

On Fixed Point Theorem of C Class Functions - B Weak Cyclic Mappings

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Abstract: New B - \mathfrak{U} -cyclic weak contraction C -class function concept has been introduced, B - \mathfrak{U} -cyclic weak F - φ - ϕ -contraction types of mapping are defined and the existence of fixed point for such types has been proved. These results mainly generalize fixed point theorems in some previous research papers.

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Introduction

First, the class Φ is all non-decreasing mappings $\phi: [0, \infty) \rightarrow [0, \infty)$ characterized by $\phi(t) = 0$ if and only if $t = 0$.

Bilgili *et al.* (2014; Karapinar and Sadarangani, 2012; Du and Karapinar, 2013) discussed the concept of Φ -weakly cyclic contraction mappings and proved fixed point theorems for mappings on Banach spaces. While Harjani *et al.* (2013) considered cyclic weak Φ -contraction on compact metric spaces, the considered mapping ϕ need not be continuous, another fixed point treating the concept is given in (Karapinar and Sadarangani, 2012). The fixed point theorem given in (Karapinar and Sadarangani, 2012) focused on a wider class of metric spaces.

Jleli *et al.* (2014; Karapinar *et al.*, 2012b) generalized the results to cyclic (φ, ϕ) -weak contractions in some other metric spaces.

Many results have been proved in different situations and settings for the purpose of generalization of the Banach contraction principle for contraction mappings and for non-expansive mappings, (Hardy and Rogers, 1973; Gregus, 1980; Kaewcharoen and Kirk, 2006; Kannan, 1971; Kirk, 1965; Park, 1980; Rhoades, 1977; 2001; Sahar Mohamed Ali Abou Bakr, 2013; Wong, 1975; Rhoades, 2009; Ćirić, 2006).

Recent results related to cyclic weak $(\varphi$ - $\phi)$ -contraction mappings appeared in (Sahar Mohamed Ali Abou Bakr, 2017) for mappings with weak cyclic representation in complete metric spaces and weakly complete normed spaces, the considered mapping φ need not be additive, the author gave some examples.

On the other side, Morales and Rojas (2009) defined BZ type mappings or the B -Zamfirescu mappings, the mappings of any of the following types:

- BB -type or B -Banach contraction
- BK type or B -Kannan contraction
- BC type or B -Chatterjea contraction

It is proved the existence of only one fixed point for such types of mapping for the continuous, one to one and sub-sequentially convergent mapping B .

For generalized cyclic weak φ - ϕ contraction types mappings, (Bilgili and Karapinar, 2013; Hussain *et al.*, 2014; Karapinar and Rakocevic, 2013; Karapinar *et al.*, 2012a; 2013).

Mathematical Preliminaries

First, in the sequel, (X, d) is the space X with a metric d and the class \mathfrak{A} is the class of all finite collections of nonempty closed subsets of X , $\mathfrak{A} = \{A_i\}_{i=1}^j$ such that $X = \bigcup_{i=1}^j A_i$:

$$\mathfrak{A} := \left\{ \mathfrak{A} = \{A_i\}_{i=1}^j : X = \bigcup_{i=1}^j A_i \right\}$$

The mappings B and S are self mappings on X . We have the following definitions.

Definition 1 (Bilgili and Karapinar, 2013)

B is known as weak Φ -contraction if and only if there is a continuous function $\phi \in \Phi$ such that:

$$d(B(x), B(y)) \leq d(x, y) - \phi(d(x, y)), \text{ for every } x, y \in X.$$

Definition 2 (Bilgili et al., 2014)

The element $\mathfrak{A} = \{A_i\}_{i=1}^j \in \mathfrak{U}$ is B cyclic representation of X if and only if:

$$B(A_1) \subset A_2, B(A_2) \subset A_3, \dots, B(A_{j-1}) \subset A_j \text{ and } B(A_j) \subset A_1$$

Definition 3 (Jleli et al., 2014)

B is \mathfrak{U} -cyclic Φ -weak contraction on X if and only if there are $\mathfrak{A} \in \mathfrak{U}$ and a continuous function $\phi \in \Phi$ satisfying the two conditions:

- \mathfrak{A} is a B cyclic representation of X
- $d(B(x), B(y)) \leq d(x, y) - \phi(d(x, y))$
for every $x \in A_k, y \in A_{k+1}, k = 1, 2, \dots, j$ and $A_{j+1} = A_1$.

