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Emissivity of Triangular Surfaces Determined by Differential Method: From Homogenization to Validity Limit of Geometrical Optics

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Abstract: Geometric optics approximation for emissivity from triangular surfaces was compared with exact scattering predictions from electromagnetic theory. Rigorous electromagnetic scattering theory was numerically formulated based on the differential method. We have used a numerical simulation of the emissivity of gold and tungsten for a wavelength equal 0.55 micron to explore the validity of the geometric optics. Surface parameter domains for the regions of accuracy of the geometric optics approximation are quantified and presented as functions of surface slope and roughness. Influence on the validity of the approximate method of multiple scattering, the shadowing effect and the cavity effect of metallic surface have been investigated. For the latter, our interest was focused on the mechanism that enhances the emissivity of an interface when ruling a grating. It has been seen that the mechanism responsible for the enhancement of the emissivity depends very much on the period of the grating. For gratings with a period much smaller than the wavelength, the roughness essentially behaves as a transition layer with a gradient of the optical index. For different period / wavelength ratio, we have found a good agreement between the differential method and the homogenization regime when the period was smaller than $\lambda/10$.

Key words: Periodic roughness, differential method, geometric optics approximation, homogenization regime, emissivity

INTRODUCTION

The modeling of directional monochromatic emissivity of a rough surface remains a subject of theoretical, experimental and numerical researches[1,2]. The directional nature of surface radiative properties is of interest in many thermal engineering such as semiconductor industry[3], solar energy or the computer graphics[4]. Radiative scattering from one-dimensional rough surfaces can be predicted by a number of approximations and exact solution. The exact approach based on the electromagnetic theory quantifies the directional nature of surface scattering. The geometric optics and Kirchhoff approximation are common specular approximation to electromagnetic scattering and have been extensively compared with exact solutions[5].

Among the exact methods, we can cited the integral method and the differential method. The differential method has been extensively studied. This theory has engendered wide interest because of its good physical insight and the simplicity of its mathematical resolution. The differential method is known in the literature as a rigorous coupled wave analysis (R.C.W.A). Among the authors who were interested in the study of the problem of diffusion by the approach RCWA, we can cited M.G Moharam and T.K Gaylord[6-8] who they applied this approach to a plane grating in the case of polarization TE and then, they extended it to the case of polarization TM[9]. In parallel, various versions of algorithms were proposed. However, some of the solution algorithms are unstable for relatively thick modulated layers[10,11]. Chateau and Hugonin[12] have proposed a new algorithm known as Modal Multilayer Method (MMM). This Technique is numerically stable and allows us to study a gratings with arbitrary period and depth. In this method, a surface relief gratings is divided in to a large number of thin layers parallel to the surface. The projection of propagation equation on a suitable basis of function gives a set of ordinary differential coupled equations. For gratings, the fields are then presented on both external media of the grating by the electric field complex amplitudes of the incident and reflected waves. A recursive resolution in terms of these last amplitudes allows as to determine the reflection efficiencies (ratio of reflected intensity to input intensity) in each order. The sum of all the reflected efficiencies for the propagating waves lead to emissivity (ε_λ).

Another approach of determination of the radiative properties of rough surfaces is considered as an

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approximate method and it is based on geometrical optics. The geometric optics approximation (GOA) ray traces the energy incident on rough surface until it leaves the surface, thereby including multiple scatters from various surface elements. Each surface interaction is modeled as a reflection from a locally optically smooth surface (Fresnel approximation). Therefore, in the limit of plane surface, the geometric optics approximation is reduced to the Fresnel approximation since only a single scatter from the surface occurs. These steps are used to calculated the emissivity (ε_λ).

Within this framework, several authors dealt with this problem for various forms of roughness surface[13-17]. Certain past studies were interested to determine the domain of validity of the geometric optics approximation in comparison with the integral method[5,18].

Also, we approach the study of the radiative properties of these rough surfaces when the period is much smaller than the wavelength. When the grating has one period largely lower than the wavelength, it does not diffract the light. In other words, only the order 0, which corresponds to the specular reflection or transmission, is propaged. Thus it behaves exactly like a smooth surface. However, its structure confers to him a radiative properties which are very different from those of homogeneous material. Bouchitté and Petit[19] showed that a grating of sufficiently small period can be compared to an anisotropy layer with index gradient. This equivalence between grating and layer leads to a coupling between the " multi-layer " process and the approach " roughness ". It was used by Gaylord and Al[20] and Southwell[21] to replace the traditional anti-reflecting coating by a grating. This technique was then extended to the creation of polarizes and filters band suppressor[22].

This work has established the region of validity between the geometric optics approximation (GOA) and differential method (MMM) especially in the cases of triangular surfaces of finished conductivity (gold and tungsten). We have analyzed the different physical phenomena depending on the period and the angles of incidence. We have found a satisfactory agreement between the emissivity calculated by differential method and that given on the basis of homogenization regime when the period is much smaller than the wave length.

