire mechanism consisting of three mobile elements \(m = 3\), two \(C5\) couplings and two \(C4\) couplings). The mobility of the double joint is obtained with the VIII:

\[
\begin{align*}
M_j &= (6 - f) \cdot m - \sum_{k=1}^{5} (k - f) \cdot c_k \\
f &= 1M_1 = 5m - 4c_3 - 3c_4 - 2c_5 - c_2 \\
&= 5 \cdot 3 - 4 \cdot 2 - 3 \cdot 2 - 2 \cdot 0 - 0 = 15 - 8 - 6 = 1
\end{align*}
\]

B. Flat devices (in which the rotation, translation, nuts and \(C4\) upper couplings are formed).

The planar mechanisms are the most common in the technique, being practically the most used mechanisms in the entire history of mankind. Today, however, spatial mechanisms are being diversified due to advanced technologies and the emergence of parallel mobile structures.

C. Spatial space mechanisms (whose elements can only have translational movements in space.

The mechanisms of the \(f = 4\) family are made up of elements whose movements have four common restrictions. For example, the flat wedge mechanisms (having three translational couplings, Fig. 7a), or the
press-screw type mechanisms (a rotation coupler, one translation and one screw nut, Fig. 7b), where one meets two mobile elements and three fifth-class couplings. Mobility is given by relation (IX):

\[
\begin{align*}
M_f &= (6 - f) \cdot m - \sum_{k \neq f} (k - f) \cdot c_k \\
M_f &= 2m - c_k = 2 \cdot 2 - 3 = 4 - 3 = 1
\end{align*}
\]

(IX)

Clarifications: The family mechanism \( f = 5 \) is not alone, it falls into all the other families.

The Dobrovolschi formula also applies to polyclonal mechanisms, provided that all the independent contours of the mechanism have the same family. Otherwise, the modified Dobrovolschi formula (relation X) is used, where instead of \( f \) (the apparent family) and \( k \) takes values from 1 to 5 (not limited to \( f + 1 \) to 5):

\[
M_f = (6 - f_a) \cdot m - \sum_{i=1}^{N} (k - f_{a}) \cdot c_i
\]

(X)

The apparent family is determined as an arithmetic mean of the families of all independent contours (XIII relationship):

\[
f_a = \frac{1}{N} \sum_{i=1}^{N} f_i
\]

(XI)

Independent contours are identified directly on the mechanism. The number of independent contours can also be checked with relation (XII):

\[
N = \sum_{i=1}^{N} c_i - m
\]

(XII)

As an example of a complex mechanism, the mechanism of Fig. 8, which has 8 fifth-class couplings, 6 movable elements and two independent contours, 012340 and 04560, is taken as an example of a complex mechanism. For the first independent contour, the family is \( f = 2 \) and for the second family is \( f = 3 \).

With relation XI we obtain \( f_a = 2.5 \). The mobility of the mechanism is determined by the relation (XIII) that introduces the numerical data of the problem in relation (X):

\[
M_f = (6 - f_a) \cdot m - \sum_{i=1}^{N} (k - f_{a}) \cdot c_i
\]

(XIII)

\[
= (6 - 2.5) \cdot 6 - (5 - 2.5) \cdot 8 = 3.5 \cdot 6 - 2.5 \cdot 8 = 21 - 20 = 1
\]

(a)

(b)  

Fig. 7: Family 4 mechanisms

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Results and Discussion (Structure of Parallel Mobile Mechanical Systems)

Figure 9 shows the kinematic scheme of a parallel mobile mechanical system having all 12 kinematic couplings (linking the six motorized legs of the two platforms, fixed and movable) of spherical joints (spherical spherical couplings allowing all rotations possible and do not allow any translation to occur), practically third-class couplings (C3). The kinematic motor couplers (six in number) can be built in two variants: C5 or C4.

Ball spheres in sphere (spherical joints) allow rotations in space on all three axes and stop all translations. They are more technologically difficult, more expensive and generally have shorter lives and their wear is quite fast (even if the ball sphere contact surface is large). They have the great advantage of a reduced gauge (mass and low volume), (Fig. 10). Their life can be prolonged by optimal design, by thorough machining, by proper lubrication, etc. Spherical joints are used in the machinery industry, especially in the automotive industry. They are encountered in wheel attachment systems (swivel pivots), steering system joints, rear-view mirrors, some gearbox changers, etc.

