

Accurate Orthogonal Circular Moment Invariants of Gray-Level Images

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Abstract: Problem statement: Orthogonal circular moments of gray level images such as Zernike, pseudo Zernike and Fourier-Mellin moments are widely used in different applications of image processing, pattern recognition and computer vision. Computational processes of these moments and their translation and scale invariants still an open area of research. **Approach:** a unified methodology is presented for efficient and accurate computation of orthogonal circular moment invariants. The orthogonal circular moments and their translation and scale invariants are expressed as a linear combination of radial moments of the same order in polar coordinates, where the later moments are accurately computed over a unit disk. A new mapping method is proposed where the unit disk is divided into non-overlapped circular rings; each of these circular rings is divided into a number of circular sectors of the same area. Each circular sector is represented by one point in its centre. The total number of input Cartesian image pixels is equal to the number of mapped circular pixels. **Results:** The implementation of this method completely removes both approximation and geometrical errors produced by the conventional methods. Numerical experiments are conducted to prove the validity and efficiency of the proposed method. **Conclusion:** A unified methodology is presented for efficient and accurate computation of orthogonal circular moment invariants.

Key words: Radial moments, Zernike moments, pseudo Zernike moments, Fourier-Mellin moments, translation invariants, scaling invariants, accurate computation

INTRODUCTION

Orthogonal circular moments of gray level images such as Zernike, pseudo Zenike and Fourier-Mellin are used to represent images with minimum amount of information redundancy. Based on this attractive property, these orthogonal moments are widely used in different image processing and pattern recognition applications. See for examples (Abandah and Anssari, 2009, Radhika *et al.*, 2010, Ismail *et al.*, 2010). Despite of this fact, the conventional approach for computing these circular moments and their invariants encounter two major problems. These problems are the lack of accuracy and the very time-consuming computational processes.

Different methods were proposed to overcome these problems for each kind of these moments. Remarkable works are presented to efficiently compute Zernike moments (Xin *et al.*, 2007; Hosny, 2008; 2010a; 2010b) and pseudo Zernike moments (Chong *et al.*, 2003a; Al-Rawi, 2010) Another group of works presents efficient computation of orthogonal Fourier-Mellin moments (Papakostas *et al.*, 2007; Hosny *et al.*, 2011; Walia *et al.*, 2011).

The variety of these methods motivates us to propose a unified efficient method for accurate computation of orthogonal circular moments of binary as well as gray level images.

The orthogonal circular moments are rotationally invariant by nature. This attractive property enables these moments to play an essential role in the invariant pattern recognition tasks. Invariance with respect to other geometric transformation such as translation and scaling could be achieved through the image normalization method. The literature of moment computation has two works that are dealing with translation invariants of Zernike moments (Chong *et al.*, 2003b) and the scale invariants of pseudo Zernike moments (Chong *et al.*, 2003c). Unfortunately, no work is available for translation and scaling invariants of Fourier-Mellin moments.

This study proposes a new unified method for efficient and highly accurate computation of orthogonal circular moments for binary and gray-level images.

The proposed method is extended to accurately compute both translation and scale moment invariants. The proposed method consists of two major steps. In the first step, highly accurate radial moments are

computed in polar coordinates. In the second step, all aforementioned circularly moments and their invariants are computed as linear combinations of the radial moments. A number of numerical experiments are conducted where the experimental results clearly show the efficiency of this proposed method.

The rest of the study is organized as follows: An overview of the orthogonal circular moments of images is presented. The proposed method is described in the section of material and methods. Conducted numerical experiments are described and the obtained results are analyzed in the section of results. It followed by a discussion and Conclusion. An acknowledgement of the financial support is presented. Finally, a list of recent references is presented.

Circular moments of images: Orthogonal circular moments of an image are defined over a unit disk in polar coordinates. In this subsection, a brief overview of orthogonal Zernike, pseudo Zernike and Fourier-Mellin moments is presented.

Zernike moments: The complex two-dimensional Zernike moments of order p and repetition q are defined as follow:

$$Z_{p,q} = \frac{p+1}{\pi} \times \int_0^{2\pi} \int_0^1 \overline{V_{p,q}(r,\theta)} f(r,\theta) r dr d\theta \quad (1)$$

where, $p = 0,1,2,3,\dots,\infty$ and q is positive integer selected according to the conditions: $(p-q) = \text{even}$ and $q \leq p$.