Derivation of the emissivity using differential method (MMM)

Geometry: In this study, we have considered a grating with a triangular groove surface relief (Fig. 1). An electromagnetic wave obliquely incident upon the grating produces both forward and backward-diffracted waves, as it is shown in the Fig. 1. Region 1 is a homogeneous dielectric with a relative permittivity of ε_I. Likewise, region 3 is homogeneous with a complex

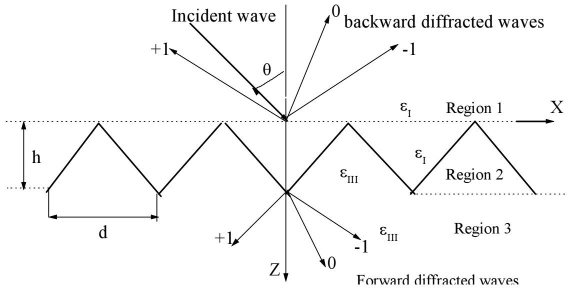

Fig. 1: Geometry of surface-relief grating

permittivity ε_{III}. Region 2 (the grating region) consists of periodic distribution of both types of materiel. In this paper, for simplicity, we have assumed that the incident light has transverse electric (TE) polarization.

The permittivity in region 2 may be expanded in a Fourier series as:

$$\varepsilon(x,z) = n^2(x,z) = \sum_{i=-\infty}^{+\infty} \tilde{n}_i(z)\exp(j\,i\,K\,x) = \varepsilon(x+d,z) \quad (1)$$

where d is the grating period, n is the refractive index, K is the magnitude of the grating vector ($K = \frac{2\pi}{d}$) and $j=(-1)^{1/2}$.

Theory of differential method: In the present analysis, the differential method MMM (Multilayer Modal Method) is adapted to the exact electromagnetic boundary value problem associated with dielectric and metallic miro-rough periodic surfaces.

In order to simplify the notation, we have introduced a modified magnetic field defined as $h = \mu c H$. We have analyzed the propagation of waves inside the grating, using the tangential components $E_y(x,z)$ and $h_x(x,z)$ of respectively the electric and modified magnetic field. These components of electromagnetic field are continuous on the boundaries. We have introduced the fundamental coupled-wave expansions:

$$E_y(x,z)=\sum_{i=-\infty}^{+\infty} E_y^{(i)}(z)\exp\left(jk_x^{(i)}x\right)$$

$$h_x(x,z)=\sum_{i=-\infty}^{+\infty} h_x^{(i)}(z)\exp\left(jk_x^{(i)}x\right) \quad (2)$$

where

$$k_x^{(0)}=k_0\cdot n_0\sin\theta \text{ et } k_x^{(i)}=k_x^{(0)}+iK,\ i\in Z \quad (3)$$

In This method, a surface relief grating is divided into a large number of thin layers parallel to the surface. By replacing the field expansions (2) and (3) in the Maxwell equation[23], we have obtained a differential system with constant coefficients according to each thin gratin k:

$$\frac{dU(z)}{dz}=\left[M_k\right]U(z) \quad (4)$$

The solution of the shift invariant system (4) between two arbitrary coordinates Z_{k+1} and Z_k ($Z_{k+1}>Z_k$) involves a matrix exponential function:

$$U(z_k)=\exp\left\{-\left(z_{k+1}-z_k\right)\left[M_k\right]\right\}U(z_{k+1}) \quad (5)$$

We shall express the matrix exponential in terms of eigenvectors and eigenvalues of [M].
Diagonalizing matrix [M], we obtain:

$$\left[M_k\right]=\left[P_k\right]\left[D_k\right]\left[P_k\right]^{-1} \quad (6)$$

Where the columns of matrix [P_k] are the eigenvectors of [M_k]and [D_k] is the diagonal matrix of the eigenvalues of [M_k].

Numerical methods for analyzing layered gratings face reveal a common difficulty associated with the inversion of matrix [7,10]. For solving the problem without numerical difficulties, we have adopted the stable algorithm presented by N. château and J.P.Hugonin[12] for transverse electric polarization (TE). The new algorithm that remains stable for gratings of any thickness by rearranging the position of the eigenvectors matrix columns in relation (6), put the eigenvalues in growing order on the diagonal of matrix [D].

