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Deformation Retracts of the Reissner-Nordstrom Spacetime and its Foldings

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ABSTRACT

Our aim in the present article is to introduce and study the relation between the deformation retract of the Reissner-Nordstrom spacetime N^4 and the deformation retract of the tangent space T_p (N^4). Also, this relation discussed after and before the isometric and topological folding of N^4 into itself. New types of conditional folding are presented. Some commutative diagrams are obtained.

Keywords: Retraction, Deformation Retracts, Folding, Reissner-Nordstrom Spacetime

1. INTRODUCTION

As is well known, the theory of foldings is always one of interesting topics in Euclidian and Non-Euclidian space and it has been investigated from the various viewpoints by many branches of topology and differential geometry (El-Ahmady, 2013a; 2013b; 2013c; 2013d; 2013e).

Most folding problems are attractive from a pure mathematical standpoint, for the beauty of the problems themselves. The folding problems have close connections to important industrial applications Linkage folding has applications in robotics and hydraulic tube bending. Paper folding has application in sheet-metal bending, packaging and air-bag folding (El-Ahmady, 2012a; 2012b; 2011). Following the great Soviet geometer (El-Ahmady and Al-Rdade, 2013), also, used folding to solve difficult problems related to shell structures in civil engineering and aero space design, namely buckling instability (El-Ahmady and Al-Hazmi, 2013). Isometric folding between two Riemannian manifold may be characterized as maps that send piecewise geodesic segments to a piecewise geodesic segments of the same length. For a topological folding the maps do not preserves lengths i.e., A map $\Im$: $M \rightarrow N$, where M and N are C^∞-Riemannian manifolds of dimension m,n respectively is said to be an isometric folding of M into N, iff for any piecewise geodesic path γ: $J \rightarrow M$, the induced path $\Im \ 0 \ \gamma$: $J \rightarrow N$ is a piecewise geodesic and of the same length as γ (El-Ahmady and El-Araby, 2010). If $\Im$ does not preserve length, then $\Im$ is a topological folding. A subset A of a topological space X is called a retract of X if there exists a continuous map r: $X \rightarrow A$ such that $r(\alpha) = \alpha$, $\forall \alpha \in A$ where A is closed and X is open (Arkowitz, 2011; Banchoff and Lovett, 2010; El-Ahmady, 2007a; 2007b, El-Ahmady, 2006; 2004a; 2004b). Also, let X be a space and A a subspace. A map r: $X \rightarrow A$ such that $r(\alpha) = \alpha$, for all $\alpha \in A$, is called a retraction of X onto A and A is the called a retract of X (El-Ahmady and Shamara, 2001). This can be re stated as follows. If i: $A \rightarrow X$ is the inclusion map, then r: $X \rightarrow A$ is a map such that $ri = id_A$. If, in addition, ri ri □ id_x we call r a deformation retract and A a deformation retract of X (El-Ahmady, 1994). Another simple-but extremely useful-idea is that of a retract. If A, $X \subset M$, then A is a retract of X if there is a commutative equation:

$$\begin{tikzcd} & Id_A & \\ A \arrow[r, "i"'] \arrow[ur, no head] & X \arrow[r, "r"'] & A \arrow[ul, no head] \end{tikzcd}$$

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If f: A→B and g: X→Y, then f is a retract of g if ri = id_A and js = id_B (Naber, 2011; Reid and Szendroi, 2011; Arkowitz, 2011; Strom, 2011; Shick, 2007). At each point p of a complete Riemannian manifold M, we define a mapping of the tangent space T_p (M) at p onto M in the following manner. If X is a tangent vector at P we draw a geodesic g(t) starting at P in the direction of X. If X has length α, then we map X into the point g(α) of the geodesic. We denote this mapping by $\exp_p$: Tp (M)→M, the map $\exp_p$ is everywhere C^∞ and in a neighborhood of p in M, it is a diffeomorphism (Kuhnel, 2006; Banchoff and Lovett, 2010).

