Fuzzy Anti-windup Schemes for NCTF Control of Point-to-point (PTP) Positioning Systems

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Abstract: The positioning systems generally need a controller to achieve high accuracy, fast response and robustness. In addition, ease of controller design and simplicity of controller structure are very important for practical application. For satisfying these requirements, NCTF (nominal characteristic trajectory following) controller has been proposed as a practical PTP positioning control. However, the effect of actuator saturation can not be completely compensated due to integrator windup because of plant parameter variations. This study presents a method to improve the NCTF controller for overcoming the problem of integrator windup by adopting fuzzy anti-windup schemes. Two fuzzy anti-windup schemes based on Mamdani and Takagi-Sugeno fuzzy system are developed and evaluated their effectiveness. The improved NCTF controller with the proposed fuzzy anti-windup schemes is evaluated through simulation using dynamic model of a rotary positioning system. The results show that the improved NCTF controller with Takagi-Sugeno-based fuzzy windup is the best scheme to compensate for the effect of integrator windup.

Keywords: Positioning, point-to-point, fuzzy, anti-windup, compensation, controller, robustness

INTRODUCTION

Motion control systems play important roles in industrial process such as machine tools, semiconductor manufacturing systems and robot systems. One type of motion control systems is point-to-point (PTP) positioning system, which is used to move a plant from one point to another point. The positioning systems generally need a controller to satisfy such requirements such as high accuracy, fast response and robustness.

Up to now many types of controllers have been proposed and evaluated for positioning systems; for example the model following type controller such as controllers with disturbance observer\(^1\)-\(^4\), time-optimal controllers\(^5\)-\(^8\) and sliding mode controllers\(^9\),\(^10\). These controllers will give good positioning performance if experts on motion control system do design the controller using the exact model and values of its parameters. It is well known that exact modeling and parameter identifications are generally troublesome and time consuming tasks. In general, advanced controllers tend to be complicated and require deep knowledge concerning controller theory and design. However, in practical applications, engineers, who are not experts in control systems, often need to design the controllers. Hence, ease of controller design and simplicity of controller structure are very important for practical applications.

In order to overcome the above-mentioned problems, nominal characteristic trajectory following (NCTF) controller has been proposed as a practical controller for point-to-point (PTP) positioning systems\(^11\). It has been shown that the NCTF control system has a good positioning performance and robustness\(^12\),\(^13\). The NCTF controller is also effective to compensate the effects of friction which is the source of positioning inaccuracy\(^14\). However, the effect of actuator saturation can not be completely compensated due to integrator windup when the plant parameters vary\(^15\). The NCTF controller gives an excessive overshoot when actuator saturation as well as parameter variations (especially inertia variation) occur in the positioning systems.

This study describes a method to improve the NCTF controller to overcome the degradation of the positioning performance due to integrator windup. Fuzzy Anti-windup schemes are introduced for the NCTF controller. Two fuzzy anti-windup schemes based on Mamdani and Takagi-Sugeno fuzzy system are developed and evaluated their effectiveness through simulation using dynamic model of a rotary positioning system.

Hence, ease of controller design and simplicity of controller structure are very important for practical applications.
simulation using dynamic model of a rotary positioning system. The simulation results confirm that the improved NCTF controller with Takagi-Sugeno-based fuzzy windup is the best scheme to compensate for the effect of integrator windup.

NCTF control system

Basic concept of NCTF control system: The structure of the NCTF control system is shown in Fig. 1. The NCTF controller consists of a nominal characteristic trajectory (NCT) and a compensator. The NCTF controller works under the following two assumptions:

* A DC or an AC servomotor is used as an actuator of the plant.
* PTP positioning systems are chosen, so $\theta_r$ is constant and $\dot{\theta}_r \equiv 0$.

Here, the objective of the NCTF controller is to make the plant motion follow the NCT and ends at the origin of the phase plane $(e, \dot{e})$.

Figure 2 shows an example of a plant motion controlled by the NCTF controller. The motion comprises two phases. The first one is the reaching phase (RP) and the second is the following phase (FP). In the reaching phase, the compensator forces the plant motion to reach the NCT as fast as possible. However, in the following phase the compensator controls the plant motion so as to follow the NCT and end at the origin. The plant motion stops at the origin, which represents the end of the positioning motion. Thus, in the NCTF control system, the NCT governs the positioning response performance.

