

Inventory Models with Power Demand and Inventory-Induced Demand with Holding Cost Functions

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Abstract: In real life customers are motivated to buy more if there is more availability of product. In this study, we develop inventory model for inventory dependent demand with different holding cost function. Mathematical formulations are discussed for two situations i.e., power demand rate with inventory dependent holding cost functions and inventory-induced demand rate with time and inventory dependent holding cost functions. The second order approximation is applied for exponential terms to find numerical values. Numerical examples are discussed to illustrate the models proposed in this study. The sensitivity analysis is given for optimal (minimum) solution with respect to key parameters is also discussed.

Keywords: Variable Holding Cost, Stock, Power Demand, Optimization, Time-Dependent

Introduction

In traditional Economic Order Quantity (EOQ) model, demand rate is considered as steady in nature. However, in reality the demand for physical products may be inventory, selling price or power-level dependent. The consumption rate for certain types of items, fluctuate with the on-hand inventory-level. Levin *et al.* (1972) discussed that large amount of products showed in a large shop will cause the buyer to purchase larger quantity. Silver and Peterson (1985), observed that “sales at the retail level tend to be proportional to stock displayed and a large amount of items displayed in a large shop will cause customers to purchase more”. Gupta and Vrat (1986) were presented a model to minimize the cost with the assumption that stock-dependent consumption rate is a function of the initial stock-level for consumption environment. Baker and Urban (1988) developed an economic order quantity model for “a power stock induced demand pattern”. After that Mandal and Phaujdar (1989) established a “production economic order quantity model for deteriorating items with uniform production and inventory-dependent demand”. Padmanabhan and Vrat (1988) presented EOQ models for inventory induced selling rate with deteriorating items and rate of selling is assumed to be a function of stock level. Gerchak and Wang (1994) presented “an inventory models for

corporate inventory-induced dependence of demand into various inventory control models and developed periodic-review inventory models with inventory level dependent demand”. Urban (1995) investigated an inventory periods differ from the stock time demand by a given amount. Alfares (2007) established the “inventory policy for an item with a inventory dependent demand rate and a storage time dependent holding cost, the holding cost per unit of the items per unit time is assumed to be an inventory function of the time spent in storage”. Datta and Pal (1990) considered “deterministic inventory system with inventory induced demand rate up to a contain level and a constant demand for the rest of the cycle with an infinite time horizon”. Pal *et al.* (1993) presented an inventory model for inventory induced demand and items deteriorate at a constant rate over time. Hwang and Hahn (2000) established an EOQ model for inventory-induced demand rate and a certain expiry date.

Several researchers considered that the holding cost is always constant. However, holding cost is time-dependent for few inventory models. Ray and Chaudhuri (1997) considered “the time value of money in to account by analyzing an inventory system with inventory induced demand rate and shortages”. Giri *et al.* (1996) presented “an inventory model for deteriorating items with shortages”. Two research papers were developed by (Chang *et al.*, 2010; Yang *et al.*, 2010) related to the inventory-dependent demand rate.

During the last two decades, several researchers have been considered that the integration of production and EOQ model and the consignment of inventory. In this direction, Banerjee (1986) established “a Joint economic lot-size model and assumed that the supplier followed a lot-for-lot shipment policy with the retailer”. Goyal (1988) further generalized Bannerjee’s model considering a batch that consists of a number of equal-sized shipments, but the production of the batch had to be finished before the shipment could start. Hill (2000) established “an optimal two-stage lot sizing and inventory batching policies”. Teng *et al.* (2011) considered an EOQ model for inventory-dependent demand under progressive payment scheme. Goyal and Chang (2009) established “an ordering-transfer model to find the retailer’s optimal order quantity and the number of transfer per order from warehouse to display area”. Yang and Wee (2003) considered “an integrated multi-lot-size production inventory model for deteriorating items”. Law and Wee (2006) established “an integrated production-inventory model for ameliorating and deteriorating items taking account of time discounting”. Other papers related to this area are (Viswanathan, 1998; Goyal and Nebebe, 2000; Chiang, 2001; Siajidi *et al.*, 2006; Hou, 2006; Ghiami *et al.*, 2013; Yang, 2014; Jiangtao *et al.*, 2013; Soni, 2013) and others.

