Discussion around the Scattering Matrix
Realization of a Microwave Filter using the
Bond Graph Approach and Scattering Formalism

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Abstract: Problem statement: Further to research works made previously and which use collectively
the scattering formalism and bond graph technique for the modeling of a physical systems often
working in high frequencies, we propose, in this article, a comparative study (discussion) for the
scattering matrix realization of a high-frequency physical system. Approach: This discussion is based,
on the one hand, on a non-causal (acausal) bond graph model which represents the starting model for the
determination of the scattering parameters. On the other hand, we shall use a causal bond graph model
richer in information and to which we shall apply what we called in former articles: the analytical
procedure of the scattering parameters exploitation with the aim of showing the importance of the
causality notion in the physical systems study by the bond graph approach, as well as the importance of
the ways and causal loops notion. Results: We will, initially, apply this discussion, to an elementary
transmission line; in the second place, the application is carried out on the equivalent circuit of a band
pass filter based on localized elements often used like microwave filters in high frequencies studies.
Conclusion: We will finish this discussion by realizing the scattering bond graph model of a quadruple
by pointing out the procedure used for the construction of this new type of bond graph model.

Key words: Transmission line, scattering bond graph model, bond pass filter, loops notion, microwave
filters, graph technique, dynamic behavior, physical systems

INTRODUCTION

The development of the bond graph technique was
articulated around two basic concepts which are the
reticulation assumption and the power continuity
principle (Mota and Mota, 2011), without forgetting the
importance of the causality concept, which reveals the
relations of cause for purpose (Cause with effect)
between the various modules of the system and returns
the bond graph model richer in information than a
simple graph model (Birkett, 2009).

The profits of the causal ways and loops make it
possible, amongst other things, to make structural
analysis, to have an estimate on the dynamic behavior
of the system, to determine the Inputs-outputs relations,
whereas the scattering formalism, through its different
properties evoked previously (Taghouti and Mami,
2010a; Taghouti and Mami, 2009), includes explicitly
the conservation laws and respect in an intrinsic way
the causal relations (Buisson et al, 2000). It thus plays a
significant role in the bond graph development and add
the interest which Paynter (1992) grants to him which
regards it as an alternative approach for the physical
systems modeling.

We will trying, in this article, to apply the new
modeling technique, described in former articles, which
uses collectively the scattering formalism and the bond
graph approach for a modeling of the physical systems
often working in high frequencies. For that purpose
and although these works on this new technique
remain limited at least as regards the bond-graphic
designers, the display of what was made in this
domain will allow us to propose, at the beginning of
our researches, a method based on a non-causal
(acausal) bond graph to calculate the scattering matrix
of the studied system.

Then and while basing itself on the fact that the
concept of causality is a concept very significant in
modeling of the physical systems since it enables us to organize the relations constitutive of the elements in an Inputs-outputs form and to analyze the variables of powers effort and flow in terms of dependence, we propose in the continuation of this study to build the “Scattering Bond Graph” model by taking account of the causality concept contrary with what was carried out in the study of Professor (Kamel and Dauphin-Tanguy, 1996) which completely cancelled the concept of causality in spite of its importance. We consider, for the construction of this new model, as starting point the conventional bond graph model which will enable us to calculate the scattering parameters by the application of the analytical exploitation procedure explained in our previously papers and which uses the causal ways notions and the Masson rule applied to the transformed, reduced and causal bond graph model and for objective to apply to the found scattering matrix, which is not actually a transfer matrix, the procedure described in the previously papers (Taghouti and Mami, 2010a) in order to have the famous model “Scattering Bond Graph” of the studied system.

Calculation of the scattering matrix from a non-causal (acausal) bond graph representation: The starting point of this method is the acausal bond graph of a physical or electrical studied system brought back in the elementary shape to two branches including only one “0-junction” and “1-junction” associated with an equivalent impedance and admittance in cascades (Kamel et al., 1993).