Definition 4

B is \mathfrak{U} -cyclic φ - ϕ -weak contraction on X if and only if there are $\mathfrak{A} \in \mathfrak{U}$ and $\varphi, \phi \in \Phi$, with φ continuous such that the following are true:

- \mathfrak{A} is B cyclic representation of X
- $\varphi(d(B(x), B(y))) \leq \varphi(d(x, y)) - \phi(d(x, y))$
 $\forall x \in A_k; y \in A_{k+1}, k = 1, 2, \dots, j$ and $A_{j+1} = A_1$

This paper generalizes the \mathfrak{U} -cyclic φ - ϕ -weak contraction types to new B - \mathfrak{U} -cyclic weak F - φ - ϕ contraction type, this new C -class of weak contraction mapping is defined step by step next.

Definition 5

Let $\mathfrak{A} = \{A_i\}_{i=1}^j$ be an element in \mathfrak{U} . Then \mathfrak{A} is BS -cyclic representation of X if and only if:

$$B(S(A_1)) \subset A_2, B(S(A_2)) \subset A_3, \dots, \\ B(S(A_{j-1})) \subset A_j \text{ and } B(S(A_j)) \subset A_1$$

Definition 6

Let $\mathfrak{A} = \{A_i\}_{i=1}^j \in \mathfrak{U}$. Then the self mapping S on X is B -cyclic weak φ - ϕ contraction mapping on X if and only if there are $\varphi, \phi \in \Phi$ with φ continuous such that the following are true:

- \mathfrak{A} is a TS -cyclic representation of X

- $\varphi(d(B(S(x)), B(S(y)))) \leq \varphi(d(B(x), B(y))) - \phi(d(B(x), B(y)))$
 $\forall x \in A_i, y \in A_{i+1}, i = 1, 2, \dots, j$ and $A_{j+1} = A_1$

Remark

The weak type contraction mapping defined in (6) generalizes the definition of cyclic weak ϕ -contraction of Erdal Karapinar, Kishin Sadarangani, cyclic weak $(\varphi$ - $\phi)$ contraction of Sahar Mohamed Ali Abou Bakr and TB contraction mappings of Jose R. Morales, Edixon Rojas (Karapinar and Sadarangani, 2012; Sahar Mohamed Ali Abou Bakr, 2017; Morales and Rojas, 2009) respectively, because these are a particular cases corresponding to taking B and φ identities.

Definition 7

B is sequentially convergent if it satisfies the condition: If $\{B(x_n)\}_{n=1}^\infty$ is convergent, then $\{x_n\}_{n=1}^\infty$ is convergent.

Finally, we have the following:

Definition 8 (Ansari, 2014; Ansari et al., 2016)

The real valued mapping $F: [0, \infty) \times [0, \infty) \rightarrow R$ is C -class if it is continuous and satisfying the axioms:

- $F(u, v) \leq u$ for all $u, v \in [0; \infty]$
- If $F(u; v) = u$, then either $u = 0$ or $v = 0$

C is the set of C -class functions. Mention that some C -class function F verifies $F(0, 0) = 0$.

Examples

If $h: [0, \infty) \rightarrow [0, \infty)$ is a continuous and $h(v) = 0$ if and only if $v = 0$, then $F(u, v) = u - h(v)$ is a C -class function, in particular:

- $F(u, v) = u - v$
- $F(u, v) = u - \frac{v}{k + v}$
- $F(u, v) = u - \left(\frac{2 + v}{1 + v}\right)v$
- $F(u, v) = u - \left(\frac{1 + u}{2 + u}\right)\left(\frac{v}{1 + v}\right)$

are C -class functions.