Zernike moments with negative values of repetition q are computed using complex conjugate from the relation $Z_{p,-q} = \overline{Z_{p,q}}$. Zernike functions $V_{pq}(r,\theta)$ forms a complete set of complex-valued functions which enable the direct computation of inverse transform.

Pseudo Zernike moments: The complex two-dimensional pseudo Zernike moments of order p and repetition q are defined as follows:

$$A_{p,q} = \frac{p+1}{\pi} \times \int_0^{2\pi} \int_0^1 \overline{W_{p,q}(r,\theta)} f(r,\theta) r dr d\theta \quad (2)$$

where, $p = 0,1,2,3,\dots,\infty$ and q is non-negative integer defined according to the condition $q \leq p$.

Pseudo Zernike moments with negative values of repetition q are computed from those of positive values by using the relation, $A_{p,-q} = \overline{A_{p,q}}$. Pseudo Zernike functions, $W_{pq}(r,\theta)$, are a complete set of complex-valued orthogonal functions.

Fourier-Mellin moments: Orthogonal Fourier-Mellin moments of order p and repetition q are defined as follows:

$$F_{p,q} = \frac{p+1}{\pi} \times \int_0^{2\pi} \int_0^1 Q_p(r) e^{-iq\theta} f(r,\theta) r dr d\theta \quad (3)$$

where, $\hat{i} = \sqrt{-1}$, $p = 0,1,2,3,\dots,\infty$ and q is non-negative integer. Similar to Zernike and pseudo Zernike, Fourier-Mellin moments of negative repetition are obtained directly from that of positive repetition.

Central radial moments: Central radial moments of order p with repetition q are defined as follows (Chong *et al.*, 2003b):

$$\hat{R}_{p,q} = \sum_{m=0}^u \sum_{n=0}^v \binom{u}{m} \binom{v}{n} (-A)^m (-\bar{A})^n R_{\alpha,\beta} \quad (4)$$

Where:

$$\begin{aligned} u &= (p+q)/2, v = (p-q)/2 \\ \alpha &= (u-m) + (v-n) \\ \beta &= (u-m) - (v-n) \\ A &= x_0 - \hat{i}y_0, \bar{A} = x_0 + \hat{i}y_0 \end{aligned} \quad (5)$$

The centroid of the input image is defined as follows:

$$\begin{aligned} x_0 &= \frac{\sum_{i=1}^N \sum_{j=1}^N x_i f(x_i, y_j)}{\sum_{i=1}^N \sum_{j=1}^N f(x_i, y_j)} \\ y_0 &= \frac{\sum_{i=1}^N \sum_{j=1}^N y_j f(x_i, y_j)}{\sum_{i=1}^N \sum_{j=1}^N f(x_i, y_j)} \end{aligned} \quad (6)$$

Scale invariants of radial moments: Scale invariants of radial moments of order p with repetition q are defined as follows Chong *et al.*, 2003c):

$$\hat{R}_{p,q} = R_{p,q} / (R_{0,0})^{\frac{p+1}{2}} \quad (7)$$

MATERIALS AND METHODS

Orthogonal circular moments are defined in polar coordinates over a unit disk. Zernike, pseudo Zernike and Fourier-Mellin moments and their translation and scale invariants could be represented as a linear combination of radial moments of the same order or less. A generic formula of the two-dimensional circular moments and their invariants of order p with repetition q is defined as follows:

$$C_{p,q} = \frac{p+1}{\pi} \sum_{k=T}^p B_{p,q,k} R_{k,q} \quad (8)$$

where, p = 0,1,2,3,...∞ and q is positive integer. The different values of the parameter T and the coefficient matrix B_{pqk} of the aforementioned circular moments are showed in Table 1.