Design of NCTF controller: The NCTF controller is designed based on a simple open-loop experiment of the plant as follows:

* Open-loop-drive the plant with stepwise inputs and measure the displacement and velocity responses of the plant. Figure 3a shows the stepwise inputs and the velocity and displacement responses due to the stepwise inputs. In this study, the rated input to the actuator $u_r$ is used as the height of the stepwise inputs.
* Construct the NCT by using the plant responses.

The velocity and displacement responses are used to determine the NCT. Since the main objective of PTP system is to stop the plant at a certain position, a deceleration process is used, see curve A, Fig. 3(a). The $h$ in Fig. 3 represents the maximum velocity. From the curve in the area A and $h$ in Fig. 3(a), the NCT in Fig. 3(b) is determined. Since the NCT is constructed based on the actual responses of the plant, the NCT includes nonlinearity effects such as friction and saturation. The
important NCT information, which will be used to
design the compensator, are NCT inclination \( m \) near the
origin and maximum error rate of \( h \). In this case, from
the relationship between plant dynamics of Eq. (1) and
Fig. 3(b), it is clear that the inclination near origin \( m \)
and the maximum error rate \( h \) relate with parameters of
the plant as follows\[^{12,15}\]:

\[ K = -\frac{h}{u_r} \]  \hspace{1cm} (1)

\[ \alpha = -m \]  \hspace{1cm} (2)

* Design the compensator based on the NCT
information.

Here, the following PI compensator is adopted due
to its simplicity:

\[ u = K_p u_p + K_i \int u_p \, dt \]  \hspace{1cm} (3)

where \( K_p \) and \( K_i \) are proportional and integral gains
respectively. Using the PI compensator parameters \( K_p \)
and \( K_i \) and the NCT characteristic near the origin (Fig.
3b), the transfer function of the closed-loop positioning
system controlled by the NCTF controller can be
approximated as follows\[^{11-15}\]:

\[ \frac{\Theta(s)}{\Theta_i(s)} = G(s) = G_1(s)G_2(s) \]  \hspace{1cm} (4)

where

\[ G_1(s) = \frac{\alpha}{s + \alpha} \]  \hspace{1cm} (5a)

\[ G_2(s) = \frac{2\zeta \omega_n + \omega_n^2}{s^2 + 2\zeta \omega_n + \omega_n^2} \]  \hspace{1cm} (5b)

\[ K_p = \frac{2\zeta \omega_n}{K \alpha} \]  \hspace{1cm} (5c)

\[ K_i = \frac{\omega_n^2}{K \alpha} \]  \hspace{1cm} (5d)

where \( K \) and \( \alpha \) are positive constants which relate to
the plant dynamics. Meanwhile \( \zeta \) and \( \omega_n \) are damping
factor and natural frequency respectively. When \( \zeta \) and
\( \omega_n \) are large enough, \( G(s) \) becomes nearly equal to \( G_1(s) \),
which represent the condition when the plant motion
follows the NCT as the objective of the NCTF control
system. Moreover, large \( \zeta \) and \( \omega_n \) also make the closed-
loop system robust to friction or inertia variation of the
plant in continuous systems\[^{19}\]. Finally, by using \( \zeta \) and
\( \omega_n \) as design parameters and considering Eqs. (2) and
(3), the PI compensator parameters are designed as
follows:

\[ K_p = \frac{2\zeta \omega_n u_r}{m h} \]  \hspace{1cm} (6)

\[ K_i = \frac{\omega_n^2 u_r}{m h} \]  \hspace{1cm} (7)

Here, \( \omega_n \) and \( \zeta \) are design parameters which should
be decided by the designer. Generally speaking, a
higher \( \omega_n \) and a larger \( \zeta \) are preferable in the design of
PI compensator parameters. However physical
constraint of the motor driver and digital
implementation of the NCTF controller limits the
design parameters to maintain the closed-loop stability
as follows\[^{12}\]:

\[ \omega_n \leq \sqrt{\frac{\alpha K S_R}{h}} \]  \hspace{1cm} (8)

\[ \zeta \omega_n \leq \frac{2}{3T} \]  \hspace{1cm} (9)

where \( S_R \) and \( T \) are motor driver slew rate and sampling
time. Detailed discussion on the theoretical background
of the NCTF control system can be seen in\[^{12,14,15}\].

Due to the fact that the NCT and the compensator
are constructed from a simple open-loop experiment of
the plant, the exact model including the friction
characteristic and the identification task of the plant
parameters are not required to design the NCTF
controller. Therefore, the controller design is simple
and easy to implement in practical situation.