This paper, we present holding cost as two different functional form of demand rate (i.e., (i) inventory and (ii) time and inventory dependent). The main objective of this paper is to find minimum total inventory cost per cycle time. In the section 2, the assumption and notation related to this study are presented. Then, we provide mathematical formulation for two different circumstances and show that the total inventory cost is minimum (i.e., total inventory cost is convex function with respect to cycle time). We also discuss numerical examples to validate the model. A sensitivity analysis is provided with respect to parameters of the system. At the last, conclusion and future research are given.

Assumption and Notations

Assumptions

The following assumptions are considered to construct the mathematical formulation:

- The demand rate is (i.e., $R(I(t)) = \alpha\{I(t)^\beta\}$), for model I and stock-dependent (i.e., function of stock-level depend on demand for model II
- Shortage is not allowed
- Lead time is negligible
- The inventory system involves only one item
- Only one item is considered for inventory system

Notations

In addition the following notations are considered in the whole manuscript:

k	=	Ordering cost per order
α	=	Constant annual demand rate
$I(t)$	=	Inventory level at any instant ‘ t ’
H	=	Holding cost parameter/time
T	=	Replenishment time
B	=	Demand elasticity, $0 < \beta < 1$
TIC	=	Optimal total inventory cost/time
TIC_1^*	=	Minimum total inventory cost per unit time for model one
TIC_2^*	=	Optimal total inventory cost/time for model two
$T = T_1^*$	=	Optimal cycle time for model I
$T = T_2^*$	=	Optimal cycle time for model II
Q	=	Order quantity
$Q = Q_1^*$	=	Optimal order quantity for model I
$q = q_2^*$	=	Optimal order quantity for model II

Mathematical Formulation

The main objective of this work is to minimize the total inventory cost/time, which contains two components i.e., (i) ordering cost (k) and (ii) Holding Cost (HC). The holding cost is the integral of $h.I(t)dt$ and demand rate $\alpha\{I(t)^\beta\}$ for model I and integral of $h.t.I(t)dt$; demand rate is $\{\alpha + \beta I(t)\}$ for model II:

Case I: Power Dependent Demand and Holding Cost Function

In this model, the total inventory cost/time is expressed as:

$$TIC = \frac{k}{T} + \frac{1}{T} \int_0^T h.I(t)dt \quad (1)$$

For this model the rate of inventory is considered to be inventory level decrease, the rate of variation of inventory level is given by:

$$\frac{dI(t)}{dt} = -\alpha\{I(t)\}^\beta, \quad \alpha > 0, 0 < \beta < 1, 0 \leq t \leq T \quad (2)$$

With the conditions $I(T) = 0$.

The solution of Equation 2 with the above condition is given by is given by:

$$I(t) = \{\alpha(1 - \beta)(T - t)\}^{1/(1-\beta)}, 0 < \beta < 1 \quad (3)$$

and the corresponding order quantity is:

$$Q = \{\alpha(1 - \beta)T\}^{1/(1-\beta)} \quad (4)$$

Solving Equation 4 for T , we have:

$$T = \frac{Q^{(1-\beta)}}{\alpha(1-\beta)} \quad (5)$$

Using Equation 3 in Equation 1 and solving, we get:

$$TIC = \frac{k}{T} + \frac{h\{\alpha(1-\beta)\}^{1/(1-\beta)} T^{1/(1-\beta)} (1-\beta)}{(2-\beta)} \quad (6)$$

Using Equation 5 and 6, becomes:

$$TIC = \frac{k\alpha(1-\beta)}{Q^{(1-\beta)}} + \frac{hQ(1-\beta)}{(2-\beta)} \quad (7)$$