1.1. Main Results

The Reissner-Nordström spacetime N^4 is given by the following metric (El-Ahmady and Al-Rdade, 2013; Hartle, 2003; Griffiths and Podolsky, 2009; Straumann, 2003) Equation 1:

$$ds^2 = -(1-\frac{2m}{r}+\frac{e^2}{r^2})dt^2 + (1-\frac{2m}{r}+\frac{e^2}{r^2})^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{1}$$

where, m represents the gravitational mass and e the electric charge of the body.

The coordinates of Reissner-NordstrÖm spacetime N^4 are given by Equation 2:

$$\begin{aligned} x_1^2 &= C_{1-}\left(1-\frac{2m}{r}+\frac{e^2}{r^2}\right)t^2 \\ x_2^2 &= C_{2+}(r^2+4mr)+(4m-e^2)\,\text{In}(r^2-2mr+e^2) \\ &+(8m^2-4e^2m)\frac{1}{\sqrt{e^2-m^2}}\tan^{-1}\frac{r-m}{\sqrt{e^2-m^2}} \\ x_3^2 &= C_3 + r^2\theta^2 \\ x_4^2 &= C_{4+}r^2\sin^2\theta\phi^2 \end{aligned} \tag{2}$$

where, C_1,C_2,C_3 and C_4 are the constant of integration .

The Reissner-Nordström space time N^4 geodesic equations for the metric (1) are given by the following Equation 3-6:

$$\frac{du^1}{d\tau}+\frac{e^2-mr}{r(r^2-2mr+e^2)}(u^1)^2-\frac{(r^2-2mr+e^2)}{r}(u^2)^2 - \frac{r^2-2mr+e^2}{r}\sin^2\theta(u^3)^2 + \frac{\left(\frac{m}{r^2}-\frac{e^2}{r^3}\right)(r^2-2mr+e^2)}{r^2}(u^4)^2=0 \tag{3}$$

$$\frac{du^2}{d\tau}+\frac{2}{r}u^1u^2 \_ \sin\theta\cos\theta(u^3)^2=0 \tag{4}$$

$$\frac{du^2}{d\tau}+\frac{2}{r}u^1u^3+2\cot\theta u^2u^3=0 \tag{5}$$

$$\frac{du^4}{d\tau}+\frac{2\left(=m+\frac{e^2}{r}\right)}{r^2-2mr+e^2}u^1u^4=0 \tag{6}$$

where, τ is an affine parameter. Suppose that $\gamma(\tau_0)=(r_0,\theta_0,\frac{\pi}{2},t_0),(r,\theta,\phi,t)$ corresponding (u^1,u^2,u^3,u^4), for all τ where $\phi=\frac{\pi}{2}$. Then Equation 7:

$$\frac{d\phi}{d\tau}=0=u^3 \tag{7}$$

Under the condition u^3 = 0 the above equations become Equation 8-11:

$$\frac{du^1}{d\tau}+\frac{e^2-mr}{r(r^2-2mr+e^2)}(u^1)^2-\frac{(r^2-2mr+e^2)}{r}(u^2)^2+\frac{\left(\frac{m}{r^2}-\frac{e^2}{r^3}\right)(r^2-2mr+e^2)}{}=0 \tag{8}$$

$$\frac{du^2}{d\tau}+\frac{2}{r}u^1u^2=0 \tag{9}$$

$$\frac{du^3}{d\tau}=0 \tag{10}$$

$$\frac{du^4}{d\tau}+\frac{2\left(-m+\frac{e^2}{r}\right)}{r^2-2mr+e^2}u^1u^4=0 \tag{11}$$

Integrating Equation (9), we get Equation 12:

$$\mu^2=\frac{\varpi_2}{r^2} \tag{12}$$

Also, integrating Equation (11), we get Equation 13:

$$\mu^4=\frac{\varpi_1 r^2}{r^2-2mr+e^2} \tag{13}$$

where, $\bar{\omega}_1$ and $\bar{\omega}_2$ are the constant of integrations. Substituting (7), (12) and (13) in (2), we get:

$$-x_1^2+x_2^2+x_3^2+x_4^2=(1-\frac{2m}{r}+\frac{e^2}{r^2})$$
$$(\frac{\bar{\omega}_1 r^2}{r^2-2mr+e^2})^2+(r^2+4mr)+(4m\_e^2)$$
$$\text{In}(r^2-2mr+e^2)+(8m^2\_4e^2m)\frac{1}{\sqrt{e^2-m^2}}\tan^{-1}$$
$$\frac{r-m}{\sqrt{e^2-m^2}}+\frac{\bar{\omega}_2^2}{r^2}+H_1$$

Which is a hypersphere $S_1^3\subset N^4$ which is a geodesic retraction.