**Fuzzy anti-windup schemes:** As the NCTF controller
uses the PI compensator to force plant motion so that it
follows the NCT, the integrator windup up may occur in
connection with large position reference. As discussed
in reference\[^{15}\], in the case of no parameter variations,
there is no significant integrator windup due to the
effect of the saturation. The effect of the saturation is
successfully compensated by using NCTF controller.
However the integrator windup becomes a problem
when the parameters vary\[^{15}\].

In order to overcome the problem of integrator
windup, the PI compensator is improved by adopting a
fuzzy anti-windup scheme. Hence an anti-windup PI
compensator is proposed to be used instead of a pure PI
compensator. The purpose the FAW is to generate the
appropriate output to adjust the value of the integrator
part when the saturation exists. Therefore, by using the
proposed anti-windup PI compensator, it is expected
that once PI compensator output \( U(s) \) exceeds the
actuator limits, the fuzzy ant-windup scheme will
generate a signal to reduce the effect of the integrator
windup. Furthermore, two fuzzy anti-windup schemes
are proposed and evaluated their effectiveness. The first
anti-windup system is Mamdani-based fuzzy anti-
windup (MFA), which is design based on Mamdani-
The second anti-windup system is Takagi-Sugeno based fuzzy anti-windup (TFA), which is design based on Sugeno-type of fuzzy system.

**Mamdani-based fuzzy anti-windup scheme:** The proposed Mamdani-based fuzzy anti-windup (MFA) for the PI compensator is show in Fig. 4. The purpose of the MFA is to generate the signal \( U_{MFA} \) to reduce the integrate part if there is input signal (\( \Delta U \)). In the case of no saturation, the input \( \Delta U \) is equal to zero. Accordingly, the MFA will output signal \( U_{MFA} \) of zero so that it does not affect the performance of the system unless there is saturation.

The first element of the proposed MFA is the fuzzy interface which is used to convert crisp input of the control output into linguistic variables. To map the crisp inputs into related linguistic fuzzy sets, associated membership functions have to be constructed. Figure 5 shows the membership functions of the MFA which use simple and commonly used triangular membership functions. The membership function parameters \( A_i, B_i \) and \( C_i \) are determined based on the NCT parameter \( h \) and the motor input rated as follows:

\[
C_i = hK_p u_r \quad (10)
\]

\[
B_i = \frac{3}{4} C_i \quad (11)
\]

\[
A_i = \frac{1}{2} B_i \quad (12)
\]

where \( K_p, h \) and \( u_r \) are proportional gain, maximum error of the NCT and the voltage input of the driver respectively. As shown in Fig. 5, the input of the MFA contains 4 fuzzy sets (linguistic variables) namely Negative Big (NB), Negative Small (NS), Zero (Z), Positive Small (PS) and Positive Big (PB). The range of the fuzzy input depends only on NCTF controller parameters \( K_c \) and \( h \) and the saturated value of the control signal \( u_r \). Hence the design of the membership function is simple and straightforward.

The MFA output has also 4 fuzzy sets (linguistic variables) as shows in Fig. 6. They are Negative Big (NB), Negative Small (NS), Zero (Z), Positive Small (PS) and Positive Big (PB). The range of the output is based on the range of the integrator term. The output membership function parameters \( A_o, B_o \) and \( C_o \) are calculated based on the NCT parameter \( h \) and the integrator gain of the PI compensator as follows:

\[
C_o = hK_i \quad (13)
\]

\[
B_o = \frac{3}{4} C_o \quad (14)
\]

\[
A_o = \frac{1}{2} B_o \quad (15)
\]

where the \( K_i \) is the integrator parameter of the PI compensator.

Fuzzy rule base is the second element of the MFA. The fuzzy rule-base of the Mamdani-based fuzzy system is composed by IF-THEN rules like *IF antecedent 1 THEN Consequent 1 [16]*. The following fuzzy rules are derived and used to reduce the effect of the integrator windup:

- \( \text{IF} \ \Delta U \ \text{is} \ \text{NB} \ \text{THEN} \ U_{MFA} \ \text{is} \ \text{PB} \)
- \( \text{IF} \ \Delta U \ \text{is} \ \text{NS} \ \text{THEN} \ U_{MFA} \ \text{is} \ \text{PS} \)
- \( \text{IF} \ \Delta U \ \text{is} \ \text{PS} \ \text{THEN} \ U_{MFA} \ \text{is} \ \text{NS} \)
- \( \text{IF} \ \Delta U \ \text{is} \ \text{NB} \ \text{THEN} \ U_{MFA} \ \text{is} \ \text{NB} \)

