System Availability in the Presence of Estimating Common-Cause Time-Varying Failure Rates

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Abstract: This study presents a method for calculating the availability of a system depicted by availability block diagram, with identically distributed components, in the presence of estimating common cause hazard, we use the Marshall and Olkin formulation of the multivariate exponential distribution. That is, the components are subject to failure by Poisson failure processes that govern simultaneous failure of a specific subset of the components. A model is proposed for the analysis of systems subject to common-cause failures that are not considered to have a constant rate but that are assumed to obey a uniqueness of maximum likelihood estimators of the 2-parameter Weibull distribution. The method for calculating the system availability requires that a procedure exists for determining the system availability from component availabilities, under the statistically independent component assumption. The study includes an example to illustrate the method.

Key words: System Availability, Component Availabilities, Common-cause Failures

INTRODUCTION

Common-Cause (CC) hazards are the failure of multiple components due to a single occurrence or condition. For example, contaminate fluid causes two pumps to fail that are operated in “parallel”. In this event, the availability of the “parallel” configuration with redundancy is less than a similar configuration with statistically independent components.

In this study, CC hazard is simultaneous hazards of multiple components due to a CC.

Most CC failure models assume that the shocks have constant (time-independent) rates of occurrence, leading to variants of the multivariate exponential distribution\(^1\)-\(^4\).

Many parameterization has been developed\(^5\), yet most of them are equivalent to (or special cases of) the general multivariate exponential model\(^6\).

There are two fundamentally different approaches for incorporating CC failure into system analysis: explicit and implicit method\(^7,\ 8\).

The component failure probability density function could be described by different models, such as the Weibull distribution calculated from either complete failure data or from the behavior of the parameter Maximum Likelihood Estimates (MLE) of a 2-parameter Weibull distribution\(^9\).

It is the main objective of the present study to utilize the hazard rates, extracted from operational experience, to calculate the availability of a system depicted by an availability block diagram with Weibull distribution components, in the presence of common-caused hazards. Availability formulae for a configuration of a definite number of components are provided.

Uniqueness of MLE of the 2-parameter Weibull Distribution: We select an appropriate hazard rate for each constituent component in the system and evaluate its characteristic parameters. The function for the reliability of each component can then be easily derived.

The hazard function of a component following a 2-parameter Weibull distribution can be described by:

\[
h_j(t) = \frac{\beta_j}{\alpha_j} t^{\beta_j - 1}, \quad j = 1, 2, \ldots, n
\]

The likelihood function is:

\[
L = \prod_{i=1}^{m} f(t_i) \prod_{i=1}^{m} R(t_i)
\]

\[
= \left\{ \prod_{i=1}^{m} \frac{\beta_j}{\alpha_j} t_i^{\beta_j - 1} \exp\left[-\left(\frac{t_i}{\alpha_j}\right)^{\beta_j}\right] \right\}
\]

\[
\left\{ \prod_{i=1}^{m} \exp\left[-\left(\frac{t_i}{\alpha_j}\right)^{\beta_j}\right] \right\}
\]

The partial derivatives of the natural log of the likelihood function are:

\[
\beta_j = 1/[\sum_{i=1}^{m} \left(\frac{t_i^{\beta_j}}{\alpha_j}\right) - \sum_{i=1}^{m} \frac{\ln(t_i)}{\tau}] (3.1)
\]

\[
\alpha_j = \left[\sum_{i=1}^{m} \left(\frac{t_i^{\beta_j}}{\tau}\right)\right]^{1/\beta_j} (3.2)
\]
Since (3.1) involves $\beta_j$ alone, iterative methods are usually directed at solving (3.1); and substituting the resulting value into (3.2) to find $\alpha_j$.

For censorting, $t_i$ is a recorded failure time for $i \leq \tau$ and $t_i = \tau$ for $1 \leq i \leq m$. When all $t_i$ ($1, 2, \ldots, m$) are available, the data are complete; complete data are a special case of right censoring for $\tau = m$.

Our empirical investigations suggest that choosing

$$\hat{\beta}_j = \left( \frac{V + (V - (\tau/m)V)}{2} \right)^{-1}$$

Where:

$$V = \lim_{\hat{\beta} \to \infty} \left[ \sum_{i=1}^{m} \hat{t}_j \ln(t_j) - \sum_{i=1}^{m} \frac{\ln(t_j)}{\tau} \right]$$

$$= \frac{\sum_{i=1}^{m} \ln(t_j)}{\tau}$$

works well. For complete data, this approximation simplifies to $\hat{\beta} = 2/V$; (4) provides a quick approximation to $\hat{\beta}$ and can be used as an initial estimate of $\hat{\beta}$ for iterative MLE routines.

