Efficiency Decomposition with Enhancing Russell Measure in Data Envelopment Analysis

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Constraint (1b) is referred to as the normalizing equation (Dyson et al., 2001). Model (1) is an input-oriented DEA model, since it assumes that inputs are under the control of DMU_j which aims to maximize its outputs and it adopts a radial efficiency measurement. If h_j is equal to one, then DMU_j is classified as efficient and inefficient otherwise.

The CCR model assumes that the production function exhibits constant returns-to-scale. Banker et al. (1984) added an additional constant variable in order for it to permit variable returns-to-scale and this is known as the BCC model. It should be noted that CCR and BCC models are based on the radial measure. A radial efficiency measurement uses a line in input space from the origin O to the point P being evaluated. If P is not efficient then this line will cross the frontier at P' and the ratio of OP'/OP gives the efficiency score (Fig. 1). DMUs that lie on the surface determine the envelope and are deemed efficient, whilst those that do not are deemed inefficient. A DEA measurement that is not from the origin O is referred to as non-radial efficiency measure.

MATERIALS AND METHODS

Russell measure: Russell (1985) incorporated radial and non-radial efficiency models in performance evaluation called the Russell measure. Model (2) shows the original formula in input oriented form:

\[
\begin{align*}
\text{Min } & \sum_{i=1}^{m} \theta_i \\
\text{s.t. } & \theta_i x_{ij} - \sum_{k=1}^{n} \lambda_k x_{ik} \geq 0, \ r = 1,2,\ldots, m \\
& \sum_{k=1}^{n} \lambda_k y_{ik} \geq y_{ik}, \ i = 1,2,\ldots, s \\
& 0 \leq \theta_i \leq 1, \ 0 \leq \lambda_k, \ i = 1,2,\ldots, s, \\
& r = 1,2,\ldots, m, \ k = 1,2,\ldots, n
\end{align*}
\]

Where:
\( \theta_i \) and \( \lambda_k \) = Dual variables

DMU_j = Efficient if and only if \( \sum_{i=1}^{m} \theta_i = 1 \)

Later, Fare et al. (1985) extended the Russell measure in an additive way, shown as Model (3) and referred to as the “Russell graph measure”, which simultaneously minimizes the input efficiency measure and maximizes the output inefficiency measure as follows:

\[
\begin{align*}
\text{Min } & R(\sum_{i=1}^{m} \theta_i, \sum_{i=1}^{n} / \varphi_i) = \sum_{r=1}^{m} \theta_i + \sum_{i=1}^{s} \varphi_i, \\
\text{s.t. } & \theta_i x_{ij} \geq \sum_{k=1}^{n} \lambda_k x_{ik}, \ r = 1,2,\ldots, m \\
& \varphi_i y_{ij} \leq \sum_{k=1}^{n} \lambda_k y_{ik}, \ i = 1,2,\ldots, s, \\
& 0 \leq \theta_i \leq 1, \ 0 \leq \varphi_i, \ 0 \leq \lambda_k, \\
& i = 1,2,\ldots, s, \ r = 1,2,\ldots, m, \\
& k = 1,2,\ldots, n
\end{align*}
\]

The variables \( \theta_i \) and \( \varphi_i \) represent the measures of input efficiency and output inefficiency of DMU_j, respectively (Cooper et al., 2007).

The basic concept of Russell measure is that it assumes the weights (coefficients) are all the same for each input (output), i.e., \( \frac{1}{m+s} \) and therefore it utilizes the mean of all input/output efficiency scores as the efficiency index of DMU_j. However, this efficiency measurement is subjective and may render bias. This study enhances the Russell measure by decomposing the normalizing equation.

A non-radial measurement: If we decompose the normalizing Eq. 1b into m components, i.e., \( v_1 x_{ij} = \alpha_i, \ v_2 x_{ij} = \alpha_2, \ldots, v_m x_{mj} = \alpha_m \) and \( \sum_{r=1}^{m} \alpha_r = 1 \), then each input will be associated with a different dual variable and Model (1) can be converted into Model (4) as a non-radial DEA model:

\[
\begin{align*}
\text{Max } & e_i = \sum_{i=1}^{m} u_i y_{ij} \\
\text{s.t. } & v_i x_{ij} = \alpha_i, \ r = 1,2,\ldots, m \\
& \sum_{i=1}^{m} u_i y_{ik} - \sum_{r=1}^{n} v_r x_{rk} \leq 0, \ k = 1,2,\ldots, n \\
& u_i, v_r \geq 0, \ \alpha_i \geq 0, \ i = 1,2,\ldots, s, \\
& r = 1,2,\ldots, m
\end{align*}
\]

where, \( \sum_{r=1}^{m} \alpha_r = 1 \). The dual problem of Model (4) is shown as Model (5):

\[
\begin{align*}
\text{Min } & \quad f_r = \sum_{r=1}^{m} \alpha_r \theta_r \\
\text{s.t. } & \quad 0, x_{rj} - \sum_{k=1}^{n} \lambda_k y_{rk} \geq 0, r = 1,2,\ldots,m \\
& \quad \sum_{k=1}^{n} \lambda_k y_{rk} - y_i \geq 0, i = 1,2,\ldots,s \\
& \quad 0, \text{ unrestricted, } \lambda_k \geq 0, \quad k = 1,2,\ldots,n, \\
& \quad r = 1,2,\ldots,m
\end{align*}
\]