When the reflected $b_f^{(i-\gamma)}$ and transmitted $f_L^{(i-\gamma)}$ field complexes amplitudes are known, the diffraction efficiencies (ratio of diffracted intensity to input intensity) may be directly determined. Then the diffraction efficiencies in region 1 and 3 are:

$$\eta_B^{(i-\gamma)}=\frac{k_{Fz}^{(i-\gamma)}}{k_{Fz}^{(0)}}\left|b_F^{(i-\gamma)}\right|^2;$$

$$\eta_F^{(i-\gamma)}=\frac{k_{Lz}^{(i-\gamma)}}{k_{Fz}^{(0)}}\left|f_L^{(i-\gamma)}\right|^2 .\ \text{avec}: k_{Fz}^{(i)}=\left[k_0^2 n^2-k_x^{(i)2}\right]^{1/2} \quad (7)$$

Surface radiative properties: As our interest is the region 1, the precedent general approach has been used to derive the monochromatic directional hemispherical reflectivity and it is given by function of the angle of incidence θ :

$$\rho'^{o}_{\lambda}(\theta)=\sum_{i=-N}^{i=+N}\eta_B^{(i-\gamma)} \quad (8)$$

Using Kirchhoff's law and conservation of energy, the absorptivity is identified to the directional monochromatic emissivity[15].

$$\varepsilon_\lambda(\theta)=1-\sum_{i=-N}^{i=+N}\eta_B^{(i-\gamma)} \quad (9)$$

Derivation of the emissivity using geometrical optics: The geometric optics or ray tracing approximation to the electromagnetic theory predictions of surface scattering is a multiple scattering solution which traces energy from incidence until it leaves the surface. Each scattering is treated as a Fresnel reflection at the local point of interaction. In the limit of plane surface, the geometric optics approximation becomes the Fresnel or specular approximation.

Thus, the geometric problem consists in calculating the coordinates of each rebound point and the local reflection angle, from the first reflection point to the last one successively. Since the surface groove is assumed to the locally optically smooth plane, only a single and specular reflection occurs at each interaction point.

An incident monochromatic pencil of parallel rays strikes the surface under the angle of incidence θ. We consider M equally spaced point on the line parallel to X-axis. An incident ray, determined by the entry point X_{Sj} , j integer from 1 to M, includes a set of N_j local reflection points $\left\{S_i\right\}_{1\leq i\leq N_j}$ inside the relief surface. Remaining energy is simply proportional to ρ_λ. After N reflection events, the remaining energy is proportional to $\rho_\lambda(\varphi_1)\rho_\lambda(\varphi_2)\times\cdots\times\rho_\lambda(\varphi_{Nj})$, where φ_i designates the local reflection angle at the interaction point S_i and $\rho_\lambda(\varphi_i)$ is the Fresnel reflection factor.

At each reflection event, a fraction $1-\rho_\lambda$ of the energy of the beam is absorbed, so, the absorptivity is equal to: $1-\rho_\lambda(\varphi_1)\rho_\lambda(\varphi_2)\times\cdots\times\rho_\lambda(\varphi_{Nj})$.

The absorptivity and emissivity derived from geometrical optics are equal to:

$$\varepsilon_\lambda(\theta,X_{Sj})=1-\prod_{i=1}^{n}\rho_\lambda(\varphi_i) \quad (10)$$

where $\varepsilon_\lambda(\theta,X_{Sj})$ is the local emissivity in the point X_{Sj}.

So, we have evaluated the contribution of the ray that impinges on the surface at a particular point X_{Sj}. The next step is to average the emissivity over the grating surface of period d according to a position X_{Sj} as follow:

$$\varepsilon_\lambda(\theta)=\frac{1}{d}\int_0^d\varepsilon_\lambda(\theta,X_{Sj})dX \quad (11)$$

where ε_λ is the directional monochromatic emissivity.

Concept of homogenization: We treat now the radiative behavior of grating with a period much smaller than the wavelength. Under these conditions, the roughness is equivalent to a superposition of layers of given effective indices using the theory of the effective mediums[24].
For TE polarization, the effective dielectric constant of $k^{iéme}$ layer is given by:

$$\varepsilon_{eff,k}=f_k\cdot\varepsilon_3+(1-f_k)=\left(n_{eff,k+1}\right)^2 \quad (12)$$

where f_k is the factor of filling.

It is established that the shape of the field in the medium of incidence results from the shape of the field in that of transition by simple matrix products[25]:

$$U_1(z_1) = A\,U_{M+2}(z_{M+1}). \qquad (13)$$

The Matrix $A = T_1 \cdot \prod_{j=2}^{M+1} C_j T_j$ characterizes the studied system and it is obtained from the matrices of transition T_j and the matrices of layer C_j [26].

We define the coefficient of reflection by:

$$r = \frac{\text{Reflected field complex amplitude}}{\text{Incident field complexe amplitude}} \qquad (14)$$

The emissivity is given by the following relation:

$$\varepsilon_\lambda(\theta) = 1 - |r|^2 \qquad (15)$$

RESULTS AND DISCUSSIONS

In the present paper, we have studied the validity of the geometric optics approximation (GOA) in comparison with differential method (MMM) in terms of directional monochromatic emissivity. We have determined by two methods the emissivity of gold (Au) and tungsten (W), of respective refractive indexes $n_u = 0.48 + i2.45$ and $n_W = 3.5 + i2.73$ for cylindrical surfaces with a triangular profile corresponding to wavelength equal to 0.55 microns. The directional monochromatic emissivity of the surfaces depends on the incidence angle, the profile of the surface, the height **h** and the period **d**, in addition to the nature of material.