Again, substituting (7), (12) and (13) in (3), we get the following curves geodesic retraction $S_1 \subset N^4$ $(u^1(\mu))^2=\varpi_1^2+\left(k-\frac{\varpi_2^2}{r^2(\mu)}\right)\left(1-\frac{2m}{r(\mu)}+\frac{e^2}{r^2(\mu)}\right)$ where k = - 1 corresponds timelike geodesics and also k =0 corresponds to null geodesics.

Then, the following theorem has been proved.

Theorem 1

Types of the geodesic retraction of Reissner-Nordstrom spacetime N^4 are hypersphere retraction and curves retraction.

Theorem 2

The deformation retract of (N^4-(p_1,q_1)) onto $S_1^3\subset(N^4-(p_1,q_1))$ under the exponential map is an induced deformation retract of T_{p1} (N^4) onto $\exp^{-1}(S_1^3)\subset T_{p1}(N^4-q_1)$. Any isometric folding $F:N^4\to N^4$ such that F (x_1,x_2,x_3,x_4) = (x_1,x_2,$|x_3|$,x_4) induces the same deformation retract of T_{p1} (N^4) under the condition $x_3 = 0$, which makes the equation:

$$\begin{array}{ccc} D^4_{p_1}(\pi)-p_1 & \xrightarrow{F_2} & D^4_{p_1}(\pi)-p_1 \\ \exp^{-1}\uparrow & & \uparrow\exp^{-1} \\ N^4-(p_1,\ q_1) & \xrightarrow{F_1} & N^4-\ (p_1,\ q_1) \end{array}$$

Commutative, where $\left(D^4_{p1}(\pi)-p1\right)$ is an open ball of radius π and center at p_1.

Proof

The parametric equation of the Reissner-Nordström space time N^4 is given:

$$\xi=(\sqrt{\ }(C_1-(1-\frac{2m}{r(\mu)}+\frac{e^2}{r^2(\mu)})t^2(\mu))$$
$$\sqrt{\ }(C_{2+}(r^2(\mu)+4mr(\mu))+4(m\_e^2)$$
$$\text{In}(r^2(\mu))-2mr(\mu)+e^2)$$
$$+(8m^2-4e^2m)$$
$$\frac{1}{\sqrt{e^2-m^2}}\tan^{-1}\frac{r(\mu)-m}{\sqrt{e^2-m^2}}$$
$$\sqrt{\ }(C_3+r_2(\mu)\theta^2(\mu))$$
$$\sqrt{\ }C_{4+}r^2(\mu)\sin^2\theta(\mu)\phi^2(\mu))$$

By using lagrangian equations:

$$\frac{d}{ds}\left(\frac{\partial T}{\partial G_i}\right)-(\frac{\partial T}{\partial G_i})=0,\ i=1,2,3,4$$

where, $T=\frac{1}{2}ds^2$ we obtain the deformation retract of (N^4-(p_1, q_1)) given by:

$$S_1^3=(\sqrt{\ }C_1,\sqrt{\ }C_2+(4m-e^2)\text{In}(e^2)+$$
$$(8m^2-4e^2m)\frac{1}{\sqrt{e^2-m^2}}$$
$$\tan^{-1}\frac{-m}{\sqrt{e^2-m^2}},\sqrt{\ }C_3\sqrt{\ }C_4)$$