Fig. 8: Complex mechanism with two independent contours

Fig. 9: The joints between the legs and the platforms must normally be all spherical kinematic spheres, that is, third class (C3) kinematic couplers
For a system parallel to 12 spherical articulations (C3) and 6 fifth-order (C5) engine couples (C5), the mobility of the system (spatial mechanism) is calculated by the general formula (1), (for a family spatial mechanism 0):

$$M_m = 6 \cdot m - 5 \cdot C_4 - 4 \cdot C_5 - 3 \cdot C_6 - 2 \cdot C_7 - 1 \cdot C_8$$

$$= 6 \cdot m - 5 \cdot C_4 - 3 \cdot C_5 = 6 \cdot 13 - 5 \cdot 6 - 3 \cdot 12$$

$$= 78 - 30 - 36 = 12$$

where, \( m \) represents the number of movable elements of the mechanism (system), in this case \( m \) is equal to 13, since the six movable legs are each formed by two elements (i.e., \( 6 \cdot 2 = 12 \)) and one of the platforms is also mobile (representing the thirteenth mobile element of the system).

Out of the 12 mobility degrees of the system, only 6 are active (representing the linear movements of linear motors). The other six degrees of mobility are passive (does not indicate the need to use additional actuators to achieve them). They are basically materialized by six additional six-foot rotation movements, each leg consisting of two kinematic elements, considered to be a solid, freely rotatable between its two spherical joints (through which it is connected to the two platforms, the fixed one from the base and the upper movable), (Fig. 11).

Although in general this passive rotation is random (cinematic is not necessary), however, it helps to improve the dynamic movement (movement) of the mechanism (system).

In fact, cylindrical (C4) engine couplings are used in place of the translational drive couplings (C5) which, besides the translational movement, also allow a relative rotation movement between the two rods of the motor coupler. Linear actuators are built in such a way that each allows a relative rotation movement between the two active bars. The motor movement is the linear motion, but a relative rotation motion within the motorcycle is also allowed.

In this situation, the six fifth-class couplings (C5) disappear and they are completely replaced by Class IV
cylindrical mobile joints (C4) (Fig. 12). The formula of degree of mobility takes the look (2):

\[ M = 6 \cdot m - 5 \cdot C_1 - 4 \cdot C_2 - 3 \cdot C_3 - 2 \cdot C_4 - 1 \cdot C_5 \]

\[ = 6 \cdot m - 4 \cdot C_3 - 3 \cdot C_1 = 6 \cdot 13 - 4 \cdot 6 - 3 \cdot 12 \]

\[ = 78 - 24 - 36 = 18 \]

The mechanism increases mobility, but only six of these mobilities are active (they refer to the linear movements imposed by the six actuators). In this case we have 12 passive rotation movements.

Both variants are not only functional but also have a better dynamic.

They were used by Stewart at first. He then proposed a more rigid (more dynamic) and more economical system, in which six of the spherical joints (C3) were replaced by six universal joints (cardanic cross, etc.), i.e., with couplings class IV.

So, out of the 12 C3 spherical couplings, half (six C3 couplings) are left to use, while six others will be of the fourth class (universal joints) and together with the Cylinder Engines (C4) they will achieve at the Stewart platform 12 C4 couplings. Mobility will be given by formula (3):

\[ M = 6 \cdot m - 5 \cdot C_1 - 4 \cdot C_2 - 3 \cdot C_3 - 2 \cdot C_4 - 1 \cdot C_5 \]

\[ = 6 \cdot m - 4 \cdot C_3 - 3 \cdot C_1 = 6 \cdot 13 \]

\[ = 4 \cdot 12 - 3 \cdot 6 = 78 - 48 = 18 = 12 \]

He immediately imposed himself and although it was thought that by replacing all universal spherical joints, the system would no longer work, yet somebody tried and saw that it was going and so and so it remained. The vast majority of Stewart's parallel platforms today have 12 universal joints and 6 cylindrical engine couplings, all of them being Class 4 kinematic couplings.

The C3 joints and the C5 sprockets disappear and only universal joints and cylindrical motors, all of the C4 kinematic class, remain (Fig. 13).

The universal joints used can be constructively of several ways (Fig. 14).

The formula for calculating mobility is now written in much simplified form (4):

\[ M = 6 \cdot m - 5 \cdot C_1 - 4 \cdot C_2 - 3 \cdot C_3 - 2 \cdot C_4 - 1 \cdot C_5 \]

\[ = 6 \cdot m - 4 \cdot C_3 = 6 \cdot 13 - 4 \cdot 18 = 78 - 72 = 6 \]

Although it seems the most rigid (dynamic) mechanism with only six degrees of mobility, all assets, representing the six linear motions of the six actuators, this system without additional, passive, rotating mobilities has succeeded in imposing a more judicious (both economically and financially but also technologically, being easier to achieve, cheaper and more reliable, Fig. 13 and 15).
Linear motors (actuators) are often hydraulic. They can also be electric, pneumatic, etc., but the most used are the hydraulic ones.