If $h: [0, \infty) \rightarrow [0, 1]$ is a continuous function, then $F(u, v) = u - h(v)$ is a C -class function, in particular we have the following:

- $F(u, v) = mu$ for some $m \in [0, 1]$
- $F(u, v) = \frac{u}{(1 + v)^r}$ for some $r \in [0, \infty)$

- $F(u,v) = u \log_{v+a} a, a > 1$
- $F(u,v) = \frac{u}{(1+u)^r}$ for some $r \in [0, \infty)$

In addition to the following:

- $F(u,v) = \log(v + a^u)/(1 + v)$, for some $a > 1$
- $F(u,v) = \ln(1 + a^u)/2$, for $e > a > 1$. Indeed $F(u, v) = u$ implies that $u = 0$
- $F(u,v) = (u + l)^{l/(1+v)^r} - l, l > 1$, for $r \in [0, \infty)$
- $F(u,v) = \sqrt[r]{\ln(1 + u^n)}$

Now; the generalized C-class of TS cyclic weak (ϕ, ϕ) -contraction mappings are defined as:

Definition 9

S is \mathfrak{A} -B-cyclic F - ϕ - ϕ -weak contraction mapping on X if and only if there are $\mathfrak{A} = \{A_i\}_{i=1}^j \in \mathfrak{A}, \phi, \phi \in \Phi$ with ϕ continuous and $F \in C$ satisfying:

- \mathfrak{A} is a BS-cyclic representation of X

$$\begin{aligned} & \phi(d(B(S(x)), B(S(y)))) \\ & \leq F(\phi(d(B(x), B(y))), \phi(d(B(x), B(y)))) \end{aligned} \quad (2.1)$$

for every $x \in A_k, y \in A_{k+1}, k = 1, 2, \dots, j$ and $A_{j+1} = A_1$

Remark

The contraction type mapping defined in definition (9) is a generalization of the contraction type defined in definition (6), because it is a particular case when taking $F(u, v) = u - v$.

Main Results

The results of this work are depending on Propositions (1) and (2) below.

Proposition 1

Let S be \mathfrak{A} -B-cyclic F - ϕ - ϕ -weak contraction on X . Then:

$$\inf \{d(B(S(x)), B(x)) : x \in X\} = 0$$

Proof

Choose $x_0 \in X$ and focus on the iterated sequence:

$$B(x_{n+1}) = B(S(x_n)) = B(S^n(x_0)) \text{ for } n = 0, 1, 2, \dots$$

If there is a natural number n_0 such that $S(x_{n+1}) = S(x_n) \forall n \geq n_0$, then $S(x_n)$ is fixed of S . This insures that such infimum is zero. Suppose that $B(S(x_{n+1})) \neq B(S(x_n))$ for all $n = 0, 1, 2, \dots$. Then, the contraction condition yields:

$$\begin{aligned} & \phi(d(B(x_n), B(x_{n+1}))) = \phi(d(BS(x_{n-1}), B(Sx_n))) \\ & \leq F(\phi(d(B(x_{n-1}), B(x_n))) = \phi(d(B(x_{n-1}), B(x_n)))) \quad (3.1) \\ & \leq \phi(d(B(x_{n-1}), B(x_n))) \end{aligned}$$

Since ϕ is non-decreasing function, we see that:

$$d(B(x_n), B(x_{n+1})) \leq d(B(x_{n-1}), B(x_n)) \forall n \in N$$

This proves that the sequence $\{d(B(x_n), B(x_{n+1}))\}_{n \in N}$ is a non-decreasing, hence the limit:

$$\lim_{n \rightarrow \infty} d(B(x_n), B(x_{n+1}))$$

exists and it is equal to the infimum of the sequence, say r :

$$\begin{aligned} r & = \lim_{n \rightarrow \infty} d(B(x_n), B(x_{n+1})) = \inf \{d(B(x_n), B(x_{n+1})) : n \in N\}, \\ r & \leq d(B(x_n), B(x_{n+1})) \forall n \in N \end{aligned}$$