Highly accurate radial moments: It is clear that, the computational process of radial moments, R_{kq}, is the core part in the whole computational process of these orthogonal circular moments and their invariants. Radial moments of order k with repetition q are defined in polar coordinates over a unit disk as follows:

$$R_{k,q} = \int_0^{2\pi} \int_0^1 r^k e^{-iq\theta} f(r, \theta) r dr d\theta \quad (9)$$

where, $\hat{i} = \sqrt{-1}$, k=0,1,2,3,...∞ and q is any positive or negative integer. For input image of size N×N as depicted in Fig. 1, a new image mapping is proposed where all of the required computational processes are done in the polar coordinates. This new mapping method is a modification of the method described by Hosny *et al.* (2011) where the total number of input image pixels is equal to the total number of mapped circular sector pixels as depicted in Fig. 2. The new mapping method is very simple and could be achieved by applying the following steps:

- The unit disk is divided into, N/2, concentric, non-overlapped circular rings
- Each circular ring is divided into, (8i+4), circular sectors where i = 0,1,2,...,N/2-1 and i=0 refers to the innermost circular ring
- The different values of the angle θ could be created by using the following pseudo-code:

```

for i = 0 to N/2-1
  for j = 0 to (8i+4)-1
    θi,j = 2π(j+0.5)/(8i+4)
  endfor
endfor
    
```

Table 1: Circular moments and their coefficients

	Coefficient matrix, B _{pqk}	Parameter T
Zernike (Hosny, 2008)	$(-1)^{\binom{p-k}{2}} \binom{p+k}{2}!$ $\binom{p-k}{2}! \binom{k+q}{2}! \binom{k-q}{2}!$	T = q && (p-q) is even
Pseudo Zernike (Al-Rawi, 2010)	$(-1)^{(p-k)} (p+k+1)!$ $(p-k)!(k+q+1)!(k-q)!$	T = q
Fourier-Mellin (Papakostas <i>et al.</i> , 2007)	$(-1)^{(p+k)} (p+k+1)!$ $(p-k)!k!(k+1)!$	T = 0

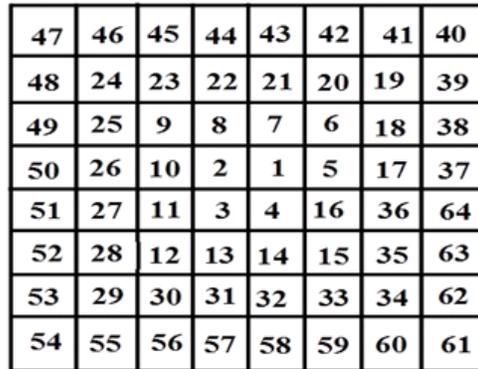


Fig. 1: Original square image pixels

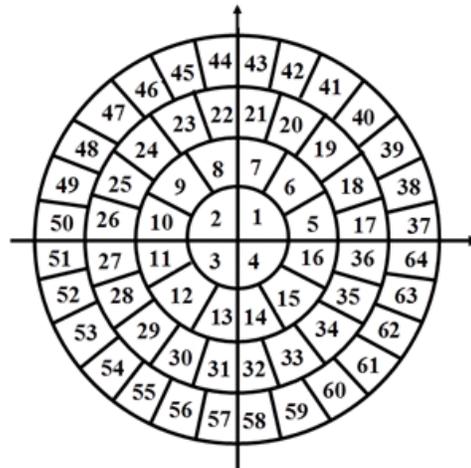


Fig. 2: Mapped circular image pixels

The radial moments of order k and repetition q are:

$$R_{k,q} = \sum_i \sum_j f(r_i, \theta_{i,j}) H_{p,q}(r_i, \theta_{i,j}) \quad (10)$$

With:

$$H_{k,q}(r_i, \theta_{i,j}) = I_k(r_i) I_q(\theta_{i,j}) \quad (11)$$

Where:

$$I_k(r_i) = \int_{U_i}^{U_{i+1}} r^k r dr \quad (12)$$

$$I_q(\theta_{i,j}) = \int_{V_{i,j}}^{V_{i,j+1}} e^{-iq\theta} d\theta \quad (13)$$

Both integrals in Eq. 12 and 13 are evaluated analytically without any kind of approximation as follows:

$$I_k(r_i) = \left(\frac{U_{i+1}^{k+2} - U_i^{k+2}}{p+2} \right) \quad (14)$$

$$I_q(\theta_{i,j}) = \begin{cases} \frac{\hat{i}}{q} (e^{-iqV_{i,j+1}} - e^{-iqV_{i,j}}), & q \neq 0 \\ V_{i,j+1} - V_{i,j}, & q = 0 \end{cases} \quad (15)$$