Differentiating Equation 7 w.r.t. Q two times, we get:

$$\frac{d(TIC)}{dQ} = \frac{h(1-\beta)}{(2-\beta)} - k\alpha(1-\beta)^2 Q^{(\beta-2)} \quad (8)$$

$$\frac{d^2(TIC)}{dQ^2} = k\alpha(1-\beta)^2(2-\beta)Q^{(\beta-3)} > 0 \quad (9)$$

The necessary and sufficient condition that $Q = Q_1^*$ is minimum is that $\frac{d(TIC)}{dQ} = 0$ and $\frac{d^2(TIC)}{dQ^2} > 0$. The minimum order quantity is obtained by solving $\frac{d(TIC)}{dQ} = 0$, we get:

$$Q = Q_1^* = \frac{\{k\alpha(1-\beta)(2-\beta)\}^{1/(2-\beta)}}{h^{1/(2-\beta)}} \quad (10)$$

and the corresponding optimal (minimum) TIC is:

$$TIC_1^* = h^{(1-\beta)/(2-\beta)} \{k\alpha(1-\beta)(2-\beta)\}^{1/(2-\beta)} \quad (11)$$

Model II: Inventory Dependent Demand Rate and Time Dependent Holding Cost Function

The demand rate is considered to be the rate of inventory-induced dependent decrease in this model, the rate of variation of inventory level is given by:

$$\frac{dI(t)}{dt} = -\{\alpha + \beta I(t)\}, \alpha > 0, 0 < \beta < 1, 0 \leq t \leq T \quad (12)$$

Under initial conditions $I(T) = 0$

The solution of Equation 12 with the above initial condition is:

$$I(t) = \frac{\alpha}{\beta} \{e^{\beta(T-t)} - 1\} \quad (13)$$

and the order quantity is:

$$Q = I(0) = \frac{\alpha}{\beta} (e^{\beta T} - 1) \quad (14)$$

The Equation 14, can be rewritten as:

$$T = \frac{1}{\beta} \log\left(1 + \frac{\beta Q}{\alpha}\right) \quad (15)$$

Total inventory cost/time is:

$$TIC = \frac{k}{T} + \frac{1}{T} \int_0^T h.t.I(t)dt \quad (16)$$

$$TIC = \frac{k}{T} - \frac{h\alpha}{\beta^2} - \frac{h\alpha}{\beta^3 T} + \frac{h\alpha}{T\beta^2} e^{\beta T} - \frac{hT\alpha}{2\beta} \quad (17)$$

Using Equation 15 and 17, becomes:

$$TIC = \frac{k\beta}{\log\left(1 + \frac{\beta Q}{\alpha}\right)} - \frac{h\alpha}{\beta^2} + \frac{hQ}{\beta \log\left(1 + \frac{\beta Q}{\alpha}\right)} - \frac{h\alpha}{2\beta^2} \log\left(1 + \frac{\beta Q}{\alpha}\right) \quad (18)$$

The optimal (minimize) value of Q is obtained by putting, $\frac{d(TIC)}{dQ} = 0$, provided $\frac{d^2(TIC)}{dQ^2} > 0$. The minimum

value of $Q = Q_2^*$ are $\frac{d(TIC)}{dQ} = 0$. But it is difficult to

handle Equation 18 to find closed form solution. The second order approximation is used for log arithmetic term and again using second order Taylor's series approximation, Equation 18 reduces to:

$$TIC = k\alpha \left(\frac{1}{Q} + \frac{\beta}{2\alpha} + \frac{\beta^2 Q}{4\alpha^2} \right) + \frac{hQ^2}{2\alpha} \quad (19)$$

