Controller Based Observer in Switched System with Norm Bounded Uncertainty

Mongi Besbes and Elyes Maherzi
Department of Electrical Engineering,
High School of Technology and Computer Science, University of Carthage, Tunisia

Abstract: Problem statement: This study discusses the robust stabilization of norm bounded discrete switched systems. Approach: The proposed method is using the second Lyapunov approach and the poly-quadratic function concept. The stabilization conditions are written through linear matrix inequality relations. The control law is based on a static output feedback with the use of a switched observer. The synthesis conditions of the controller are written in the form of linear matrix inequalities difficult to resolve by current numerical solvers. That’s why relaxations are proposed to mitigate the pessimism of LMI conditions obtained. Results: The poly-quadratic Lyapunov approach provides a constructive way to tackle uncertainty in the switched framework. The feasibility is illustrated by the example of discrete uncertain switched systems. Conclusion: With these results, the study of stability can be achieved for arbitrary switching laws, state-dependent, time dependent or generated by a controller. However, the implementation of the control law is possible only if the switching status is well known in real time.

Key words: Matrix inequalities, robust stabilization, arbitrary switching laws, inequality relations, systems combine continuous, stability analysis

INTRODUCTION

Many natural and artificial systems work in different operating modes, each with its own dynamic. The car changes from one dynamic mode to another with every change of speed, the human heart switches between different modes depending on the emotional state of the person. These systems concoct continuous dynamics with both synchronous or asynchronous discrete events. Such class of systems is called hybrid systems (Liberzon, 2003)

The literature has shown a growing interest in switched systems. Switched systems are hybrid systems defined by a set whose elements are dynamic continuous and/or discrete time models with commutation law which define, in time, the jumps between the elements, leading to a non stationary dynamic system. Some recent results are given in (Daafouz et al., 2001; 2002a) where a sufficient (but relatively non restrictive compared to the quadratic approach) stability condition for discrete switched systems is provided using the poly-quadratic approach recently proposed by (Daafouz and Bernussou, 2001) to analyze stability and stabilization control of Linear “Time Varying systems”.

Other approaches are concerned with the determination of an act of switching between several controllers guaranteeing the stability of systems (Skafidas et al., 1999; Liberzon, 2003; Hespanha and Morse, 2002) which operating constraints require switching between multiple controllers.

This study proposes an extension of this works in the case when the switches are made between uncertain LTI systems. The control investigated is of state feedback, observer based and dynamic controller.

MATERIALS AND METHODS

Let us consider a discrete autonomous switched system (1) where each subsystem is vitiated by a norm bounded uncertainty (Maherzi et al., 2007; Zhou and Khargonekar, 1987; 1988), this system can be described by the following equalities (1):

\[ x(k+1) = \sum_{i=1}^{M} \xi_i^i (A_i + \Delta A_i)x(k) \]  

(1)

With Eq. 2 and 3:

\[ \Delta A_i = D_i F_i E_i \]  

(2)

And:

\[ F_i^T F_i \leq \gamma_i I \]  

(3)
Where:

\( \ell = \) Switching index

\( M = \) Number of subsystems Eq. 4 and 5:

\[
\xi_n^\ell = \begin{cases} 
1 & \text{if the state matrix } A \in (A_\ell + \Delta A_\ell) \\
0 & \text{if not}
\end{cases}
\]

\( \xi_n^\ell \geq 0; \sum_{\ell=1}^{M} \xi_n^\ell = 1; \xi_n^\ell = [\xi_1^\ell, ..., \xi_N^\ell]^T \)

Analysis of stability: By using Theorem developed by (Daafouz and Bernussou, 2001) the system (1) is poly-quadratically stable if and only if there are \( N \) symmetric positive definite matrices \( S_1, ..., S_N \) and \( N \) symmetric positive definite matrices \( G_1, ..., G_N \) matrices of appropriate dimensions satisfying Eq. 6:

\[
\begin{bmatrix}
G_i + G_i^T - S_i, G_i^T (A_\ell + \Delta A_\ell) \n
\end{bmatrix} > 0 \quad \forall \ell \in (\epsilon \times \epsilon)
\]

This is equivalent to the following inequality Eq. 7 and 8:

\[
\begin{bmatrix}
G_i + G_i^T - S_i, G_i^T A_i \n
\end{bmatrix} > 0
\]

Knowing that Eq. 9:

\[
\begin{bmatrix}
0, G_i^T \Delta A_i \n
\end{bmatrix} \leq \begin{bmatrix}
G_i^T E_i^T E_i G_i, 0 \\
0, \gamma_i D_i D_i^T
\end{bmatrix}
\]

So we can say that if Eq. 10:

\[
\begin{bmatrix}
G_i + G_i^T - S_i, G_i^T A_i \n
\end{bmatrix} > 0
\]

Then inequality (7) is true. Thereby lead to the following conditions Eq. 11:

\[
\begin{bmatrix}
G_i + G_i^T - S_i, G_i^T E_i^T E_i G_i, G_i^T A_i^T \n
\end{bmatrix} > 0
\]

Applying the Schur complement leads to the following proposal.