Zhou et al. (2007) stated that \( \alpha_r, r = 1,2,\ldots,m \), are the normalized user-specified weights for adjusting the rth input. Based on this argument, \( \alpha_r \)'s are decision variables when incorporated into the non-radial DEA model and their linear combination should be equal to one. However, this may result in infinitely many solutions in solving Models (4) and (5) and thus this study proposed the following procedure to provide a reasonable mechanism in the determination of \( \alpha_r \)'s:

1. Run the CCR model to obtain the values of \( v_r^* \), \( r = 1,2,\ldots,m \)
2. Calculate the value of \( \alpha_r \) by letting \( \alpha_r = v_r^* x_{rj} \)
3. Run Model (5)

The major characteristic of the procedure is that it applies the radial model to obtain the best weights of inputs for each DMU and thus the decision maker can utilize these to specify the \( \alpha_r \) values in the non-radial DEA model. Chen and Ali (2004) argued that the efficiencies between Models (1), (2), (4) and (5) are the same and thus the procedure provides a reasonable mechanism to determine the \( \alpha_r \) values.

Here \( \alpha_r \) is the weighted volume of input \( r \) and \( \alpha^r \theta^r \), can thereby be interpreted as the contribution to the current efficiency of input \( r \). Therefore, \( \theta_r \) is the efficiency index of input \( r \). In other words, the efficiencies of DMUs enable the decision maker to decompose them, in a way similar to the Russell measure. Based on the proposed procedure, one can attain the explicit efficiency index and mitigate the shortcoming of Russell measure.

RESULTS AND DISCUSSION

An illustration: The purpose of this illustration is to demonstrate the proposed technique and point out the bias of efficiency evaluation by the Russell measure. Suppose there are four evaluated DMUs with two inputs and two outputs (Table 1).

For DMU_A, the efficiency score by using Model (1) is \( h^*_A = 0.7432 \), where \( v^*_1 = 0.0255 \) and \( v^*_2 = 0.0368 \). Hence, \( \alpha_1 = 15v^*_1 = 0.3376 \), \( \alpha_2 = 18v^*_2 = 0.6624 \), \( \theta_1 = 1.1111 \) and \( \theta_2 = 0.5556 \). By utilizing Russell measure, the efficiency of DMU_A is equal to 0.8334 (Min RA = (1.1111+0.5556)/2 = 0.83335). It is obvious that the Russell measure render bias since the efficiency score of DMU_A is different from that by the CCR model. Additionally, based on the proposed technique, we present three findings.

Finding 1: In Russell measure, it assumed that the coefficients of the efficiency indices are the same. However, it is not necessary that all efficiency indices correspond to equivalent weights. By utilizing the proposed technique, decision makers can determine the best weight for each input (output) and therefore identify the outstanding input (output) according to the efficiency indices. As can be seen in the illustration, the efficiency index of input \( X_2 \) is superior to that of \( X_1 \) for DMU_A, since \( \theta_2 = 0.8421 \) is larger than \( \theta_1 = 0.6232 \).

Finding 2: The characteristic of DEA is that it allows DMUs to select the factor weights which are the most favorable for them in calculating their efficiency scores. This flexibility generally classifies many DMUs as efficient. Based on the marginal efficiency indices of inputs (or outputs), decision makers can determine the relative importance of factors and incorporate the relative importance relationships into the DEA model as constraints to attain a higher discrimination level.

<table>
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<tr>
<th>DMU</th>
<th>( X_1 )</th>
<th>( X_2 )</th>
<th>( Y_1 )</th>
<th>( Y_2 )</th>
<th>( v^*_1 )</th>
<th>( v^*_2 )</th>
<th>( u^*_1 )</th>
<th>( u^*_2 )</th>
<th>CCR efficiency</th>
</tr>
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<tbody>
<tr>
<td>A</td>
<td>15</td>
<td>18</td>
<td>10</td>
<td>8</td>
<td>0.0225</td>
<td>0.0368</td>
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<td>B</td>
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<td>12</td>
<td>12</td>
<td>12</td>
<td>10^-6</td>
<td>0.0833</td>
<td>10^-6</td>
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<tr>
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<td>20</td>
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<td>0.0247</td>
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<td>10^-6</td>
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</tr>
<tr>
<td>D</td>
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<td>14</td>
<td>20</td>
<td>0.0167</td>
<td>0.0222</td>
<td>10^-6</td>
<td>0.0500</td>
<td>1.0000</td>
</tr>
</tbody>
</table>
Finding 3: To obtain the values of $v_r, r=1,2,...,m$, it is usually found that some $v_r$’s are equal to zero and thus the marginal efficiencies of these inputs are of zero. To avoid this situation, decision makers can utilize assurance region (Thompson et al., 1990) in the CCR model.

CONCLUSION

This study is concerned with the measurement of efficiency from a DEA perspective. Because of the similarity to the Russell measure, we have called it the enhanced Russell measure. Using Russell measure may render bias when measuring efficiency by means of inputs/outputs efficiencies. To avoid this situation, this study applies the non-radial DEA model to obtain the efficiency indices through identifying the $\alpha_r$ values. The major advantages of the proposed technique are that it can not only provide a reasonable mechanism in the determination of $\alpha_r$’s but also enhance the Russell measure. In addition, it can assist the decision maker in determining the relative importance of factors to improve the discrimination level of DMUs efficiencies and ensure the optimal scores of DMUs as the CCR efficiency.

REFERENCES