On the other side, we have the same conclusions for the two sequences $\{\phi(d(B(x_n), B(x_{n+1})))\}_{n \in N}$ and $\{\phi(d(B(x_n), B(x_{n+1})))\}_{n \in N}$, consider the two positive real numbers ϕ_0 and ϕ_0 given as follows:

$$\begin{aligned} \phi(r) & \leq \phi_0 = \lim_{n \rightarrow \infty} \phi(d(B(x_n), B(x_{n+1}))) \\ & = \inf \{\phi(d(B(x_n), B(x_{n+1}))) : n \in N\} \end{aligned}$$

and:

$$\begin{aligned} \phi(r) & \leq \phi_0 = \lim_{n \rightarrow \infty} \phi(d(B(x_n), B(x_{n+1}))) \\ & = \inf \{\phi(d(B(x_n), B(x_{n+1}))) : n \in N\} \end{aligned}$$

The limit of the inequalities (3.1) as $n \rightarrow \infty$ gives:

$$\phi_0 \leq F(\phi_0, \lim_{n \rightarrow \infty} \phi(d(B(x_{n-1}), B(x_n)))) \leq \phi_0$$

and, therefore:

$$\phi_0 = 0, \text{ or } \lim_{n \rightarrow \infty} \phi(d(B(x_{n-1}), B(x_n)))$$

Now; suppose that $r > 0$, we have $\phi(r) > 0$, thus:

$$0 < \phi(r) \leq \phi(d(B(x_{n-1}), B(x_n))) \forall n \in N$$

Letting $n \rightarrow \infty$ in the last inequalities, we get the following contradiction:

$$0 < \phi(r) \leq \lim_{n \rightarrow \infty} \phi(d(B(x_{n-1}), B(x_n))) = 0$$

that is; the assumption $r > 0$ is not true, hence $r = 0$, thus:

$$\lim_{n \rightarrow \infty} d(B(x_{n+1}), B(x_n)) = \inf \{d(B(x_{n+1}), B(x_n)) : n \in N\} = 0$$

Hence; $\{x_n\}_{n=1}^{\infty} \subset X$ such that:

$$\lim_{n \rightarrow \infty} d(B(S(x_n)), B(x_n)) = 0$$

This is sufficiently proved that $\inf\{d(B(S(x)), B(x)) : x \in X\} = 0$.

Proposition 2

Let S be \mathcal{U} - B -cyclic F - ϕ - ϕ -weak contraction mapping on X . Then the iterated sequence $\{B(x_n) = B(S^n(x_0))\}_{n \in N}$ is Cauchy.

Proof

We determine for a given $\epsilon > 0$ a natural number $n_0 \in N$ satisfying; if $m, n > n_0$ with $x_n \in A_i$ and $x_m \in A_{i+1}$ for some $i \in \{1, 2, \dots, j\}$ ($n-m \equiv 1(j)$) then $d(B(x_n), B(x_m)) < \epsilon$ gives a contradiction.

Suppose that there is $\epsilon > 0$ satisfying; any $n_0 \in N$ yields $m, n > n_0$ with $B(x_n) \in A_i$ and $B(x_m) \in A_{i+1}$ for some $i \in \{1, 2, \dots, j\}$ ($n-m \equiv 1(j)$) satisfying:

$$\epsilon < d(B(x_n), B(x_m))$$

since ϕ and ϕ are non-decreasing, we see that:

$$0 < \phi(\epsilon) \leq \phi(d(B(x_n), B(x_m)))$$

and

$$0 < \phi(\epsilon) \leq \phi(d(B(x_n), B(x_m))) \tag{3.2}$$

and since ϕ is continuous, we see that:

$$0 < \phi(\epsilon) \leq \phi\left(\lim_{n, m \rightarrow \infty} (d(B(x_n), B(x_m)))\right) \tag{3.3}$$

using the contractivity condition of S we see that:

$$\begin{aligned} & \phi(d(B(x_{n+1}), B(x_{m+1}))) \\ &= \phi(d(B(S^{n+1}(x_0)), B(S^{m+1}(x_0)))) \\ &\leq F\left(\phi(d(B(S^n(x_0)), B(S^m(x_0))))\right), \tag{3.4} \\ & \phi(d(B(S^n(x_0)), B(S^m(x_0)))) \\ &\leq \phi(d(B(x_n), B(x_m))) \end{aligned}$$