Proposition 1: By using Theorem developed by (Daafouz et al., 2002b) the system described by Eq. 1 is a poly-quadratically stable “if and only if there are \( N \) symmetric positive definite matrices \( S_1, ..., S_N \) and \( N \) symmetric positive definite matrices \( G_1, ..., G_N \) matrices” of appropriate dimensions satisfying Eq. 12:

\[
\begin{bmatrix}
G_i + G_i^T - S_i, G_i^T A_i^T, G_i^T E_i^T \\
A_i G_i, S_i - \gamma_i D_i D_i^T
\end{bmatrix} > 0
\]

Synthesis of dynamic control based on a switched observer: This part aims to design switched observer to reconstruct the unknown states of the system (1), these states will then be used to formulate a stabilizing control law. The ultimate goal would be to stabilize the observation error and system states (Liu and Duan, 2005).

Form of switched observer: Let us consider an observer of the form Eq. 13:

\[
\dot{\hat{x}}(k+1) = A_{\ell}(\hat{x}(k)) + B_u u(k) + L_e(y(k) - \hat{y}(k))
\]

The gains observation have to be calculated to guarantee the convergence to zero of the observation error Eq. 14:

\[
\epsilon(k) = x(k) - \hat{x}(k)
\]

The evolution of the observation error is described by the following Eq. 15:

\[
\epsilon(k+1) = \sum_{\ell=1}^{M} \xi_n^\ell(k) \left( A_\ell + \Delta A_\ell - \hat{A}_\ell \right) \epsilon(k)
\]

Control and stabilization of the system: The new augmented system is Eq. 16-22:
\[
\begin{align*}
\phi(x(k+1)) &= \sum_{\ell=0}^{\infty} \phi_s(\ell) \phi(x(k)) \\
\phi_1 &= \begin{pmatrix} A_1 + \Delta A_1 + B_1 k_1 \\ \Delta_1 + \Delta A_1 \\ \hat{A}_1 - L_1 C_1 \end{pmatrix} \\
\phi_k &= \begin{pmatrix} A_k + \Delta A_k + B_k k_k \\ \Delta_k + \Delta A_k \\ \hat{A}_k - L_k C_k \end{pmatrix} \\
\Delta_i &= A_i - \hat{A}_i
\end{align*}
\]

Let:

\[
\Phi_i = \sum_{l=0}^{\infty} \Phi \Psi + \Delta_i + \Delta_i - \hat{A}_i
\]

Find a solution to the LMIs (26) while respecting the equality constraints (28) and (29) is a non convex problem a solution exists only if the following rank conditions are verified Eq. 30:

\[
\begin{align*}
\text{rank}[G_{i,3}^\top G_{i,3}] &= \text{rank}[U_i G_{i,3}^\top V_i G_{i,3}^\top] = \text{rank}[F_i G_{i,3}^\top] \\
\end{align*}
\]

So to find solutions to the LMIs (25) that satisfy the constraints (28) and (29) it is possible to apply an iterative algorithm of solving the LMI (25) iteratively by testing the rank condition (28) and (29) at each stage (Halabi, 2005).

Another approach would be to fix.

Proposition 2: The system is poly-quadratically stable “if and only if there are N symmetric positive definite matrices \(S_1, \ldots, S_N\) and N symmetric positive definite matrices \(G_1, \ldots, G_N\) of appropriate dimensions satisfying Eq. 25-29:

\[
\begin{align*}
\Gamma &= \begin{pmatrix} G_{i,3}^\top(A_i + B_i k_i) + R_i - G_{i,3}^\top B_i k_i + G_{i,3}^\top A_i - R_i - U_i C_i \\
G_{i,3}^\top(A_i + B_i k_i) + F_i - G_{i,3}^\top B_i k_i + G_{i,3}^\top A_i - F_i - V_i C_i \end{pmatrix} \\
\Psi &= \begin{pmatrix} G_{i,3}^\top D_i + G_{i,3}^\top D_i \\
G_{i,3}^\top D_i + G_{i,3}^\top D_i \end{pmatrix} \\
\end{align*}
\]