Now let us consider the series impedance and the parallel admittance in reduced variables as the Fig. 1a and Fig. 1b shows it below.

\[ w_i = \frac{v + i}{2} \]  \hspace{1cm} (1)

\[ w_r = \frac{v - i}{2} \]  \hspace{1cm} (2)

The bond graph representations (Fig. 2) associated respectively to the Fig. 1a and b above will be as follows:

The scattering matrices \( S_{\text{series}} \) (often noted \( S_s \)) and \( S_{\text{parallel}} \) (often noted \( S_p \)) associated to the two representations below can be written, while being based with the Eq. 1 and 2 and with the Kirchhoff rules, in the following form Eq. 3 and 4:

\[ S_s = \begin{bmatrix} \frac{z}{z+2} & \frac{2}{z+2} \\ \frac{z}{z+2} & \frac{z}{z+2} \end{bmatrix} \]  \hspace{1cm} (3)
The equivalence out of wave matrixes is given respectively by the below expressions Eq. 5 and 6:

\[
\begin{align*}
S_y &= \begin{bmatrix}
-\frac{y^2}{y+2} & \frac{2}{y+2} \\
\frac{2}{y+2} & -\frac{y}{y+2}
\end{bmatrix} \\
W_z &= \begin{bmatrix}
\frac{z+2}{2} & -\frac{z}{2} \\
\frac{z}{2} & \frac{2-z}{2}
\end{bmatrix} \\
W_p &= \begin{bmatrix}
\frac{y+2}{2} & \frac{y}{2} \\
-\frac{y}{2} & \frac{2-y}{2}
\end{bmatrix}
\end{align*}
\] (4)

The product of the two wave matrixes below gives us the total wave matrix \( W_{sp} \) of the system in cascade in the following form Eq. 7:

\[
W_{sp} = W_z \cdot W_p = \begin{bmatrix}
\frac{zy+z+y+2}{2} & \frac{zy-z+y+2}{2} \\
\frac{zy+z-y+2}{2} & -\frac{zy-z+y+2}{2}
\end{bmatrix}
\] (5)

By applying the transformations between wave matrixes and scattering matrixes, we can have the total scattering matrix form \( S_{sp} \) of the system in cascade, such as Eq. 8:

\[
S_{sp} = \frac{1}{zy+z+y+2} \begin{bmatrix}
zy+z-y & 2 \\
2 & -zy+z-y
\end{bmatrix}
\] (6)

The determination of the scattering matrix of a complex physical system is made by referring to the Eq. 8 and by the organization of the system in the hierarchical arborescence form allowing finding the elementary structure of equivalent impedance and admittance in cascade (stunt). Now in the case of the electric systems working in high frequencies, the organization in the hierarchical arborescence form is almost impossible, because, like example, we can never return an antenna with micro-strip lines or multi-coats (multilayer) antennas with distributed elements under the shape of serial impedance in cascade (stunt) with a parallel admittance. We shall discuss during this study the case of the characteristic impedances of these systems and the possibilities of returning them in the form of impedance and admittance in cascade (stunt).

Comparison to the analytical exploitation procedure: Application in a transmission elementary line: Let us consider now the transmission line and its elementary variation represented like indicates it the following Fig. 3.

The electric study of the transmission lines (Beck et al., 1995) is possible that has to leave an equivalent model with localized elements and which represents a linear variation which the dimensions are much smaller than the used wavelength guided (\(l<<\lambda)\). Under these constraints, it is possible to model the line as a stake in cascade (stunt) of elementary quadripole (Matthaei et al., 1980) as indicates it the Fig. 4 below.

Where:
- \( R \) = Linear electrical resistance
- \( L \) = Linear inductance
- \( C \) = Linear capacity
- \( G \) = Linear conductance

The elementary structure of the equivalent linear impedance and linear admittance in cascade is given by the following expressions Eq. 9:
The normalization of these expressions compared to a normalization resistance (often we shall consider the internal resistance of the generator as normalization resistance by taking into accounts the condition of impedance adaptation) allows us to write Eq. 10 and 11:

\[
\begin{align*}
  z &= r + \tau_L \cdot s \\
  y &= g + \tau_C \cdot s 
\end{align*}
\]  

(10)

Where:

\[
\begin{align*}
  r &= \frac{R}{R_o}, \quad g = \frac{1}{r} \\
  \tau_L &= \frac{L}{R_o}, \quad \tau_C = C \cdot R_o 
\end{align*}
\]  

(11)

From the acausal bond graph model of Fig. 5 given below as well as the expression of Eq. 8, we can directly deduce the scattering parameters of the elementary line variation describes previously. Thus we can write Eq. 12:

\[
\begin{align*}
  S_{11} &= \frac{\tau_C \tau_L \cdot s^2 + \left[ \frac{\tau_C (1 + g)}{\tau_L (1 - r)} \right] \cdot s + g(r - 1) + r}{\tau_C \tau_L \cdot s^2 + \frac{\tau_C (1 + r)}{\tau_L (1 + g)} \cdot s + r(1 + g) + g + 2} \\
  S_{12} &= \frac{\tau_C \tau_L \cdot s^2 + \frac{\tau_C (1 + r)}{\tau_L (1 + g)} \cdot s + r(1 + g) + g + 2}{2} \\
  S_{21} &= \frac{\tau_C \tau_L \cdot s^2 + \frac{\tau_C (1 + r)}{\tau_L (1 + g)} \cdot s + r(1 + g) + g + 2}{2} \\
  S_{22} &= \frac{-\tau_C \tau_L \cdot s^2 + \left[ \frac{\tau_C (1 + r)}{\tau_L (1 + g)} \right] \cdot s - g(r + 1) + r}{\tau_C \tau_L \cdot s^2 + \frac{\tau_C (1 + r)}{\tau_L (1 + g)} \cdot s + r(1 + g) + g + 2}
\end{align*}
\]  

(12)

The causality assignment on this bond graph results in an effort-flow causality seen by the system as indicated on Fig. 6. While referring to our old works (Taghouti and Mami, 2009; 2010a; 2010b), the analytical Inputs-outputs relations can take, then, the following form Eq. 13:

\[
\begin{align*}
  \phi_1(t) &= H_{11}(s)\epsilon_1(t) + H_{12}(s)\varphi_1(t) \\
  \varphi_2(t) &= H_{21}(s)\epsilon_1(t) + H_{22}(s)\varphi_2(t)
\end{align*}
\]  

(13)

The scattering matrix will take the following form Eq. 14:

\[
\begin{align*}
  S &= \begin{bmatrix} 1 - H_{11} - H_{22} & 2H_{12} \\ 1 + H_{11} - H_{22} + \Delta H & -1 - H_{44} - 2H_{21} \end{bmatrix} \\
  \Delta H &= H_{12} - H_{22} - H_{21}
\end{align*}
\]  

(14)

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  \Delta H &= H_{12} - H_{22} - H_{21}
\end{align*}
\]  

(14)

We detect, on this reduced and causal bond graph model, by going through the causal ways, a single causal loop \( B_1 \) where the associated integro-differential operator is Eq. 15:

\[
B_1 = \frac{1}{z \cdot y}
\]  

(15)
The integro-differential operator associated with the
determiner of the reduced bond graph model is Eq. 16:

\[ \Delta = \frac{1}{z \cdot y} \]  

(16)

The operator associated with the causal way
connecting the reduced variable \( \varepsilon_1 \) with the \( P_1 \) port to
the variable \( \phi_1 \) with the same port is Eq. 17:

\[ H_{11}(s) = \frac{y}{z \cdot y - 1} \]  

(17)

The operator associated with the causal way
connecting the reduced variable \( \phi_2 \) with the \( P_2 \) port to
the variable \( \varepsilon_1 \) with the \( P_1 \) port is Eq. 18:

\[ H_{12}(s) = \frac{1}{z \cdot y - 1} \]  

(18)

The operator associated with the causal way
connecting the reduced variable \( \varepsilon_2 \) with the \( P_1 \) port to
the variable \( \phi_2 \) with the \( P_2 \) port is Eq. 19:

\[ H_{22}(s) = \frac{-z}{z \cdot y - 1} \]  

(19)

While referring to Eq. 14, the scattering parameters
of the \( S \) matrix according to the linear reduced
impedance and admittance are such as Eq. 21:

\[
S_{11} = \frac{z \cdot y - y + z}{z \cdot y + y + z - 2} \\
S_{12} = \frac{2}{z \cdot y + y + z - 2} \\
S_{21} = \frac{2}{z \cdot y + y + z - 2} \\
S_{22} = \frac{-z \cdot y - y + z}{z \cdot y + y + z - 2}
\]  

(21)

By replacing the linear impedance and admittance
by their Eq. 10, we can rewrite the scattering parameters in the following form Eq. 22:

\[
S_{11} = \frac{\tau_c \tau_r \cdot s^2 + \left[ \frac{\tau_c (1+g)}{\tau_r (r-1)} \right] \cdot s + r (g+1) - g}{\tau_c \tau_r \cdot s^2 + \left[ \frac{\tau_c (1+r)}{\tau_r (1+g)} \right] \cdot s + r (1+g) + g - 2} \\
S_{12} = \frac{2}{\tau_c \tau_r \cdot s^2 + \left[ \frac{\tau_c (1+r)}{\tau_r (1+g)} \right] \cdot s + r (1+g) + g - 2} \\
S_{21} = \frac{2}{\tau_c \tau_r \cdot s^2 + \left[ \frac{\tau_c (1-g)}{\tau_r (1+r)} \right] \cdot s + r (g-1) - g} \\
S_{22} = \frac{\tau_c \tau_r \cdot s^2 + \left[ \frac{\tau_c (1+r)}{\tau_r (1+g)} \right] \cdot s + r (1+g) + g - 2}{\tau_c \tau_r \cdot s^2 + \left[ \frac{\tau_c (1+g)}{\tau_r (r-1)} \right] \cdot s + r (g+1) - g}
\]  

(22)

We notice that the scattering parameters found by
the two methods present some differences with regard
to the sign of the numerator or the denominator
parameters, which is due to the causality assignment
and the change number of the orientation while
following the variables efforts and flow through the
information bonds of the reduced and causal bond
graph model above.