In our knowledge, the accuracy of this approximate method depends on the multiple reflection and shadowing phenomenon due to the relief of surface. For incidence angles surrounding $\theta = 0°$ (normal incidence), there is no effects of shadow and only multiple scattering is to be considered. The effect of shadowing is most important at large incident angles. In order to study the influence of these effects on the regions of validity of the GOA, our calculations were performed with the angle of incidence $\theta = 1°$ for the former effect and with $\theta = 60°$ for the latter.

It is clear that for a height h of the grating, when the period d increases sufficiently, angle β defined by $tg\beta = \frac{2h}{d}$ tends towards zero, so the directional emissivity $\varepsilon'_\lambda(\theta)$ tends towards the emissivity of the plane surface. Thus, we should expected that the two curves displaying emissivity $\varepsilon'_\lambda\left(d/\lambda\right)$ versus d/λ, for fixed θ and h/λ, obtained by the differential method and the geometric optics approximation presented the

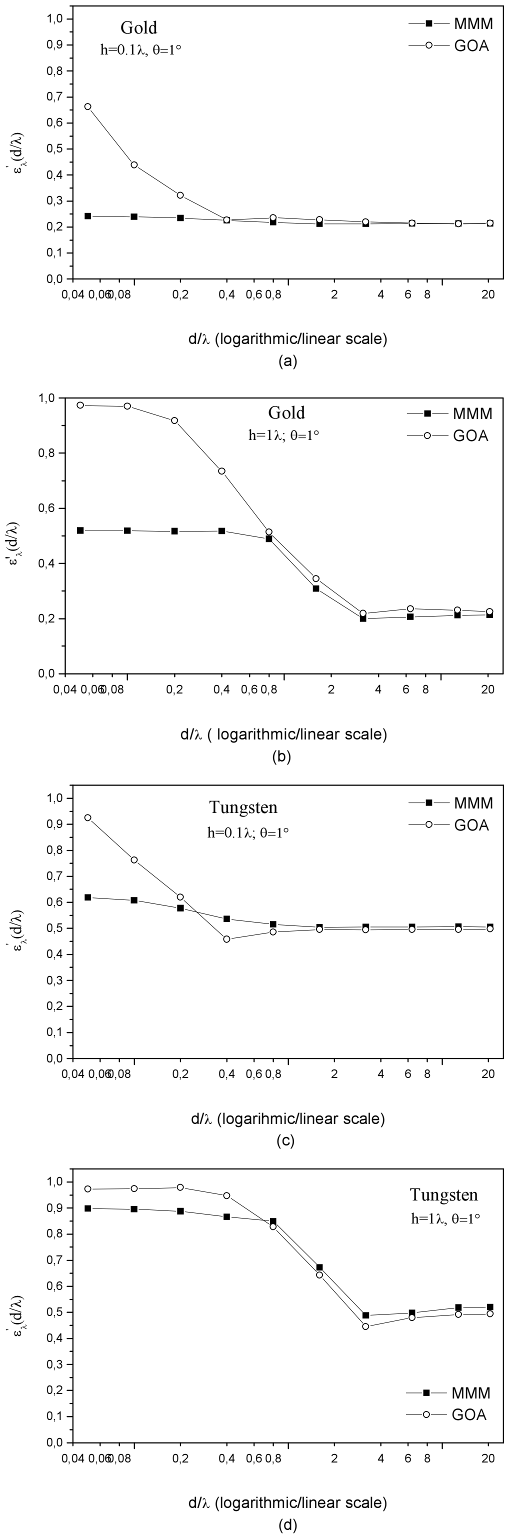

Fig. 2: Comparison of differential method (MMM) solutions with geometric optics approximation (GOA) for the directional monochromatic emissivity $\varepsilon'_\lambda\left(d/\lambda\right)$ of triangular surfaces. Case of the angle of incidence $\theta = 1°$

same horizontal asymptotic behavior, for d greater than λ. By comparing these two curves, we can calculate the limit value $\left(d/\lambda\right)_{lim}$ which quantifies the validity of the GOA for constants d/λ and θ.