[Note that the approximations are valid for $\beta T < 1$ and $(\beta Q)/\alpha < 1$. The second order approximation for log arithmetic term and then Taylor's series approximation, is given by:

$$\log\left(1 + \frac{\beta Q}{\alpha}\right) = \frac{\beta Q}{\alpha} - \frac{\beta^2 Q^2}{2\alpha^2},$$

$$\frac{\alpha}{\beta Q} \left(1 - \frac{\beta Q}{2\alpha}\right)^{-1} = \frac{\alpha}{\beta} \left(\frac{1}{Q} + \frac{\beta}{2\alpha} + \frac{\beta Q}{4\alpha^2}\right) \quad \text{and}$$

$$T = \frac{1}{\beta} \log\left(1 + \frac{\beta Q}{\alpha}\right) \approx \frac{Q}{\alpha} \left(1 - \frac{\beta Q}{2\alpha}\right), \text{ approximately].}$$

Differentiating Equation 19, with respect to Q two times, we get:

$$\frac{d(TIC)}{dQ} = k\alpha \left(-\frac{1}{Q^2} + \frac{\beta^2}{4\alpha^2} \right) + \frac{hQ}{\alpha} \quad (20)$$

and:

$$\frac{d^2(TIC)}{dQ^2} = \frac{2k\alpha}{Q^3} + \frac{h}{\alpha} > 0 \quad (21)$$

The optimal (minimum) $Q = Q_2^*$ is obtained by solving $\frac{d(TIC)}{dQ} = 0$, we get:

$$4hQ^3 + k\beta^2Q^2 - 4k\alpha^2 = 0 \quad (22)$$

The optimal (minimum) order quantity value $Q = Q_2^*$ is obtained by solving Equation 22 for Q and putting $Q = Q_2^*$ in Equation 19, the corresponding optimal (minimum) total inventory cost $TIC = TIC_2^*$ is obtained.

Numerical Examples

In this section the present study provides the following numerical examples to validate the results are illustrated in the above section 3:

Example 1: Case I: Consider the inventory elements such as $\alpha = 600$ units/year, $k = \$500$ / order, $\beta = 0.1$, $h = 1$. We obtain.

$Q = Q_1^* = 1012.336$ units, $T = T_1^* = 0.938423$ year and $TIC = TIC_1^* = \$1012.335$.

Example 2: Case II: Let $\alpha = 6000$ units per year, $k = \$500$ /order, $\beta = 0.1$, $h = 10$. We obtain $Q = Q_2^* = 1016.4$ units, $T = T_2^* = 0.1986233$ year and $TIC = TIC_2^* = \$3724.57$.

Sensitivity Analysis

In this section, we introduce sensitivity analysis with the variation of different parameters. Let us take the parameter values of the inventory system as Sensitivity analysis is performed by varying parameters, the effect of key parameters ' h ' ' k ' and ' β ' on the optimal solution, the set of values of ' h ', ' k '

and ' β ' are assumed to be ' h ' = 1,5,10,-----,80, ' β ' = 0.2, 0.3, 0.4, 0.5 and ' k ' = 510, 530, 550, 580.

The following managerial phenomena can be obtained from Table 1 and 2:

- From Table 1a, we see that when holding cost parameter increases, Q_1^* decreases and TIC_1^* increases. That is, changing ' h ' causes a opposite change in Q_1^* and positive change in TIC_1^*
- From Table 1a, we see that, if holding cost parameter increases Q_1^* decreases and increase in total inventory cost TIC_1^* , keeping β constant
- From Table 1b, we see that increase of β results, increase in optimal order quantity Q_1^* and total inventory cost TIC_1^* . That is, change ' β ' causes positive change in Q_1^* and TIC_1^* , keeping h constant
- From Table 1c, we see that increase of holding cost parameter h results decrease in optimal order quantity Q_1^* and increase in total cost TIC_1^* , keeping k constant
- From Table 1c, If holding cost parameter h increases, Q_2^* and TIC_2^* will decrease, keeping k constant. That is, changing ' h ' causes a positive change in Q_1^* and TIC_1^* , keeping k constant
- From Table 1c, we observe that when ordering cost ' k ' increases, the optimal order quantity Q_1^* and total inventory cost TIC_1^* , both will increase, keeping holding cost parameter h constant. That is, change in ' k ' causes Q_1^* and TIC_1^* , keeping h constant
- From Table 2a, we see that increase of holding cost parameter results increase in optimal order quantity Q_2^* and total inventory cost TIC_2^*
- From Table 2a, we see that increase of k leads increase in order quantity Q_1^* and total inventory cost TIC_2^*
- From Table 2b, we see that increase in initial demand α results increase in optimal order quantity Q_2^* and total inventory cost TIC_2^* . That is change in α causes positive change in Q_2^* and TIC_2^*
- From Table 2b, we see that increase in β results increase in optimal total inventory cost TIC_2^* while optimal order quantity Q_2^* approximately constant. That is change in β causes positive change in TIC_2^* and no change in Q_2^*