Now; using inequalities (3.2), (3.3) and letting $n, m \rightarrow \infty$ in (3.4) with $n-m \equiv 1(j)$ proves the following contradiction:

$$\phi(\epsilon) \leq F(\phi(\epsilon), \phi(\epsilon)) \leq \phi(\epsilon) \tag{3.5}$$

This proves that $F(\phi(\epsilon), \phi(\epsilon)) = \phi(\epsilon)$, hence either $\phi(\epsilon) = 0$ or $\phi(\epsilon) = 0$, consequently $\epsilon = 0$, therefore this sequence is having Cauchy subsequence, this fact with Proposition (1) completes the proof.

Finally; we have:

Theorem 1

If (X, d) is complete, B is one to one continuous self sequentially convergent mapping on X and S is \mathcal{U} - B -cyclic F - ϕ - ϕ -weak contraction mapping on X . Then S owns fixed points. Moreover; we have the following:

- If $\{A_i\}_{i=1}^j$ is representation of X and z is fixed of S , then $B(z) \cap_{i=1}^j A_i$
- Arbitrarily two consecutive sets of $\{A_i\}_{i=1}^j$ cannot contain two different fixed points

Proof

Using Proposition (2), the sequence $\{B(S^n(x_0))\}_{n \in N}$ is Cauchy, the completeness of X shows that there is a point $y \in X$ such that:

$$\lim_{n \rightarrow \infty} B(S^n(x_0)) = y \tag{3.6}$$

Such a limit point is lying in the set $\cap_{i=1}^j A_i$, because each $A_i, i = 1, 2, \dots, j$ contains in finitely many members of the infinite sequence $\{B(S^n(x_0))\}_{n \in N}$, hence y is a limit point for A_i for each $i = 1, 2, \dots, j$, giving that A_i is closed for each $i = 1, 2, \dots, j$ shows that $y \in A_i$ for each $i = 1, 2, \dots, j$ (as closed set contains all its limit points), thus:

$$y \in \cap_{i=1}^j A_i$$

Since B is sequentially convergent mapping, the sequence $\{S^n(x_0)\}_{n=1}^\infty$ has a convergent subsequence $\{S^{n_k}(x_0)\}_{k=1}^\infty$, hence there is $z \in X$ such that:

$$\lim_{k \rightarrow \infty} S^{n_k}(x_0) = z$$

Using the continuity of B gives that:

$$\lim_{k \rightarrow \infty} B(S^{n_k}(x_0)) = B(z) \tag{3.7}$$

Using (3.6) and (3.7) proves that $B(z) = y$. We will see that such a z is fixed of S . In fact; there is $i \in \{1, 2, \dots, j\}$ such that $z \in A_i$, for the number $i + 1$ we get $n_i \in N$ such that:

$$\{x_{n_i}, x_{n_i+1}, x_{n_i+2}, \dots\} \subset A_{i+1}.$$

Therefore z and $\{x_{n_i}, x_{n_i+1}, x_{n_i+2}, \dots\}$ are lying in two consecutive sets, on the other side we have:

$$\begin{aligned} d(B(S(z)), B(z)) &\leq d(B(S(z)), B(x_{n_{i+1}})) \\ &+ d(B(x_{n_{i+1}}), B(z)) \tag{3.8} \\ &= d(B(S(z)), B(S(x_{n_i}))) + d(B(S(x_{n_i})), B(z)). \end{aligned}$$

Hence we use the contraction property of S as:

$$\begin{aligned} &\varphi(d(B(S(z)), B(S(x_{n_i})))) \\ &\leq F(\varphi(d(B(z), B(x_{n_i}))), \varphi(d(B(z), B(x_{n_i})))) \tag{3.9} \\ &\leq \varphi(d(B(z), B(x_{n_i}))) \\ &= \varphi(d(B(z), B(S(x_{n_i})))) \end{aligned}$$