With retraction $R_1,R_1:(N^4-(p_1,q_1))\to S_1^3$, then $\exp^{-1}$ (N^4-q_1) is an open ball $D^n_{p1}(\pi)\subset T_{p1}(N4)$. If F_1 is a deformation retract of (N^4 – (p_1,q_1)) onto a geodesic retraction $S_1^3,F_1:\{N^4-(p_1,q_1)\}\times I\to_{\{N^4-(p1,q1)\}}$ such that:

$$F_1(x,v)=(1-v)(\sqrt{\ }(C_1-(1-\frac{2m}{r(\mu)}+\frac{e^2}{r^2(\mu)})t^2(\mu)),$$
$$\sqrt{\ }(C_2+(r^2(\mu))+4mr(\mu)+4m-e^2)\text{In}(r^2(\mu)-2mr(\mu)+e^2)$$
$$+(8m^2-4e^2m)\frac{1}{\sqrt{e^2-m^2}}\tan^{-1}\frac{r(\mu)-m}{\sqrt{e^2-m^2)}}\sqrt{\ }(C_3+r^2(\mu)\,\theta^2(\mu)),$$
$$\sqrt{\ }(C_{4+}r^2(\mu)\sin^2(\mu)\phi^2(\mu))+v(\sqrt{\ }(C_{2+})(4m-e^2)\text{In}(e^2)+$$
$$(8m^2-4e^2m)\frac{1}{\sqrt{e^2-m^2}}\tan^{-1}\frac{-m}{\sqrt{e^2-m^2)}}\sqrt{\ }C_3\sqrt{\ }C_4)$$
$$F_1(x,0)=(\sqrt{(C_{1-}}(1-\frac{2m}{r(\mu)}+\frac{e^2}{r^2(\mu)})t^2(\mu))\sqrt{(C_{2+}}(r^2(\mu)_{+4}$$
$$mr(\mu))+(4m\_e^2)\text{In}(r^2(\mu)_{-2}mr(\mu)+e^2)+(8m^2{}_{-4}e^2m)$$

$$\frac{1}{\sqrt{e^2-m^2}}\tan^{-1}\frac{r(\mu)-m}{\sqrt{e^2-m^2)}}\sqrt{(C_3+r^2(\mu))}\,\theta^2(\mu))\sqrt{(C_{4+}r^2(\mu)}$$ and

$\sin^2\theta(\mu)\phi^2(\mu))$

$$F_1(x,1)=(\sqrt{C_1},\sqrt{(C_{2+}}(4m-e^2)\text{In}(e^2)\sqrt{C_4})+(8m^2-4e^2m)$$

$$\frac{1}{\sqrt{e^2-m^2}}\tan^{-1}\frac{-m}{\sqrt{e^2-m^2)}}$$

also $\begin{matrix}\exp^{-1}(\sqrt{C_1}\sqrt{(C_{2+}}(4m-e^2)\text{In}(e^2)+\\(8m^2-4e^2m)\dfrac{1}{\sqrt{e^2-m^2}}\tan^{-1}\dfrac{-m}{\sqrt{e^2-m^2}}\sqrt{C_3},\sqrt{C_4})\end{matrix}$ is a hypersphere $S_1^3\subset T_{p1}(N^4)$ of radius $\frac{\pi}{2}$ then there is an induced deformation retract of $\left(D_{p1}^4(\pi)-p_1\right)$ defined by $F_2:\{D_{p1}^4(\pi)-p_1\}\times I\to\{D_{p1}^4(\pi)-p_1\}$ such that

$$F_2((x_1,x_2,x_3,x_4),v)=(x_1,x_2,x_3,x_4)(1-v)+\frac{\pi(x_1,x_2,x_3,x_4)}{v2\,|(x_1,x_2,x_3,x_4)|}$$

Where $\begin{matrix}F_2((x_1,x_2,x_3,x_4),0)=(x_1,x_2,x_3,x_4)\\F_2((x_1,x_2,x_3,x_4),1)=\dfrac{\pi(x_1,x_2,x_3,x_4)}{2\,|(x_1,x_2,x_3,x_4)|}\end{matrix}$ Which is

$S_1^3\subset D_{p1}^4(\pi)$, also $\begin{aligned}&\exp^{-1}\{F_1(x,1)\}=F_2((x_1,x_2,x_3,x_4),1)=S_1^3\\&=F_2\{D_{p1}^n(\pi)-p_1\}=F_2\{\exp^{-1}(N^4-q_1)\}\\&\Rightarrow\exp^{-1}oF_1=F_2\,o\exp^{-1}\end{aligned}$

and the following equation is commutative:

$$\begin{array}{ccc}\left(D_{p_1}^4(\pi)-p_1\right) & \xrightarrow{F_2} & \left(D_{p_1}^4(\pi)-p_1\right)\\ \exp^{-1}\uparrow & & \uparrow\exp^{-1}\\ \left(N^4-(p_1,\ q_1)\right) & \xrightarrow{F_1} & \left(N^4-\ (p_1,\ q_1)\right)\end{array}$$