Furthermore, the fuzzy inference engine and the defuzzifier are used to carry out all the fuzzy operations and hence to determine the \( U(s)_{MFA} \) on the activated rules along with the membership degrees of the associated fuzzy inputs. Mamdani fuzzy inference engine is used to decide the MFA output. Moreover, the Mamdani inference engine works base on Mamdani implication (Min operation) and disjunctive aggregator (Max operator), while the MFA output is converted into crisp output based on the Centroid method.
Takagi-sugeno-based fuzzy anti-windup scheme: Figure 7 shows the proposed Takagi-Sugeno-based fuzzy anti-windup (TFA). The main objective of the fuzzy (TFA) Anti-windup is to generate the suitable signal to reduce the integrator part if there is saturation otherwise the output from the Anti-windup is zero. The first step of the TFA is to convert the crisp input to the membership value of the fuzzy set. Similar with the MFA, triangular membership functions are adopted as shown in Fig. 8. There are three linguistic variables represented by the triangular membership function namely Negative Saturation (NS), Unsaturated (US) and Positive Saturation (PS). Unlike the MFA which is signal $\Delta U$ as the input, the control signal $U$ is used as the input. Therefore the range of the fuzzy input depends on the range of $U(s)$. The membership function parameters $A$ and $B$ are determined based on the NCT parameter $h$ and the PI parameters as follows:

$$A = u_r \quad (16)$$
$$B = h K_p \quad (17)$$

Unlike the MFA which the output is fuzzy variables, the output of the TFA is a crisp variable. In general, fuzzy rule-base of the Takagi-Sugeno fuzzy system is composed by "If $x_1$ is $A$ AND $x_2$ is $B$ THEN $u_1 = f(x_1, x_2)$", where $x_1$ and $x_2$ are the antecedent and $u_1$ is the crisp consequence on $x_1$ and $x_2$. Three rules are used to cover the entire situation in the TFA, the rules are:

- $If \ U \ is \ NS \ THEN \ the \ output \ is \ U_1$
- $If \ U \ is \ US \ THEN \ the \ output \ is \ U_2$
- $If \ U \ is \ PS \ THEN \ the \ output \ is \ U_3$

where

$$U_1 = U \frac{K_h - 0.1}{B - A - 0.1} + \frac{(A + 0.1)(K_h - 0.1)}{B - A - 0.1} - 0.1 \quad (18)$$

$$U_2 = 0 \quad (19)$$

$$U_3 = U \frac{K_h - 0.1}{B - A - 0.1} - \frac{(A + 0.1)(K_h - 0.1)}{B - A - 0.1} + 0.1 \quad (20)$$

Finally, the overall output of the TFA $U_n$ is calculated based on the following equation:

$$U_n = \frac{\sum_{i=1}^{3} \mu_i U_i \times U_i}{\sum_{i=1}^{3} \mu_i} \quad (21)$$

Controller design

Simulated plant description: The NCTF controller with anti-windup PI compensator is tested using a dynamic model of the experimental rotary positioning system as shown in Fig. 9. The positioning system consists of an AC servomotor, a driver and an inertia mass (spindle). The positioning performance was examined using the detailed model in Fig. 10. Its
Table 1: Parameters of the plant

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inertia load, J</td>
<td>$1.17 \times 10^{-3}$ kgm$^2$</td>
</tr>
<tr>
<td>Motor resistance, R</td>
<td>1.2 Ω</td>
</tr>
<tr>
<td>Motor inductance, L</td>
<td>8.7 mH</td>
</tr>
<tr>
<td>Motor torque constant, $K_t$</td>
<td>0.57 Nm/A</td>
</tr>
<tr>
<td>Back-Emf constant, $K_b$</td>
<td>0.57 Vs/rad</td>
</tr>
<tr>
<td>Viscous friction, C</td>
<td>$1.67 \times 10^{-3}$ Nms/rad</td>
</tr>
<tr>
<td>Frictional torque, $\tau_f$</td>
<td>0.215 Nm</td>
</tr>
<tr>
<td>Proportional current gain, $K_{cp}$</td>
<td>26.2 V/A</td>
</tr>
<tr>
<td>Integral current gain, $K_{ci}$</td>
<td>$3.62 \times 10^3$ V/As</td>
</tr>
<tr>
<td>Proportional velocity gain, $K_{sp}$</td>
<td>$8.60 \times 10^{-2}$ As/rad</td>
</tr>
<tr>
<td>Input voltage range, $u_r$</td>
<td>± 6 Volt</td>
</tr>
</tbody>
</table>