Using the continuity of φ and taking the limit of the inequalities (3.9) as $i \rightarrow \infty$ prove the following:

$$\begin{aligned} &\lim_{i \rightarrow \infty} \varphi(d(B(S(z)), B(S(x_{n_i})))) \\ &\leq \lim_{i \rightarrow \infty} \varphi(d(B(z), B(S(x_{n_i-1})))) \\ &= \varphi(d(B(z), \lim_{i \rightarrow \infty} B(S(x_{n_i-1})))) \tag{3.10} \\ &= \varphi(d(B(z), y)) \\ &= \varphi(d(B(z), B(z))) = \varphi(0) = 0 \end{aligned}$$

Using the continuity of φ once more with the inequalities (3.10) shows that:

$$\begin{aligned} 0 &= \lim_{i \rightarrow \infty} \varphi(d(B(S(z)), B(S(x_{n_i})))) \\ &= \varphi(\lim_{i \rightarrow \infty} d(B(S(z)), B(S(x_{n_i})))) \end{aligned}$$

hence:

$$\lim_{i \rightarrow \infty} d(B(S(z)), B(S(x_{n_i}))) = 0 \tag{3.11}$$

Using (3.11) in (3.8) after taking the limit as $i \rightarrow \infty$ gives:

$$\begin{aligned} d(B(S(z)), B(z)) &\leq \lim_{i \rightarrow \infty} d(B(S(z)), B(S(x_{n_i}))) \\ &+ \lim_{i \rightarrow \infty} d(B(S(x_{n_i})), B(z)) \\ &= 0 + d(y, B(z)) = d(B(z), B(z)) = 0 \end{aligned}$$

Hence $d(B(S(z)), B(z)) = 0$, therefore, $B(S(z)) = B(z)$, since B is one to one, we get $S(z) = z$ and hence z is fixed of S .

To show that two consecutive sets of $\{A_i\}_{i=1}^j$ cannot contain two different fixed points, by contrary assume that w and z are two different fixed of S , $S(w) = w$ and $S(z) = z$ those are lying in two consecutive sets, we have the following:

$$\begin{aligned} \varphi(d(B(w), B(z))) &= \varphi(d(B(S(w)), B(S(z)))) \\ &\leq F(\varphi(d(B(w), B(z))), \varphi(d(B(w), B(z)))) \end{aligned}$$

hence $\varphi(d(B(w), B(z))) = 0$, or $\phi(d(B(w), B(z))) = 0$, that is; $d(B(w), B(z)) = 0$, consequently $B(w) = B(z)$, since B is one to one, $w = z$. This completes the proof.

We also have:

Theorem 2

Let $(X, \|\cdot\|)$ be weakly complete normed space, C be a closed convex subset of X , $\{C_i\}_{i=1}^j$ be a nonempty closed subsets of C , $C = \bigcup_{i=1}^j C_i$. If B and S are self mappings on C , $\{C_i\}_{i=1}^j$ is BS representation of C and S is weak F - φ - ϕ -contraction on C , then S has fixed points.

Proof

Using Proposition (2) the sequence of iterates $\{B(S^n(x_0))\}_{n \in N}$ is Cauchy, using the weak completeness assumption of X there exists $x \in X$ such that:

$$w\text{-}\lim_{n \rightarrow \infty} B(S^n(x_0)) = x$$

Since C is closed convex subset of X , the sequence $\{B(S^n(x_0))\}_{n \in \mathbb{N}}$ converges strongly to x and $x \in C$. For the other parts of the proof use Theorem (1).

Conclusion

This paper suggests new C -class of TS cyclic weak (φ, ϕ) -contraction mappings and proved the existence of unique fixed point for such types of mappings.

Competing Interest

The authors have no competing interest.

Author's Contributions

The first author contributed in the ration of two-third in the writing of this paper.

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