**Discussion and comment:** The causality assignment on
a bond graph model does not depend solely on the type
of elements but also on the total structure of junction. In
fact, the causality is more informative than the concept of
impedance and admittance (Birkett, 2009) which loses of
its interest on a bond graph model replaced by the
concept of elementary transmittance obtained starting
from the profits of the ways and the causal loops.

Moreover, with regard to the extraction of the
scattering parameters method by the analytical
exploitation procedure describes in our previously
works (Taghouti and Mami, 2010a), where we
presented the four relations related to the various types
of causality, making it possible to determine the \( S \)
matrix. The problem arises when one deals with
situation where causality on the bond of entry, the bond
of exit or even on the two bonds is not single. In this
case, the choice of the type of causality determines
which relations to be used (Taghouti and Mami, 2009;
2010a; 2010b).

However, it would be more judicious, to facilitate
and reduce calculations, to choose the case of the
obligatory causality imposed by the inductive and
capacitive elements constituting the studied system and
if is not the case, choosing us causality making revealing a minimum number of ways and causal loops as in the case of the example of the elementary line variation of the previously Fig. 6.

Otherwise, in the method of the acausal bond graph which allows us, after a hierarchical reorganization, to obtain an impedance series and a parallel admittance in cascades about it (Redfield and Krishnan, 1993) and by looking at the Ohm’s law given by the Eq. 23 above for the calculation of impedance, we note that a causality entering effort (or outgoing flow) was implicitly taken into account resulting in a derived causality for elements I and one integral causality for the elements CL:

\[
\begin{align*}
U &= Z \cdot I \\
U &= R \cdot I \\
U &= \tau_c \cdot s \cdot I \\
U &= \frac{1}{\tau_c \cdot s} \cdot I
\end{align*}
\]

Reciprocally, in the calculation of admittance, in fact the “I” elements are in integral causality whereas the “C” elements are in derived causality. This being well cavity, independent from the effective causality which would have the bond graph model in integral causality.

**MATERIALS AND METHODS**

**Realization procedure of the scattering bond graph:**

We propose, during this article, a new type of relation enters the scattering formalism and the bond graph approach while combining both procedures described during our research works (Taghouti and Mami, 2010b; Taghouti and Mami, 2009; ), by leaving from a reduced and causal bond graph model (Amara and Scavarda, 1991) to reach a particular type of bond graph model called “Scattering bond graph” who makes to appear explicitly the various of power waves (Kamel and Dauphin-Tanguy, 1996; Kamel et al., 1993).

Contrary to the scattering formalism often used in problems of waves distribution (optics, hyper-frequency) (Wake, 1998), a bond graph model is an unified representation by numerous domains of the physics and like the scattering bond graph is associated with a classic bond graph model, we tried, in this article, to preserve our studies in the frequency field while choosing to work like beginning, on high-pass filters with localized or distributed elements (Taghouti and Mami, 2009; 2010a; 2010b) considering the realization of this type of bond graph amounts changing field of study while passing from the frequency field to the temporal field.

**Scattering bond graph of a quadripole:** The most general form of the scattering matrix “S” of a quadripole is given by the Eq. 24 and 25 below:

\[
S = \frac{1}{d(s)} \begin{bmatrix}
    b_1^1 s^n + \cdots + b_0^1 \\
    b_1^2 s^n + \cdots + b_0^2 \\
    \vdots \\
    b_1^n s^n + \cdots + b_0^n
\end{bmatrix}
\]

Where:

\[
d(s) = a_n s^n + a_{n-1} s^{n-1} + \cdots + a_0
\]

The S matrix presented above is a 2-2-matrix having a particular form whatever the complexity of the expressions of the equivalent series impedance or parallel admittance if we work by an acausal bond graph model (Kamel and Dauphin-Tanguy, 1996; Kamel et al., 1993).

Indeed, if the system does not have any active source, then the quadripole is known as reciprocal, moreover, if the system is supposed without loss the S matrix is orthogonal.