Case of the ratio h/λ=0.1 and h/λ= 1: We start by presenting and discussing the results concerning the symmetrical triangular grooves with height h/λ=0.1 and h/λ= 1. We compare the curves of emissivity $\varepsilon'_\lambda\left(d/\lambda\right)$ calculated by the two methods for the angles of incidence $\theta=1°$ and $\theta=60°$.

a. Case of the angle of incidence $\theta=1°$: Figure 2a and 2b show curves displaying the directional monochromatic emissivity ε'_λ of gold grating versus the ratio d/λ (logarithmic/linear scale) for a heights equal to 0.1λ and 1λ. In order to study the different regimes, we have varied the period d of the grating in the range $\left[\frac{\lambda}{20}, 20\lambda\right]$. It seem that the ray tracing approach GOA yields the same results as the MMM method for large periods but fails for periods smaller than $\left(d/\lambda\right)_{lim}=0.4$ corresponding to h equal to 0.1λ (Fig. 2a) and for periods smaller than $\left(d/\lambda\right)_{lim}=0.8$ corresponding to h equal to 1λ (Fig. 2b). These two limits can also be expressed by the inequality: $\beta<\beta_{lim}$, where β_{lim} is about 26.56° for $h=0.1\lambda$ and about 68.19° for $h=1\lambda$. In the case $h=0.1\lambda$, the agreement between the two methods corresponds to cavities in V inside of which an incident ray under the angle 1° undergoes only one reflection according to a local angle equal to (β+ 1°). Whereas for $h=1\lambda$, this agreement exceeds the single scattering domain limited to approximately $\left(d/\lambda\right)_{lim}>6.4$, to cover cases of multiple scattering defined by ratio d/λ included between 0.8 and 6.4 corresponding to angles β located between approximately 30° and 70°. As example, for $h=1\lambda$ and $d=0.8\lambda$ the number of point of impact associated with an incident ray inside the cavity in V is equal to four for the total incident beam.

We can concluded from the first two figures that we can groove on a plane surface a symmetrical cavities in V with positive slope 2h/d inside of which the GOA remains valid until this slope reaches value 0.5 for $h=0.1\lambda$ and the value 2.5 for $h=1\lambda$.

Thus, in these two cases ($h=0.1\lambda$ and $h=1\lambda$) and for $\theta=1°$ the domain of validity in term of slope of surface is more extended when the height h of the grating is higher. Then, we show that the common asymptotic behavior of the exact and approximate solutions is well illustrated in these figures. The horizontal asymptote of emissivity given by the limit value of $\varepsilon'_\lambda\left(d/\lambda\right)$, corresponds exactly to that of plane surface according to the normal incidence. It is equal to 0.21 as shown by the indicatrix of emissivity of this surface (Fig. 3a) calculated by the relation of Fresnel deduced from the electromagnetic theory of Maxwell[15]. Also, the Fig. 2a and 2b showed no agreement between the two methods when the slope is higher than 0.5 for $h=0.1\lambda$ and when the slope exceeds 2.5 for $h=1\lambda$.

In the case of tungsten (W) for the same profiles in V and the same angle of incidence 1°, there is an agreement between the two methods from d/λ equal to approximately 0.8 for the two considered heights $h=0.1\lambda$ (Fig. 2c) and $h=1\lambda$ (Fig. 2d). This agreement is practically identical to that of the case of gold and the precedent interpretations remain valid here.

Asymptotic limit of emissivity, corresponding to the low values of β ($\frac{d}{\lambda}$ great height), is well that of the plane surface according to the normal and it is equal to 0.51 (Fig. 3b). For the symmetrical V profiles and for incidences angles surrounding $\theta=0°$ (normal incidence), it is established that the GOA lead to values of emissivity which tend towards the unit when angle β tends towards $\frac{\pi}{2}$ as well for conducting materials as dielectric. This is explained by the fact that an incident ray under the angle 1° is trapped and undergoes a large number of reflections inside the cavity (62 for $\frac{h}{\lambda}=1$ and $\frac{d}{\lambda}=0.05$) which behaves then like a black body. This fact is seen in Fig. 2b and 2d. The emissivity increases gradually as the period decreases. The mechanism responsible for this enhancement of the emissivity is very simple and can be explained by the fact that since there is multiple scattering (or ray trapping) there is more absorption. We may call this a cavity effect[26]. For smaller periods, the effect is different. So far, we shall examine this case in homogenization regime.

b. Case of the angle of incidence $\theta=60°$: For the same symmetrical V profile of gold or tungsten and for angle of incidence θ equal to 60°, there is an