If F_1: $N^4\to N^4$ is an isometric folding and any folding homeomorphic to this type of folding:

$$F_1(x_1,x_2,x_3,x_4)=(x_1,x_2,|x_2|,x_4)$$

Then F_1 (F_1 (x,1)) = F_1 (x,1), there is an induced isometric folding $F_2:(D_{p1}^4(\pi)-p1)\to(D_{p1}^4(\pi)-p_1)$ such that $F_2:\left(S_1^3\left(\frac{\pi}{2}\right)\right)=S_1^3\left(\frac{\pi}{2}\right)\text{i.e.}F_1(F_2(x_1,x_2,x_3,x_4),1)=$.

$F_2(x_1,x_2,x_3,x_4),1\text{ if }F_1(F_1)=F_1$

Let F_μ is the set of all types of folding homeomorphic to F_1 under the condition:

$$F_1(x_1,x_2,x_3,x_4)=(x_1,x_2,|x_3|,x_4)$$

Then the deformation retract of any $F\in F_\mu$ (N^4) is invariant, i.e.,

$F_1(f)=S_1^3$, the induced invariant deformations retract:

$$F_2\{\bar{F}(D_{p1}^4(\pi)-p1)\}=S_1^3\left(\frac{\pi}{2}\right)$$

Theorem 3

Under the condition t = e = m = 0, the deformation retract of $S_1^2-(p_1,q_1)$ onto $S_1^1\subset S_1^2-(p_1-q_1)$, under the exponential map is an induced deformation retract of $T_{p1}(S_1^2)$ onto $\exp^{-1}(S_1^1)\subset T_{p1}(S_1^2-q_1)$. Any isometric folding $F:S_1^2\to S_1^2$ such that F(x_1, x_2, x_3) = (x_1|x_2|x_3) induces the same deformation retract of $T_{p1}(S_1^2)$, which makes the equation:

$$\begin{array}{ccc}D_{p_1}^2(\pi)-p_1 & \xrightarrow{F_2} & D_{p_1}^4(\pi)-p_1\\ \exp^{-1}\uparrow & & \uparrow\exp^{-1}\\ S_1^2-(p_1,\ q_1) & \xrightarrow{F_1} & S_{1_1}^2-\ (p_1,\ q_1)\end{array}$$

Commutative, where $D_{p1}^2(\pi)$ is an open ball of radius π and of center at p1.

Theorem 4

Any isometric folding $F:S_1^3\subset N^4\to S_1^3\subset N^4$ such that F(p) = p,p is any point on $S_1^3\subset N^4$. There is an induced isomtric folding of the tangent space $T_{p1}(S_1^3)$ such that the following equation is commutative:

$$\begin{array}{ccc}T_{p_1}\left(S_1^3\right) & \xrightarrow{\bar{F}} & T_{p_1}\left(S_1^3\right)\\ \exp^{-1}\uparrow & & \uparrow\exp^{-1}\\ \left(S_1^2-q_1\right) & \xrightarrow{F} & \left(S_1^2-q_1\right)\end{array}$$

q1 is the conjugate point of $p1,p_1,q_1\in S_1^3\subset N^4\text{i.e.},\exp^{-1}oF=\bar{F}\,o\exp^{-1}$.

Proof

Since q_1 is a conjugate point to p_1, then $\exp^{-1}$: $(S_1^3-q_1)\to T_{p1}(S_1^3)$ under this map $(S_1^3-q_1)$ mapped onto an open ball $D_{p1}^3\subset T_{p1}(S_1^3)$, p_1 is the center of the ball with radius π.