Proposition 2: The system is poly-quadratically stable “if and only if there are N symmetric positive definite matrices \(S_1, \ldots, S_N\) and N symmetric positive definite matrices \(G_1, \ldots, G_N\) of appropriate dimensions satisfying Eq. 25-29:
With:

\[ \hat{A}_r = A_r - \Delta_r \]  
\[ \Delta_r = (G_{r,4})^{-1} F_r \]  
\[ L_r = G_{r,4}^T V_r \]

**RESULTS**

It is clear that these methods are conservative, because the control gains and observers are calculated separately and we imposed a special structure in the matrix variable \( G \). However these methods are interesting because they allow a relaxation of conditions LMIs to be solved by the freedom granted to the matrices \( \hat{A}_r \), required for the construction of the observers.

**Illustration example:** It’s a benchmark example, commonly used in the literature, which we added an uncertainty of norm bounded type.

\[
\begin{bmatrix}
 x_1(k+1) \\
 X(k+1) \\
 x_3(k+1) \\
 x_4(k+1)
\end{bmatrix} =
\begin{bmatrix}
 A & B & k_1 & k_2 \\
 C_1 & C_2 & C_3 & C_4
\end{bmatrix}
\begin{bmatrix}
 x_1(k) \\
 x_2(k) \\
 x_3(k) \\
 x_4(k)
\end{bmatrix}
\]

Based on the condition (5.47) we obtain the following observers:

\[ \hat{A}_r = A_r + B_r K_r \]

\[ L_r = \begin{bmatrix}
 0.2203 & 0.0207 & 0.4472 \\
 0.1076 & 0.8584 & 0.8031 \\
 0.7844 & 0.9053 & 0.4856 \\
 0.3740 & 0.5572 & \\
 0.4396 & 1.9716 & \\
 1.0580 & 1.7440 &
\end{bmatrix} \]

One of the two actuators can fail, in this case the matrix \( B \) may take three different values:

\[
B_1 = \begin{bmatrix}
 0.6135 & 0.6538 \\
 0.2749 & 0.4899 \\
 0.8807 & 0.7741 \\
\end{bmatrix}
\]

\[
B_2 = \begin{bmatrix}
 0.6135 & 0.0000 \\
 0.2749 & 0.0000 \\
 0.8807 & 0.0000 \\
\end{bmatrix}
\]

\[
B_3 = \begin{bmatrix}
 0.000 & 0.6538 \\
 0.000 & 0.4899 \\
 0.000 & 0.7741 \\
\end{bmatrix}
\]
The conditions obtained from the formulation norm bounded uncertainties have the advantage of reducing the number of LMIs to be solved compared to the polytopical formulation of uncertainty which, admittedly, is less conservative but generates much of LMIs to solve.

Figure 1 and 2 shows the evolution of controls $u_1$ and $u_2$ for the uncertainty range with a loss of control $u_1$ between times $[15s \to 20s]$ and loss of control $u_2$ between times $[0s \to 10s]$.

![Fig. 1: Evolution of actuator 1](image1)

![Fig. 2: Evolution of actuator 2](image2)

![Fig. 3: System output $Y_1$](image3)

![Fig. 4: System output $Y_2$](image4)

Figure 3 and 4 show the system stability despite uncertainties and actuator failures.

**DISCUSSION**

The poly-quadratic Lyapunov approach provides a constructive way to tackle bounded uncertainty in the switched framework. The controller synthesis conditions are dependent on information provided by observers and the feasibility of LMIs generated by the calculations. We believe that the proposed conditions are not pessimistic and against this approach may offer alternatives to conventional methods of synthesis of current regulator for this particular type of system.

**CONCLUSION**

With these results, the study of stability can be achieved for arbitrary switching laws, state-dependent, time dependent or generated by a controller. However, the implementation of the control law is possible only if switching law status is known in real time (Gao et al., 2004; Hetel et al., 2008).
In terms of outlook, an important point, to seek, is to develop conditions based even on partial knowledge of the switching law without necessarily having a thorough knowledge of real-time switching.

Information obtained in advance of the switching law can afford to give less restrictive conditions, not having to account for any switching law possible. Indeed, it is possible in some practical applications, such as control systems via computer networks, to estimate an interval containing the delay as a function of various parameters of the network. It will be interesting, in this case, to define conditions of stabilization not only taking into account the parameter uncertainties but also uncertainties about the delay.

REFERENCES