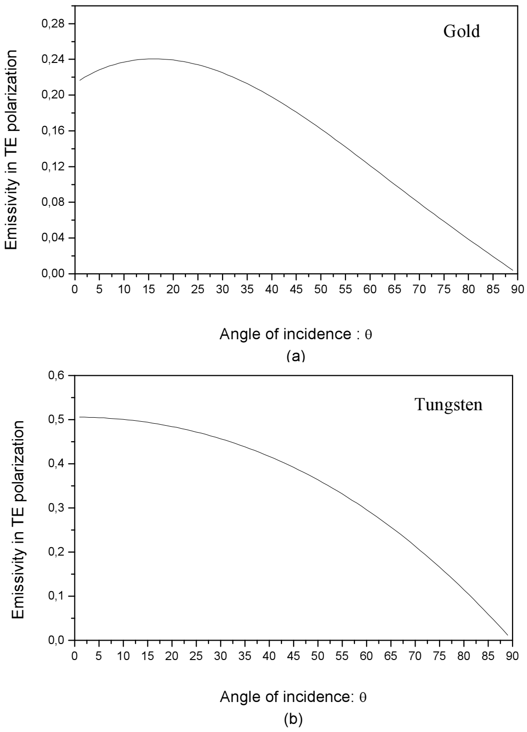

Fig. 3: Emissivity of plane surface

agreement between the two methods if the ratio $\frac{d}{\lambda}$ is greater than 1.6 and the height of the grating is equal to 0.1λ (Fig. 4a and 4b). In these cases, the condition of validity of the GOA is expressed by an angle β lower than 10° according to a simple reflection under a local angle equal to ($60° \pm \beta$) for any incidence. Therefore, it is clear that there domains of validity in term of slope are much more extended than those of the angle of incidence $\theta=1°$.

The asymptotic limit value of $\varepsilon_\lambda'\left(d/\lambda\right)$ is about 0.121 for gold (Fig. 3a) and 0.296 for tungsten (Fig. 3b) and it is in agreement with that given by the Fresnel's formulae. We have already pointed out that the angle of incidence $\theta=60°$ has been chosen to study the influence of the shadowing effect on the validity of the GOA. However, at this angle of incidence, this effect occurs for surfaces having grooves with slopes greater than 30°. The shadowing effect is then important and the GOA is not valid. Besides, an analysis of multiple reflection shows that in these cases the simple scattering is rather frequent. The shadowing effect combined with a multiple reflection prohibit validity of the geometric method. Thus for $h=0.1\lambda$, the absence of this effect for $\theta=60°$ and $\beta<30°$, always realized for $\theta=1°$, must be associated with a simple reflection in order to validate the GOA.

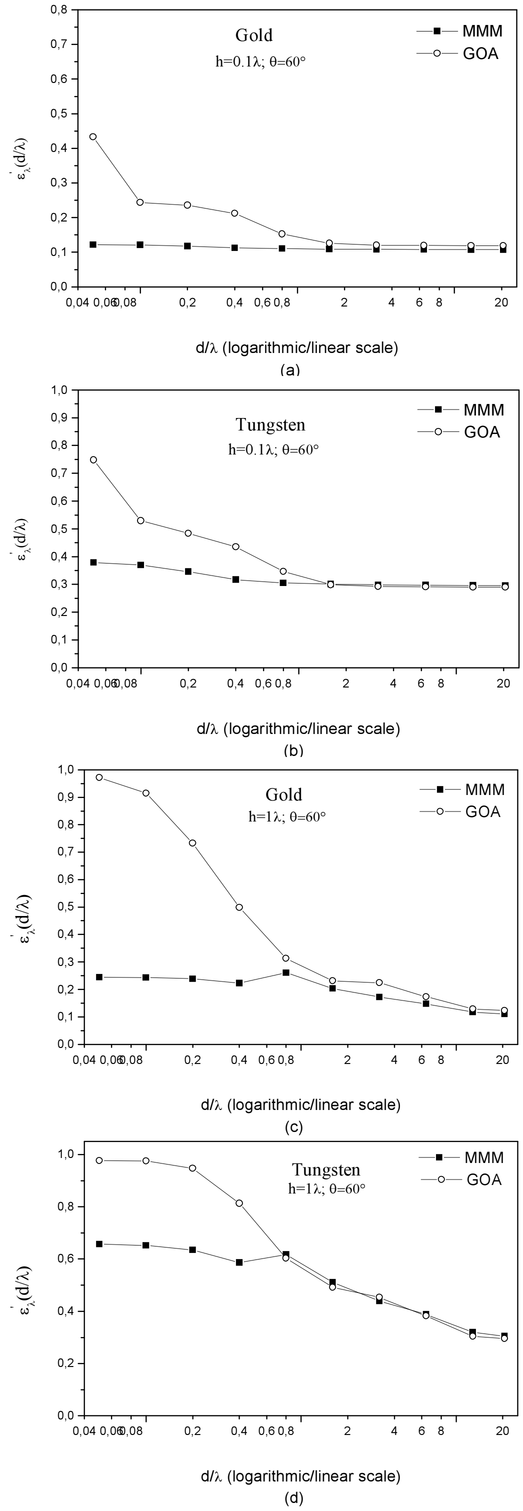

Fig. 4: Comparison of differential method (MMM) solutions with geometric optics approximation (GOA) for the directional monochromatic emissivity $\varepsilon_\lambda'\left(d/\lambda\right)$ of triangular surfaces. Case of the angle of incidence $\theta = 60°$