The vectors of upper and lower limits could be defined as follows:

$$U_{i+1} = \frac{(2i+1)}{N} \quad (16a)$$

$$U_i = \frac{2i}{N} \quad (16b)$$

$$V_{i,j+1} = \theta_{i,j} + \frac{\pi}{(8i+4)} \quad (17a)$$

$$V_{i,j} = \theta_{i,j} - \frac{\pi}{(8i+4)} \quad (17b)$$

It is clear that, the kernels, $I_k(r_i)$ and $I_q(\theta_{i,j})$ are image-independent. Therefore, both kernels could be pre-computed and stored for any future use.

For efficient computation of highly accurate radial moments, one-dimensional cascade is implemented for fast computation and two types of symmetry properties are applied for memory reduction. Both of these approaches were successfully employed in the theme of

moment computation.

Equation 10 will be rewritten in a separable form as follows:

$$R_{k,q} = \sum_i I_k(r_i) Y_{iq} \quad (18)$$

$$Y_{iq} = \sum_j I_q(\theta_{i,j}) f(r_i, \theta_{i,j}) \quad (19)$$

It must be noted that, an 8-way symmetry property could be applied to achieved 87% of memory saving. For detailed description of this symmetry, the readers are referred to the study (Hosny *et al.*, 2011)

RESULTS

A number of numerical experiments are conducted to prove the validity and the efficiency of the proposed method. Two standard gray level images as displayed in Fig. 3 are used where the full set of Zernike and orthogonal Fourier-Mellin are computed by using the proposed method and the traditional approximated ZOA methods. Mean Square Error (MSE) is used in the evaluation process. The computed sets of moments are used to reconstruct the input image and then, the MSE are computed. The values of the MSE are plotted.

Two numerical experiments are conducted. In the first experiment, the 'house' gray level image of size 128x128 is used, where the full set of orthogonal Fourier-Mellin moments up to order Max = 20 is computed by using the proposed method and the ZOA method. The MSE of both methods are computed, plotted and depicted in Fig. 4. It is clear that, the MSE of the proposed method decreased as the moment order increased. On the other side, the MSE of the ZOA method decreased for few moment orders and suddenly increased. This is evidence on the accuracy of the proposed method.

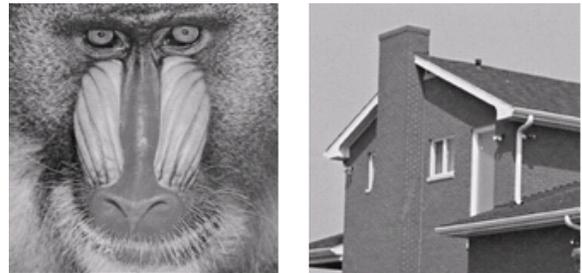


Fig. 3: Standard gray level images

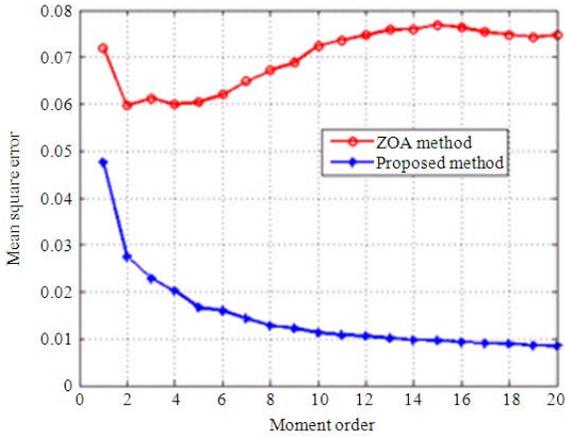


Fig. 4: Mean square errors: Orthogonal Fourier-Mellin moments

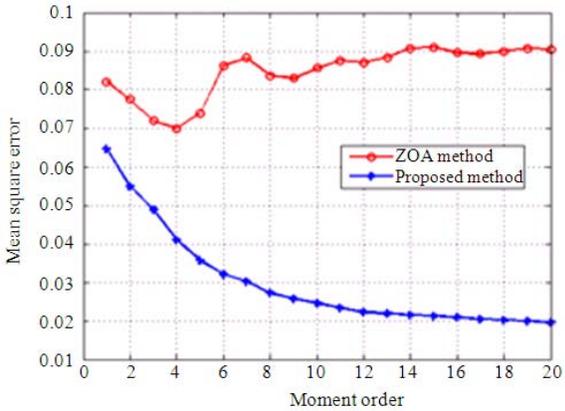


Fig. 5: Mean square errors: Zernike moments

In the second experiment, the full set of Zernike moments are computed with the proposed and the ZOA method. The MSE of both methods are computed, plotted and depicted in Fig. 5. As expected, a similar result is reached.