Table 1. (a) Values of optimal values of $T = T_1^*$, $Q = Q_1^*$ and $TIC = TIC_1^*$ for different values of ' h ', taking remaining parameters same as in example 1

h	Q_1^*	T_1^*	TIC_1^*	h	Q_1^*	T_1^*	TIC_1^*
5	433.956	0.437832	2169.78	40	145.256	0.163503	5810.257
10	301.306	0.315293	3013.068	50	129.160	0.147103	6458.031
15	243.405	0.260197	3651.074	60	117.342	0.134932	7040.558
25	186.023	0.204275	4650.578	70	108.198	0.125430	7573.884
35	155.832	0.174179	5454.130	80	100.855	0.117742	8068.421

Table 1. (b) Variation of optimal values of order quantity $Q = Q_1^*$, cycle time $T = T_1^*$ and total inventory cost $TIC = TIC_1^*$ with the variation of parameter 'h' and demand elasticity ' β ', keeping all the remaining elements same as in example 1

$h \downarrow$	$\beta \rightarrow$	0.2	0.3	0.4	0.5
30	Q_1^*	204.257	249.678	308.262	383.155
	T_1^*	0.09399	0.0556	0.03114	0.01631
	TIC_1^*	6128.05	7491.11	9247.85	11494.6
35	Q_1^*	187.503	228.034	279.948	345.735
	T_1^*	0.08777	0.05219	0.02939	0.01549
	TIC_1^*	6562.61	7981.2	9798.19	12100.7
40	Q_1^*	174.097	210.808	257.533	316.287
	T_1^*	0.08271	0.04939	0.02796	0.01482
	TIC_1^*	6963.87	8432.32	10301.3	12651.5
45	Q_1^*	163.07	196.697	239.256	292.402
	T_1^*	0.07849	0.04705	0.02675	0.01425
	TIC_1^*	7338.13	8851.36	10766.5	13158.1
50	Q_1^*	153.798	184.876	224.008	272.568
	T_1^*	0.0749	0.04506	0.02571	0.01376
	TIC_1^*	7689.92	9243.82	11200.4	13628.4
55	Q_1^*	145.867	174.796	211.054	255.788
	T_1^*	0.0718	0.04332	0.02481	0.01333
	TIC_1^*	8022.67	9613.81	11608	14068.3
60	Q_1^*	138.983	166.075	199.883	241.372
	T_1^*	0.06907	0.0418	0.02401	0.01295
	TIC_1^*	8339	9964.5	11993	14482.3

Table 1. (c) Variation of optimal solution of $Q = Q_1^*$, $T = T_1^*$ and $TIC = TIC_1^*$ with the variation of holding cost parameter 'h' and ordering cost 'k', taking remaining parameters same as in example 1