Table 1: Emissivity computed using the homogenization regime and those calculated by the differential method and geometrical optics $\frac{d}{\lambda} = 0.05$

		Au			W		
		MMM	Homogenization	GOA	MMM	Homogenization	GOA
h	θ						
$h=0.1\lambda$	1°	0,2416	0,2425	0,6624	0,6185	0,6233	0,9245
	10°	0,2380	0,2388	0,6433	0,6128	0,6175	0,9169
	50°	0,1559	0,1565	0,4994	0,4586	0,4629	0,8250
	60°	0,1220	0,1225	0,4332	0,3787	0,3826	0,7481
$h=1\lambda$	1°	0,5190	0,5199	0,9734	0,8979	0,8965	0,9736
	10°	0,5145	0,5103	0,9799	0,8938	0,8924	0,9799
	50°	0,3196	0,3165	0,9740	0,7524	0,7512	0,9761
	60°	0,2448	0,2451	0,9712	0,6572	0,6561	0,9764

When the height of the grating is equal to λ there is practically an agreement between the GOA and the MMM for $\frac{d}{\lambda}$ higher than 0.8 as well for gold and tungsten (Fig. 4c and 4d). This limit also results in angles β lower than approximately 70° like the case of the angle of incidence 1°. However, for surfaces with $h=1\lambda$, the shadowing phenomenon does not prohibit the validity of the geometric method.

Limiting slope $\left(\frac{2h}{d}\right)_{\lim}$ function of the angle of incidence θ: In order to study the validity domain of the geometric optics, we have considered again the preceding study in the cases of gold for heights $h=0.1\lambda$ and $h=1\lambda$, for the angles of incidence in the range [10°, 80°] with a step of 10°. The two curves of emissivity $\varepsilon'_{\lambda}\left(d/\lambda\right)$ for fixed h and θ, calculated by the two methods and taking forms similar to those of the curves presented on Fig. 2a and 2b, allow as to determine the value of limiting ratio $\left(d/\lambda\right)_{\lim}$ from which the GOA is valid. In order to obtain eight values of $\left(d/\lambda\right)_{\lim}$, for each fixed height ($h=0.1\lambda$ and $h=1\lambda$), we have studied the variation of ε'_{λ} as a function of $\frac{d}{\lambda}$. For this ratio, we have associated the limiting slope $\left(\frac{2h}{d}\right)_{\lim}$ for fixed h. Figure 5 shows curves displaying the limiting slope versus the cosine of the angle of incidence or emission θ, for a fixed height h. This curves delimits two regions. The entire region below the curve is the region of validity of the geometric optics, whereas in the above region the use of the MMM or another exact method is necessary. This graph shows that the domain validity of the geometric optics approximation is more extended for $h=\lambda$ thane for $h=0.1\lambda$.

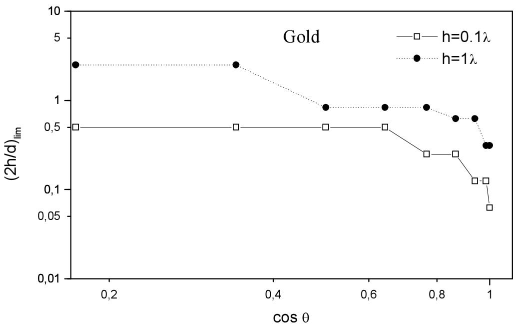

Fig. 5: Differential method of triangular surfaces domain plot with region validity for the geometric optics approximation in terms of limiting slope $\left(2h/d\right)_{\lim}$

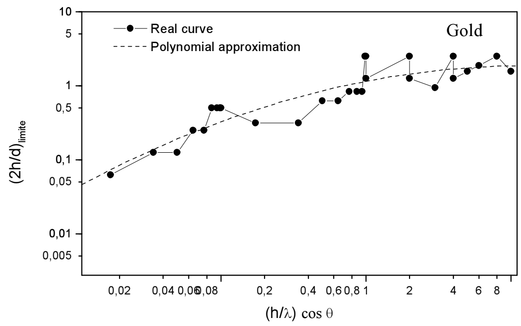

Fig. 6: Ratio $\left(2h/d\right)_{\lim}$ as a function of $\left(h/\lambda\right)\cos\theta$

Limiting slope $\left(\frac{2h}{d}\right)_{\lim}$ function of $\left(\frac{h}{\lambda}\right)\cos\theta$: By adopting the preceding step, for fixed $\frac{h}{\lambda}$ and θ while varying the ratio d/λ, we have determined the limit of validity of the GOA in comparison with the MMM for the angles of incidence 1° and 60° and for the ratio $\frac{h}{\lambda}$ going up to value 10 with a step equal to 2. However, we include the points determined from the curves of