Their advantages (hydraulic actuators in particular, but also parallel systems in general) are primarily represented by high operating speeds (like actuator systems from specialized tractors), high speeds while keeping a good dynamics. Balancing them is simpler (for hydraulic systems, which act by default not only as engines but also as hydraulic shock absorbers, simultaneously). Parallel systems (generally) are faster, more dynamic, better balanced, quieter and especially "more rigid and more precise" compared to serial structures.

Where high rigidity and high accuracy are required, it will be considered (from the outset) the use of a parallel mobile mechanical system (for medical, brain, or spinal cord operations, for example in toxic, chemical, nuclear, heavy industry, etc.).

Although it seems exaggerated, in some of the aforementioned environments (on spinal surgery), devices based on super rigid parallel platforms were introduced at the request of specialists by supplementing the six engine legs with three more, thus resulting in nine legs (Fig. 16).

We now have nine feet, each of which contains two moving kinematic elements and three C4 couplings.

The number of mobile elements, \( m \), now stands at \( 9 \times 2 + 1 = 19 \). The kinematic couples are only of the fourth class, \( C_4 = 9 \times 3 = 27 \). The formula of the mechanism (system) mobility is given by the relationship (5):

\[
M_a = 6 \cdot m - 5 \cdot C_5 - 4 \cdot C_4 - 3 \cdot C_3 - 2 \cdot C_2 - 1 \cdot C_1
\]

\[
= 6 \cdot 19 - 5 \cdot 27 - 4 \cdot 27 - 3 \cdot 19 - 2 \cdot 1 - 1 \cdot 0
\]

\[
= 114 - 108 = 6
\]
The system with only six degrees of mobility (all assets) will work the same as the one presented in the present work with the six lateral actuators and the three additional legs will not be additional hydraulic motors, but only additional hydraulic shock absorbers; they will be virtually pulled up permanently by the upper movable platform and will always resist the movement (they will make a brake and a further damping). The rigidity of the system will increase significantly.

Although it looks much more complex (at first glance), this system is the same as the classic one (with six lateral actuators) and the calculations are the same as the classic Stewart system presented.

The three additional legs achieve only better stability, support, braking and especially increased rigidity of the entire system.

If nine effective actuators are to be implemented, then the structure of the mechanism to achieve some additional mobility (at least three) must be rethought. For every universal joint transformed into a spherical one, a degree of additional mobility is obtained. To have the mechanism 9 instead of 6, three universal joints must be replaced by three spherical kinematic spheres. The most logical would be to replace the three upper legs of the extra legs. In this case, the mobility formula takes the form (6):

\[ M_6 = 6 \cdot m - 5 \cdot C_1 - 4 \cdot C_2 - 3 \cdot C_3 - 2 \cdot C_4 - 1 \cdot C_5 \]

\[ = 6 \cdot 6 - 4 \cdot C_4 - 3 \cdot C_5 = 6 \cdot 19 - 4 \cdot 24 - 3 \cdot 3 \]

\[ = 114 - 96 - 9 = 9 \]  

\[(6)\]

In this situation, the theory also changes.

Even the classical parallel systems presented have a very high stiffness and very good precision and can maintain their balance during fast moving loads with a high loading load (see photo in Fig. 17).

The load is very high, the travel speeds are high, the big and sudden inclines are not missing either. As can be seen in Fig. 17, the load is not anchored, but it is laid freely on the upper (upper) platform.

\section*{Conclusion}

The paper briefly presents how structural analysis is performed on spatial mechanisms, presenting a practical application in the mechatronics of parallel robots, especially in Stewart platforms.

Structural analysis always helps to better understand the phenomena and especially the way the mechanisms are made.

At spatial mechanisms the problems are a little more complex than those raised by plane mechanisms.

For this reason, the present paper tries to fill a gap in the field, especially as very few specialists in the theory of mechanisms of robots and machines still work today such structures on the basis of the theoretical knowledge, the modalities and working methods being approached being the most often based on approximate calculations, empirical formulas, or simply on experimental findings, woven with computerized theoretical modeling, but lacking in the essence of the underlying theory that is no longer so well-known.

The structure and geometry of the mechanisms represent basically the basic elements that need to be studied primarily when we want to analyze such a mechanism already built and especially when we want to synthesize a new one.

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Fig. 17: System parallel to six hydraulic linear load actuators in motion
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4-Contract research. GR 69/10.05.2007: NURC in 2762; theme 8: Dynamic analysis of mechanisms and manipulators with bars and gears.

5-Labor contract, no. 35/22.01.2013, the UPB, "Stand for reading performance parameters of kinematics and dynamic mechanisms, using inductive and incremental encoders, to a Mitsubishi Mechatronic System" "PN-II-IN-CTI-2012-1-0389".


Ethics

This article is original and contains unpublished material. Authors declare that are not ethical issues and no conflict of interest that may arise after the publication of this manuscript.

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