Since pseudo Zernike moments are derived from original Zernike moments and based on the big similarity between their functions, the computational process of pseudo Zernike moments is straightforward.

All computational processes are performed by using a code designed with Matlab8 and operated on a Lenovo R400 Laptop.

Invariance to different kinds of geometric transformations such as rotation, translation and scale will be tested. Invariance of the Zernike, pseudo Zernike and Fourier-Mellin moments are tested using standard gray level images.

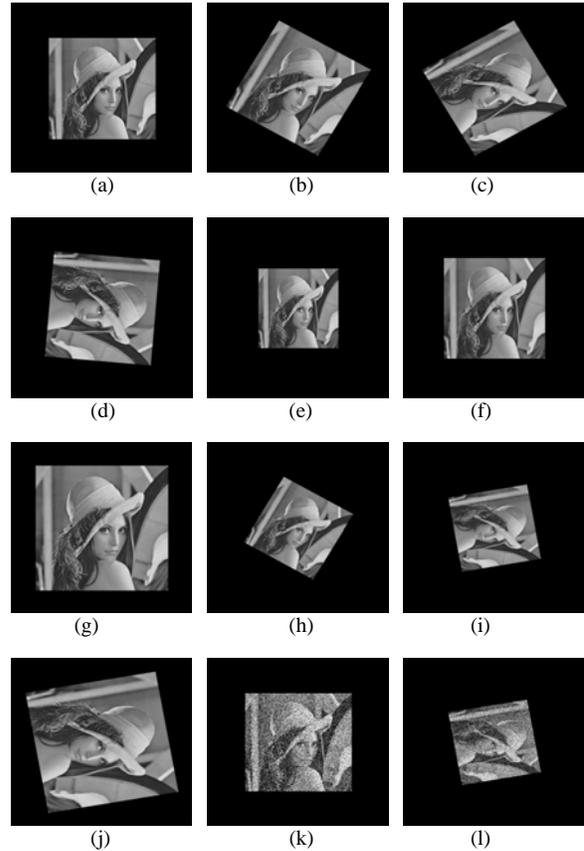


Fig. 6: (a) Original image, (b) Rotation $R = 30^\circ$, (c) $R = 60^\circ$ and (d) $R = 95^\circ$, (e) Scaling $S = 0.75$, (f) $S = 0.95$, (g) $S = 1.25$, (h) $S = 0.75$ and $R = 30^\circ$, (i) $S = 0.75$ and $R = 80^\circ$, (j) $S = 1.25$ and $R = 80^\circ$, (k) Noisy image, (l) $S = 0.75$ and $R = 80^\circ$

These numerical experiments were designed in (Hosny, 2008). The first experiment is concerned with testing invariance against rotation where the original gray level image of Lena is rotated by 30° , 60° and 95° as shown in Fig. 6b-d.

The absolute errors between the low order orthogonal moments of the original image and the rotated ones are showed in Table 2-4 for Zernike, pseudo Zernike and Fourier-Mellin moment invariants respectively. It is clear that, the absolute errors are very small whatever the rotation angle which ensure the accuracy of the proposed unified method.

Invariance against scaling for both image reduction and enlargement is tested. The image of 'Lena' is reduced by the scaling factors 0.75 and 0.95. It is also enlarged by the factor 1.25 as shown in Fig. 6e-f and 7g. The absolute errors of the low order moments for original and the scaled images are showed. As expected, the absolute errors of the proposed moment invariants are very small.