Let $F:(S_1^3 - q_1) \to (S_1^3 - q_1)$ such that $F(p_1) = p_1$ be an isometric folding, then there is an induced isometric folding $\overline{F}$ such that:

$$\overline{F}: T_{p1}(S_1^3 t) \to T_{p1}(S_1^3)$$

Let γ be any curve in $(S_1^3 - q_1)$ then $F(\gamma) = \dot{\gamma}$, since there is no conjugate point to p_1 on $(S_1^3 - q_1)$, then $\exp^{-1}(\gamma) = \beta$, then $p_1 \in \beta$, p_1 is the beginning of β, also $\exp^{-1}(\dot{\gamma}) = \dot{\beta}$. There is an induced isometric folding $\overline{F}: T_{p1}(S_1^3 - q_1) \to T_{p1}(S_1^3 - q_1)$ such that:

$$\begin{aligned}
&\overline{F}(\beta) = \overline{F}(\exp^{-1}(\gamma)) = \alpha \\
&\exp^{-1} \text{ o } F(\gamma)_\exp^{-1}(\dot{\gamma}) + \dot{\beta} \\
&\exp^{-1}(F(p_2)) = \exp^{-1}(P_3) = P_3' \\
&\overline{F}(\exp^{-1}(p_2))
\end{aligned}$$

Is the end of β and the beginning point of α is the beginning point of β, the end point of α is the end point of β, then $\alpha = \beta$, i.e.:

$$\exp^{-1} \text{o} F = \overline{F} \text{o} \exp^{-1}$$

Theorem 5

Under the condition t = e = m = 0, Any isometric folding $F: S_1^2 \to S_1^2$ such that F(p) = p, p is any point on S_1^2. There is an induced isomtric folding of the tangent space $T_{p1} S_1^2$ such that the following diagram is commutative:

$$\begin{array}{ccc}
T_{p_1}(S_1^2) & \xrightarrow{\overline{F}} & T_{p_1}(S_1^2) \\
\exp^{-1} \uparrow & & \uparrow \exp^{-1} \\
(S_1^2 - q_1) & \xrightarrow{F} & (S_1^2 - q_1)
\end{array}$$

q_1 is the conjugate point of p_1, $p_1, q_1 \in S_1^2$ i.e.:

$$\exp^{-1} \text{o} F_1 = F_2 \text{ o} \exp^{-1}$$

Theorem 6

Under the conditions in theorem (2), if the following equation:

$$\begin{array}{ccc}
\left(D_{p_1}^4(\pi) - p_1\right) & \xrightarrow{F_2} & \left(D_{p_1}^4(\pi) - p_1\right) \\
\exp^{-1} \uparrow & & \uparrow \exp^{-1} \\
\left(N^4 - (p_1, q_1)\right) & \xrightarrow{F_1} & \left(N^4 - (p_1, q_1)\right)
\end{array}$$

Is commutative and F_1 (F_1) $=F_1$, then the following equation is commutative:

$$\begin{array}{ccc}
\left(D_{p_1}^4(\pi) - p_1\right) & \xrightarrow{F_2} & \left(D_{p_1}^4(\pi) - p_1\right) \\
\exp^{-1} \uparrow & & \uparrow \exp^{-1} \\
\left(N^4 - (p_1, q_1)\right) & \xrightarrow{F_1} & \left(N^4 - (p_1, q_1)\right)
\end{array}$$

Proof

Since $\exp^{-1}$ o $F_1 = F_2$ o $\exp^{-1}$, then:

$$F_1 = \exp \text{ o} F_2 \text{ o} \exp^{-1}, F_1(F_1) = F_1, F_2(F_2) = F_2$$

We get:

$$\begin{aligned}
&\exp^{-1} \text{o} F_1 = \exp^{-1}(F_1) = \exp^{-1} \\
&(\exp \text{ o } F_2 \ \exp^{-1}) = F_2 \text{ o } \exp^{-1} = F_2 \text{ o } \exp^{-1}
\end{aligned}$$

2. CONCLUSION

In this study we achieved the approval of the important of the curves and surface in Reissner-Nordström spacetime N^4 by using some geometrical transformations. The relations between folding, retractions, deformation retracts, limits of folding and limits of retractions of the curves and surface in the Reissner-Nordström spacetime N^4 are discussed. New types of the tangent space T_p (N^4) in Reissner-Nordström spacetime N^4 are deduced.

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