Component Availability Model: Figure 1 is the state transition diagram for the 1-component availability model:

States 1. $x_1$ 2. $\overline{x}_1$

The general relation to the state probabilities as a function of time is:

$$P_j(t) = \begin{bmatrix} P_1(t) \\ P_2(t) \end{bmatrix} = \begin{bmatrix} -h(t) & \mu \\ h(t) & -\mu \end{bmatrix} \begin{bmatrix} P_1(t) \\ P_2(t) \end{bmatrix}$$

The first equation of (6), after much tedious algebra, the result is:

$$P_j(t) = \exp\left[ -\int_0^t (h(t) + \mu) dt \right]$$

$$[1 + \mu \int_0^t \exp[-\int_0^t (h(t) + \mu) dt]]$$

In general, for the given:

$$h(t) = h_j(t) = \frac{\beta_j}{\alpha_j} t^{\beta_j-1}, \text{ and } \mu = \mu_j,$$

$$A_j(t) = P_j(t) (j = 1, 2, \ldots, n)$$

A system availability analysis with common-cause hazards: $A^n_j(t)$ is the probability that the specified component is operating at time $t$, i.e. The probability that none of the processes that govern the simultaneous failure of $j$ component, $j=1, 2, \ldots, n$, includes the specific component. Based on the $S$-independence of the Poisson processes, we have:

$$A^n_j(t) = \prod_{i=1}^{n}(A_i(t))^{(1)}$$

(9)

The probability that a specific group of $k$ components out of $n$-component system are all good is:

$$A^{(k)}_n(t) = \prod_{i=1}^{n}(A_i(t))^{(1)}$$

(10)

These formulas were originally derived from Kyung[10] for constant hazard rates; similar arguments are valid for time-varying failure rates[11].

The results are $A_c(t)$ and $A_s(t)$ in terms of availabilities $A_i(t)$.

Illustrative Example: Let the given Fig. 2 is the availability block-diagram.

Fig. 1: Component Availability State-transition Diagram

Fig. 2: Availability Block-Diagram for Example
Table 1: Compute Estimate of the Parameters $\beta_j$ and $\alpha_j$, and Assuming Repair Rates $\mu_j$ for Number of Simultaneous Failures

<table>
<thead>
<tr>
<th>Number of Simultaneous Failures</th>
<th>Ordered Failure Time $t_{ij}$</th>
<th>$V_j$</th>
<th>$\beta_j 2 / V_j$</th>
<th>$\alpha_j \left( \sum_{i=1}^{10} t_{ij}^2 / 10 \right)^{10}$</th>
<th>$\mu_j$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$j=1,2,\ldots,5$ and $i=1,2,\ldots,10$</td>
<td>$1$</td>
<td>37.58 72.88 115.136 152.165 185.213</td>
<td>0.6819</td>
<td>2.42</td>
<td>138.07</td>
</tr>
<tr>
<td></td>
<td>$2$</td>
<td>31.43 36.65 73.82 96.101 111.195</td>
<td>0.948</td>
<td>2.12</td>
<td>97.220</td>
</tr>
<tr>
<td></td>
<td>$3$</td>
<td>27.35 66.83 96.101 131.145 199.222</td>
<td>0.884</td>
<td>2.26</td>
<td>128.41</td>
</tr>
<tr>
<td></td>
<td>$4$</td>
<td>24.32 41.66 79.89 98.120 180.235</td>
<td>1.117</td>
<td>1.79</td>
<td>111.66</td>
</tr>
<tr>
<td></td>
<td>$5$</td>
<td>18.26 39.53 77.93 108.135 220.253</td>
<td>1.216</td>
<td>1.64</td>
<td>118.84</td>
</tr>
</tbody>
</table>

For identically distributed components with statistically-independent failure processes, the availability $A_S(t)$ of the whole system can then be evaluated as:

$$A_S(t) = \prod_{j=1}^{5} [A_j(t)]^{1/5}$$

(11)

Where:

$$A_j(t) = A_j^{(1)}(t) = \prod_{i=1}^{k} A_i^{(1)}(t), \ k = 2,3,4,5$$

(14)

Making use of the data provided and assumed Table 1, the available functions of Eqs. (11) and (12) in terms of $A_j^{(1)}(t)$, and $A_S^{(k)}(k=2,3,4,5)$ respectively vary with time as shown in Fig. 3 (a) and 3 (b). Thus, for this case, the system availability, assuming common-cause, failures, is appreciably lower than the i.i.d system availability.

**Notation:**

- $n$ = Number of components in the system;
- $k$ = Number of good components that allow the system to operate;
- $A_i(t)$ = Availability of component $j$ at time $t$;
- $A_c(t)$ = System availability at time $t$ with CC hazard;
- $A_s(t)$ = System availability without CC hazard;
- $A_n^{(k)}(t)$ = Probability that all the components of a specific k-component subset out of an n-component system are operating at time t;
- $h_j(t)$ = Hazard rate; $h_j(t)dt$ = conditional probability of an event failing specific j components, and no others, during $(t,t+dt)$, given no such event during $(0,t)$;
- $h_j(t) = \int_0^t h_j(u)du$ : cumulative hazard function;
\binom{n}{j} = \text{Number of combinations of } j \text{ items out of possible } n \text{ items.}

\alpha_j, \beta_j = \text{Positive [scale, shape] parameter of component } j

m = \text{Number of items tested;}

\tau = \text{Failure time of item } i \text{ under test;}

R(t_i) = \text{Reliability of a single component at time } t_i;

F(t_i) = \text{Probability density function of time } t_i;

t_s = \text{Maximum test time for censoring;}

\pi_j = \text{Number of items that fail before } t_i;

\mu_j = \text{Constant repair rate for component } j;

S_i = \text{Event that component } i \text{ is good.}

REFERENCES