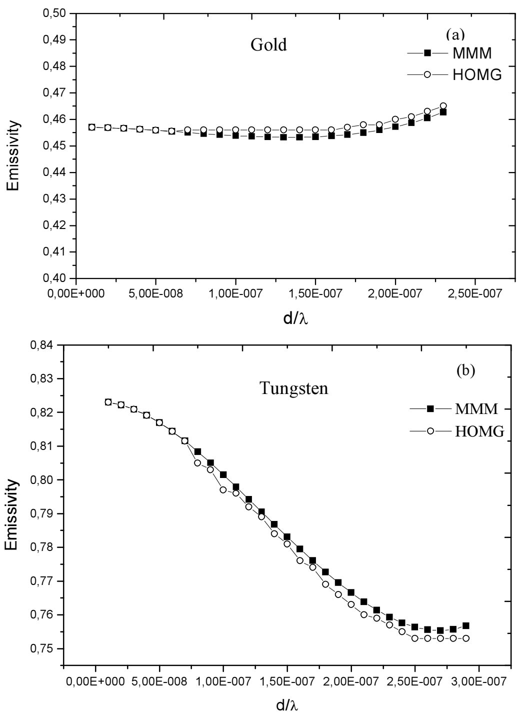

Fig. 7: The directional monochromatic emissivity ε'_λ versus d/λ given by a differential method and homogenization regime

Fig. 5. Thus, Fig. 6 gives a significant idea concerning the region of validity of the geometric optics approximation. As can be seen, in the region situated below the "cloud" of points of this figure, the geometric optics approximation is valid, whereas above this same "cloud" the use of an exact method is necessary. It is to be noted that the differential method is the unique used method for the depth of $h=10\lambda$ relative to the metallic surface.

Homogenization regime: We have compared the diffraction efficiencies of grating by using the differential method (MMM) with the homogenization regime. We have considered a symmetrical grating in V and a normal incident monochromatic wave upon the grating. We have varied the period d of the grating in the range $\left[\frac{\lambda}{80},\frac{\lambda}{2}\right]$ for two materials. Figures 7a and 7b show curves displaying the directional monochromatic emissivity ε'_λ versus $\frac{d}{\lambda}$ given by a differential method and homogenization regime. We have noticed that in the case of TE polarization and for two materials, the relative error between two methods is lower than 2% when the period is smaller than $\frac{\lambda}{10}$. Under these conditions, the grating is equivalent to a superposition of layers with given effective indices determined by the theory of the effective mediums. We have summarized the results of the comparison between the homogenization model, MMM and the GOA in Table 1 for two materials when the ratio d/λ is equal to 0.05. We note a good agreement between the homogenization regime and the differential method (MMM) in all these cases and no agreement with the geometric optics approximation (GOA). For homogenization regime, we have shown that the emissivity can be enhanced. The mechanism responsible of this enhancement depends very much on the period of the grating. For gratings with a period much smaller than the wavelength, the roughness essentially behaves as a transition layer with a gradient of the optical index. Such a layer reduces the reflection thereby increasing the absorption.

CONCLUSION

In this paper, we have determined by the differential method (MMM), the geometric optics approximation (GOA) and the homogenization regime, the emissivity of gold (Au) and tungsten (W) surfaces with a triangular profile, for a wavelength equal to 0.55 microns. To our knowledge, comparison between these three methods in term of directional monochromatic emissivity, especially for metallic surfaces is not frequent. Indeed it is known that the treatment of these cases by the integral method is of an excessively high numerical cost.

The results obtained by the exploitation of the codes elaborated for the three methods and for TE polarization are validated. The accuracy of the used numerical differential method has been tested in some cases of angles of incidence and surface parameters. However, in this paper surface parameter domains for the regions of accuracy of the geometric optics approximation have quantified and presented as a functions of surface slope and roughness.

In this respect, the influence of the multiple scattering, the shadowing phenomena and the homogenization regime is studied in several different cases.

From results obtained in this work, we have found that the domains of validity depends on the ratio d/λ. For values of d/λ larger than 2, the ray tracing model is valid provided that there is no shadowing and the simple reflection is usually required. The shadowing effect combined with a multiple reflection prohibit the validity of the geometric method. However, for surfaces with $h=1\lambda$ the shadowing phenomenon does not prohibit the validity of the geometric method. By using the ray tracing approach, we have obtained a large absorptivity which can be explained by the cavity effect (multiple scattering).

For values of d/λ smaller than 0.5, the emissivity of the rough surface is well described by an effective homogeneous index which is known as the homogenization regime. In this homogenization regime, a large absorptivity caused by the index gradient multilayer reduces reflection. We found a good agreement between the emissivity calculated by the Multilayer Modal Method (MMM) and that given on the basis of the homogenization regime.

An extension of this work to other periodic and two dimensional rough surfaces in polarization TM is to be considered for other materials and wave lengths.

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