The number of dynamic components present in the studied physical system is given by the “n” degree of d(s) denominator. Indeed, “n” will indicate the number of I and C elements in integral causality if one started from a bond graph model, whereas the elements which are in derived causality do not play a part in the dynamics of the system.

**RESULTS**

Now let us consider the equivalent circuit of a band-pass filter based on localized elements like the Fig. 7 shows it below.

The conventional and causal bond graph model of this filter intercalated between the two input-output ports P₁ and P₂ is given by the Fig. 8.

We know that in such circuit the two parallel and series inductive and capacitive elements, like the Fig. 8 indicates it above, can be replaced by an impedance series and a parallel admittance without modifying the dynamic behavior of the system. To apply the analytical exploitation procedure to the S scattering matrix, we simplify the model above in a transformed, causal and reduced bond graph model as indicates it the following Eq. 26 Fig. 9.

Where:

\[
\begin{align*}
    z &= \frac{1}{\tau_c s^8} + \tau_i s^8 \\
    y &= \tau_c s^8 + \frac{1}{\tau_l s^8}
\end{align*}
\]
The parameters of the scattering matrix will be thus in the following forms:

\[
\begin{align*}
S_{11} &= \frac{t_1 t_2 c_1 c_2 c_3^2 + c_1 t_1 t_2 - c_2}{c_2 c_3 t_1 c_1^2 - t_1 t_2 - c_3}, \\
S_{12} &= \frac{2 c_1 t_1^2}{c_1 c_2^2 + c_1 t_1 t_2 - c_2}, \\
S_{21} &= \frac{c_1 t_1 c_2^2 + c_1 t_2 - c_1}{c_1 c_2^2 + c_1 t_1 t_2 - c_2}, \\
S_{22} &= \frac{2 c_1 t_2^2}{c_1 c_2^2 + c_1 t_1 t_2 - c_2}.
\end{align*}
\]

\[S_{ij} = \frac{1}{C_2} \left( C_3 C_1^2 S_{11} + C_1 C_2^2 S_{12}^2 - C_1 C_2 S_{21} S_{22} \right) + \frac{1}{C_1} \left( C_3 C_2^2 S_{12} - C_1 C_2 S_{21} \right)
\]

\[S = S' + M_D
\]

\[M_{D} = \text{Represent the direct transmission matrix.}
\]

\[\text{The terms of the constant matrix } M_D \text{ are a function of the respective coefficients of the numerators and the common denominator thus represent the quotients of Euclidian division of the each term of the scattering matrix by its common denominator (Kamel and Dauphin-Tanguy, 1996; Kamel et al., 1993).}
\]

\[M_n = \begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{bmatrix}
\]

\[\text{The second stage consists in seeking for the new scattering matrix “S” the development in alpha-beta and building the corresponding bond graph model by using the procedure of realization of a bond graph model in the multivariable case like we explained in the previously papers.}
\]

\[\text{The direct part comes to be grafted on this bond graph using a suitable number of bonds from information connecting the entries to the various exits.}
\]

\[\text{We represent the scattering bond graph model in the case of a direct diagonal matrix by the following Fig. 10.}
\]
Fig. 10: The scattering bond graph model of the quadripole

This particular type of bond graph, consists of two identical direct chains (the S matrix of any quadripole is square and has dimension two) modeling the dynamic part (Kamel and Dauphin-Tanguy, 1996) related to the common denominator $d(s)$ and having for entries two effort sources representing the incidental waves $w_{r_1}$ et $w_{r_2}$.

The total structure of the bond graph model remains same whatever the degree of the common denominator. Only the number of I and C elements, related to the number of $\alpha_i$ and thus with the degree of $d(s)$, changes. This being, obviously, in accord with the preceding remark on n degree of $d(s)$.

The variables of exit, representing the reflected waves $w_{r_1}$ et $w_{r_2}$, are obtained using detectors judiciously placed to collect information on the level of the adequate port.

CONCLUSION

In this study, we reminded and explaining briefly, with some improvements, a procedure which uses collectively and in a clarify way the scattering formalism and the bond graph approach for the modelling of a physical system by bringing to light the power waves and their distributions on a particular type of graph bond often called “Scattering Bond Graph”.

The procedure described in this study, has allows us to have a temporal representation with dynamic elements which can have a physical interpretation (performance) and a better analyze of energy phenomena.

This procedure gave us, by means of a graphic representation, an access to the various power waves, contrary to the scattering matrix which remains a formal model difficult to interpret.

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REFERENCES