Controller design: First, the NCTF controller was designed based on the Normal Plant. Fig. 11 shows the NCT as a result of a simulated experiment. With reference to Fig. 11, the inclination $m$ and maximum error rate $h$ of the NCT are 67.4 rad and 240 rad/s respectively. The compensator parameters are designed by using $h$ and $m$ of the NCT. For the PI compensator, design parameters $\zeta$ and $\omega_n$ are chosen as 13 and 29 respectively. The values of the compensator parameters calculated using Eqs. (6) and (7) are $K_p = 0.279$ and $K_i = 0.312$.

Anti-windup design: Both the MFA and TFA are designed based on procedure discussed earlier. Thus, Fig. 12 shows the input and the output membership functions of the MFA by considering the NCTF controller parameter discussed earlier while those of the TFA are depicted in Fig. 13. The rules of the both proposed anti-windup are simply designed by using the developed rules presented earlier.

RESULTS AND DISCUSSION

Now, the performances of the positioning system controlled using the NCTF with MFA (NCTF-MFA) and TFA (NCTF-TFA) are compared to that with tracking-anti-windup (NCTF-TAW). Tracking anti-windup scheme is classical anti-windup commonly used for PID controller. Detail discussion on the implementation of the tracking anti-windup for the NCTF controller can found in reference.

Figure 14 shows the step responses to a 0.5 rad step input when the controllers are used to control Normal Object and their positioning performances are summarized in Table 2. As shown in Fig. 14, the 0.5-rad step input does not cause the saturation of the control signal. Here, all the controllers produce a similar response due to a similar bandwidth. Hence, in terms of overshoot and settling time, all of the controller give a similar performance.

Furthermore, in order to estimate the robustness of the control systems to inertia variation, all the controllers are implemented on Increased Inertia Object. Fig. 15 shows the step responses to a 0.5-rad step input when all the controllers are implemented for controlling Increased Inertia Object. Table 2 shows the positioning performance resulting from all the controllers. All NCTF controllers give similar results since there is no significant saturation of the actuator as shown Fig. 15(b). The result confirms that the use of anti-windup PI compensator does not affect the positioning performance when there is no saturation of the actuator.

Next, simulation has been done for a larger step input so that the actuator saturates. Figure 16 shows the step responses to a 5-rad step input when all the controllers are implemented for controlling Increased Inertia Object. Table 3 shows the positioning performance resulting from all the controllers. The saturation of the actuator occurs as shown in Fig. 16(b).
However, Fig. 16(a) and Table 3 show that the positioning performance of the positioning system with NCTF-TFA has less overshoot than the rest, however in terms of the settling time the NCTF-MFA is the better. In the same time, NCTF-TAW has the same performances when compared to the NCTF-MFA.

Furthermore, the remarkable finding in the pervious simulation is that the performances of the NCTF-TAW and NCTF-MFA seem to be similar in terms of overshooting and settling time. To distinguish between both of them, a 50-rad step input is considered for the next simulation study. As shown in Fig. 17 and Table 3, the performance of the NCTF-MFA in terms of overshoot and settling time is better than NCTF-TAW. On the other hand, NCTF-TFA still remains as the best one.

**CONCLUSION**

This study has documented the improvement of the NCTF controller to overcome the effects of integrator windup due to actuator saturation. Fuzzy anti-windup
PI compensator was used as compensator for the NCTF controller instead of a conventional PI compensator. Two fuzzy anti-windup schemes are designed namely Mamdani-based fuzzy anti-windup (MFA) and Takagi-Sugeno-based fuzzy anti-windup (TFA) schemes. Through simulation using dynamic model of a rotary positioning system, the effectiveness of the NCTF controller with the proposed fuzzy anti-windup PI compensator is evaluated. The effectiveness of the proposed schemes are evaluated and compared with the conventional tracking anti-windup scheme. The results confirm that the NCTF controller with the Takagi-Sugeno-based fuzzy anti-windup PI compensator is better than the others. The use of the Takagi-Sugeno-based fuzzy anti-windup PI compensator is more effective to overcome the problem due to integrator windup compared to that of the others. Moreover, the simulation results also confirm that the NCTF controller with the Takagi-Sugeno-based anti-windup PI compensator, which is design based a simple open loop experiment, gave the best positioning performance as well as performance robustness to inertia variations.

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