Table 2: Absolute values of low order Zernike moments (First experiment)

Transformation			Zernike moments						
R	S	N	X ₂₀	Z ₂₂	Z ₃₁	Z ₃₃	Z ₄₀	Z ₄₂	Z ₄₄
300	----	0	0.0787	0.0354	0.0348	0.039	0.0417	0.0143	0.0297
600	----	0	0.0784	0.0351	0.0345	0.0392	0.0415	0.0141	0.0296
950	----	0	0.0786	0.0354	0.0347	0.0391	0.0416	0.0140	0.0293
----	0.75	0	0.0787	0.0354	0.0348	0.0390	0.0417	0.0143	0.0297
----	0.95	0	0.0786	0.0354	0.0348	0.0390	0.0418	0.0143	0.0297
----	1.25	0	0.0787	0.0353	0.0347	0.0388	0.0417	0.0143	0.0297
800	0.75	0	0.0787	0.0352	0.0346	0.0390	0.0414	0.0143	0.0296
800	1.25	0	0.0783	0.0350	0.0348	0.0386	0.0417	0.0141	0.0296
800	0.75	1	0.0778	0.0343	0.0344	0.0392	0.0422	0.0138	0.0283

Table 3: Absolute values of low order pseudo Zernike moments (First experiment)

Transformation			Pseudo Zernike moments							
R	S	N	A ₂₀	A ₂₁	A ₂₂	A ₃₁	A ₃₂	A ₃₃	A ₄₂	A ₄₄
300	----	0	0.1827	0.0347	0.0185	0.0648	0.0144	0.0221	0.0251	0.1200
600	----	0	0.1825	0.0345	0.0185	0.0647	0.0144	0.0223	0.0250	0.1201
950	----	0	0.1824	0.0343	0.0184	0.0645	0.0145	0.0224	0.0253	0.1203
----	0.75	0	0.1827	0.0347	0.0185	0.0648	0.0144	0.0221	0.0251	0.1200
----	0.95	0	0.1827	0.0347	0.0185	0.0648	0.0144	0.0221	0.0251	0.1200
----	1.25	0	0.1826	0.0347	0.0185	0.0647	0.0144	0.0222	0.0251	0.1200
800	0.75	0	0.1827	0.0345	0.0185	0.0648	0.0145	0.0221	0.0252	0.1202
800	1.25	0	0.1825	0.0347	0.0183	0.0645	0.0144	0.0223	0.0253	0.1201
800	0.75	1	0.1819	0.0339	0.0181	0.0641	0.0149	0.0216	0.0245	0.1204

Table 4: Absolute values of low order orthogonal Fourier-Mellin moments (First experiment)

Transformation			Orthogonal Fourier-Mellin moments							
R	S	N	F ₂₀	F ₂₁	F ₂₂	F ₃₀	F ₃₁	F ₃₂	F ₃₃	F ₄₂
300	----	0	0.0443	0.0155	0.0101	0.0139	0.0074	0.0112	0.0235	0.0043
600	----	0	0.0442	0.0156	0.0101	0.0138	0.0072	0.0111	0.0234	0.0044
950	----	0	0.0441	0.0154	0.0103	0.0137	0.0075	0.0114	0.0233	0.0043
----	0.75	0	0.0443	0.0155	0.0101	0.0139	0.0074	0.0112	0.0235	0.0043
----	0.95	0	0.0443	0.0155	0.0101	0.0139	0.0074	0.0111	0.0233	0.0043
----	1.25	0	0.0444	0.0154	0.0102	0.0137	0.0073	0.0112	0.0235	0.0042
800	0.75	0	0.0441	0.0155	0.0101	0.0138	0.0075	0.0112	0.0232	0.0043
800	1.25	0	0.0441	0.0153	0.0101	0.0139	0.0074	0.0112	0.0235	0.0042
800	0.75	1	0.0438	0.0150	0.0094	0.0132	0.0064	0.0107	0.0239	0.0035

To test robustness against noise, a white Gaussian noise is added to the image of Lena according to the MATLAB statement; A= imnoise (A, 'Gaussian', 0, 0.05). The noisy image (Fig. 6k) is scaled with the factor 0.75 and then rotated with the angle 80° (as displayed in Fig. 6l).