$h \downarrow$	$k \rightarrow$	510	530	550	580
30	Q_1^*	170.773	174.266	177.696	182.733
	T_1^*	0.18914	0.19262	0.19603	0.20102
	TIC_1^*	5123.19	5227.97	5330.89	5482
35	Q_1^*	157.465	160.685	163.849	168.646
	T_1^*	0.17582	0.17905	0.18222	0.18702
	TIC_1^*	5511.27	5623.99	5734.71	5897.27
40	Q_1^*	146.778	149.78	152.729	157.058
	T_1^*	0.16504	0.16808	0.17105	0.17541
	TIC_1^*	5871.13	5991.21	6109.15	6282.33
45	Q_1^*	137.956	140.777	143.549	147.618
	T_1^*	0.15609	0.15896	0.16177	0.16589
	TIC_1^*	6208	6334.97	6454.23	6642.8
50	Q_1^*	130.514	133.183	135.805	139.655
	T_1^*	0.12028	0.15122	0.1539	0.15782
	TIC_1^*	6525.69	6659.15	6790.25	6982.73
55	Q_1^*	124.128	126.667	129.161	132.822
	T_1^*	0.11497	0.14454	0.1471	0.15085
	TIC_1^*	6827.06	6966.68	7103.84	7305.21
60	Q_1^*	118.572	120.997	123.379	126.876
	T_1^*	0.11033	0.13871	0.14116	0.14476
	TIC_1^*	7114.32	7259.82	7402.74	7612.59

Table 2. (a) Values of optimal values of $Q = Q_2^*$, $T = T_2^*$ and $TIC = TIC_2^*$ for different values of holding cost parameter 'h' and ordering cost 'k', taking all the elements constant as in example 2

h	Q_2^*	T_2^*	TIC_2^*	k	Q_2^*	T_2^*	TIC_2^*
15	1062.63	0.175537	4259.88	550	1255.66	0.202087	3969.79
25	896.264	0.148262	5045.93	600	1292.61	0.213114	4207.75
35	801.177	0.132638	5641.82	650	1327.56	0.218812	4439.26
60	669.426	0.110949	6747.25	700	1360.76	0.224222	4664.96
80	608.215	0.100855	7423.76	1000	1532.54	0.252161	5922.94

Table 2. (b) Variation of optimal solution of $Q = Q_2^*$, $T = T_2^*$ and $TIC = TIC_2^*$ with the variation of constant annual demand rate ' α ' and demand elasticity ' β ', keeping all the parameters same as in example 2

α	Q_2^*	T_2^*	TIC_2^*	β	Q_2^*	T_2^*	TIC_2^*
6100	1229.98	0.199603	3745.01	0.2	1216.27	0.198602	3750.33
6200	1243.28	0.198518	3765.22	0.3	1216.07	0.196517	3776.60
6300	1256.62	0.197474	3785.22	0.4	1215.77	0.194417	3803.37
6400	1269.88	0.196450	3805.01	0.5	1215.40	0.192308	3830.65
6500	1283.07	0.195447	3824.59	0.6	1214.94	0.190189	3858.44

Conclusion and Future Research Directions

In this study, we have extended Alfares (2007) model to allow for power demand rate with inventory dependent holding cost function and stock-dependent demand rate with time and inventory dependent holding cost function. Then, we have provided numerical examples for both models. In addition, we have presented the sensitivity analysis of key elements on the effect of the optimal solution. Results show that:

- An increase of ' h ' leads decrease in Q^* and TIC^*
- Increase of ' β ' leads increase of order quantity and decrease in total inventory cost
- Increase of ordering cost leads increase in order quantity and total inventory cost
- Change in β causes positive change in TIC_2^* and no change in Q_2^*
- Change in α causes positive change in Q_2^* and TIC_2^*

It is observed that changes are quite significant with the change of various system parameters. Second order approximations have been used for log arithmetic terms to find closed form optimal solution.

The models presented in this work can be extended in different ways. We may extend the demand as quadratic time dependent demand. The models can also be extended by adding for deteriorating items. In addition, the model can be generalized to allow for shortages and time value of money.

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Author's Contributions

Rakesh Prakash Tripathi: Being a supervisor, he guided the student in the write-up, read through the manuscript, reviewed and approved it before sending to a journal.

Sarla Pareek: She guided the student in the preparation, reviewed and checked it before sending to a journal.

Manjit Kaur: Being a research student, she conceived the idea and drafted the paper.

Ethics

This article is original and contains unpublished material. There are no ethical issues involved in any aspect of this paper.

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