Another group of numerical experiments are conducted where the image of recycling logo is used. These numerical experiments were designed in (Hosny, 2008) where the input test image is 3-fold rotation symmetry. The gray level image of recycle logo with size 128×128 (Fig. 7a) is rotated by 30°, 60° and 95° as shown in Fig. 7b-d. As in the previous numerical experiments, the absolute errors of low order orthogonal moments are computed and shown in Table 5.

Invariance against a combination of scaling and rotation is tested where the test image is reduced by the percentage of 75% and then rotated through the angle

80°. In another experiment, the test image is enlarged using the percentage 125% and then rotated through the angle 80° as shown in Fig. 7e and f.

Robustness against the white Gaussian noise is tested. The noisy image (Fig. 7g) is scaled with the factor 0.75 and then the noisy image is rotated by an angle 30° (Fig. 7h). The absolute errors are showed in Table 5. The acronyms 'R' and 'S' refer to rotation and scaling respectively. The values '0' means noise-free while '1' means noisy.

The conducted experiments clearly show the accuracy of the proposed unified method. All aspects of accuracy evaluation such as image reconstruction, invariance with respect to geometric transformations (rotation and scaling) and robustness against noise are considered. The proposed unified method is proved to be very effective in case of symmetric images.

Table 5: Absolute values of low order Zernike moments (Second experiment)

Transformation			Zernike moments						
R	S	N	Z ₂₀	Z ₂₂	Z ₃₁	Z ₃₃	Z ₄₀	Z ₄₂	Z ₄₄
300	----	0	0.0839	0.1382	0.0394	0.0844	0.5987	0.0336	0.0108
600	----	0	0.0838	0.1381	0.0392	0.0842	0.5985	0.0335	0.0106
950	----	0	0.0835	0.1385	0.0390	0.0841	0.5986	0.0337	0.0102
800	0.75	0	0.0832	0.1381	0.0391	0.0842	0.5986	0.0334	0.0107
800	1.25	0	0.0832	0.1381	0.0391	0.0842	0.5986	0.0334	0.0107
300	0.75	1	0.0839	0.1382	0.0393	0.0844	0.5987	0.0336	0.0108

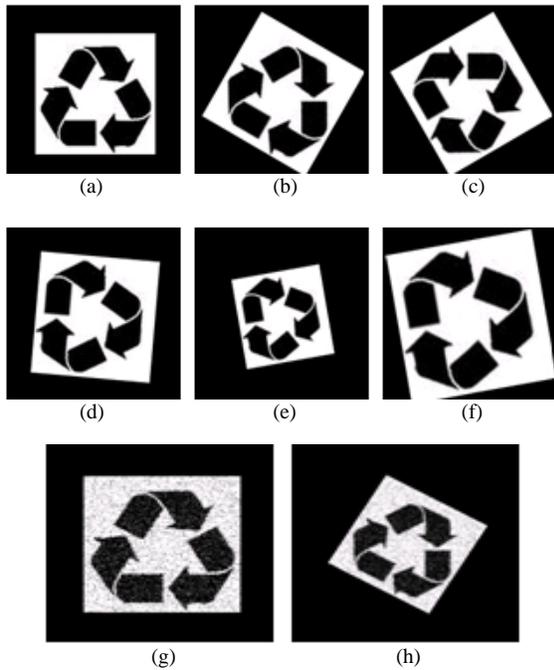


Fig. 7: (a) recycling symbol's image, (b) Rotation R = 30°, (c) R = 60°, (d) R = 95°, (e) S = 0.75 and R = 80°, (f) S = 1.25 and R = 80°, (g) Noisy image, (h) Noise and S = 0.75 and R = 30°

DISCUSSION

The conducted experiments clearly show the accuracy of the proposed unified method. All aspects of accuracy evaluation such as image reconstruction, invariance with respect to geometric transformations (rotation and scaling) and robustness against noise are considered. The proposed unified method is proved to be very effective in case of symmetric images.

CONCLUSION

This study proposes a unified method for accurate computation of circular moments for gray-scale images. A new image mapping approach is proposed. In this approach, the total number of the Cartesian pixels of the

input digital image is equal to the corresponding circular image pixels in polar coordinates. Radial moments are accurately computed by using the proposed mapping scheme and then; Zernike, pseudo Zernike and Fourier-Mellin moments are expressed as a linear combination of radial moments. The numerical experiments are performed for real images with different sizes to ensure the efficiency of the proposed method.

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