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Multiparameterized Soft Set

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Abstract: Problem statement: In 1999 Molodtsov introduced the concept of a soft set as a general mathematical tool for dealing with uncertainty. The solutions of such problems involve the use of mathematical principles based on uncertainty and imprecision. **Approach:** In this study we recall the definition of a soft set, its properties and its operations. **Results:** As a generalization of Molodtsov's soft set we introduce the definition of a multiparameterized soft set and its basic operations, namely complement, union, intersection and OR. **Conclusion:** We give examples for these concepts. Basic properties of the operations are also given.

Key words: Soft set, multisoft set, basic operations, basic properties

INTRODUCTION

Most of the problems in engineering, medical science, economics, environments. have various uncertainties. Molodtsov (1999) initiated the concept of soft set theory as a mathematical tool for dealing with uncertainties which is free from the above difficulties. After Molodtsov's work, some different operations and application of soft sets were studied by Chen *et al*. (2005), Maji *et al*. (2002, 2003). Furthermore Maji *et al*. (2001) presented the definition of fuzzy soft set and Roy and Maji (2007) presented the applications of this notion to decision making problems. As a generalization of Molodtsov's soft set we introduce the definition of a multisoft set and its basic operations, namely complement, union and intersection. We give examples for these concepts. Basic properties of the operations are also given.

Preliminaries: We recall some basic notions in soft set theory. Molodtsov (1999) defined soft set in the following way. Let U be a universe set and let E be a set of parameters. Let P (U) denote the power set of U and $A \subseteq E$.

Definition 1: (Molodtsov, 1999). A pair (F, A) is called a soft set over U, where F is a mapping given by F: $A \rightarrow P$ (U). In other words, a soft set over U is a parameterized family of subsets of the universe U for $\varepsilon \in A$, F (ε) may be considered as the set of ε-approximate elements of the soft set (F, A).

Example 1: Let us consider a soft set (F, E) which describes the ''attractiveness of houses'' that Mr. X is considering for purchase. Suppose that there are six houses in the universe U = {H_1, H_2, H_3, H4, H_5, H_6} under consideration and that E = {e_1, e_2, e_3, e_4, e_5} is a set of decision parameters. The e_i (1, 2, 3, 4, 5) stand for the parameters ''expensive'', ''beautiful'', ''wooden'', ''cheap'' and ''in green surroundings'' respectively.

Consider the mapping F given by ''houses (.)'', where (.) is to be filled in by one of the parameters $E_i \in E$, For instance, F (e_1) means ''houses (expensive)'' and its functional value is the set {$h \in U$: h is an expensive house}

Suppose that F (e1) = {h_2, h_4}, F (e_2) = {h_1, h_3}, F (e_3) = $\varnothing$, F (e_4) = {h_1, h_3, h_5} and F (e_5) = {H_1}. Then we can view the soft set (F, E) as consisting of the following collection of approximations:

$$\begin{aligned}(F,E) = \{&(\text{expensive houses, } \{h_2,h_4\}),\\ &(\text{beautiful houses, } \{h_1,h_3\}),\\ &(\text{wooden houses, } \varnothing),\\ &(\text{cheap houses, } \{h_1,h_3,h_5\}),\\ &(\text{in the green surroundings, } \{h_1\})\}.\end{aligned}$$

Each approximation has two parts: a predicate and an approximate value set.

The following definitions are due to Maji *et al*. (2003).

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Definition 2: For two soft sets (F, A) and (G, B) over U, (F, A) is called a soft subset of (G, B) if:

- $A \subset B$ and
- $\forall \varepsilon \in A$, F (ε) and G (ε) are identical approximations

This relationship is denoted by $(F, A) \tilde{\subset} (G, B)$. In this case (G, B) is called a soft superset of (F, A) and is denoted by $(G, B) \tilde{\supset} (F, A)$.

Definition 3: Two soft sets (F, A) and (G, B) over a common universe U are said to be soft equal if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A).

Definition 4: Let E = {e_1, e_2,…, e_n} be a set of parameters. The NOT set of E, denoted by $\neg E$, is defined by $\neg E = \{\neg e_1, \neg e_2, \ldots, \neg e_n\}$ where $\neg e_i$ = not e_i, $\forall_i$.

Definition 5: The complement of a soft set (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, \neg A)$ where $F^c : \neg A \to P(U)$ is a mapping given by $F^c(\alpha) = U - F(\neg \alpha), \forall \alpha \in \neg A$.

Definition 6: A soft set (F, A) over U is said to be a null soft set, denoted by Φ, if $\forall \varepsilon \in A$, F (ε) = ∅ (null-set).

Definition 7: A soft set (F, A) over U is said to be an absolute soft set, denoted by $\tilde{A}$, if $\forall \varepsilon \in A$, $F(\varepsilon) = U$.

Definition 8: The union of two soft sets (F, A) and (G, B) over a common universe U is the soft set (H, C) where $C = A \cup B$, and $\forall \varepsilon \in C$:

$$H(\varepsilon) = \begin{cases} F(\varepsilon) & \text{if} \quad \varepsilon \in A - B, \\ G(\varepsilon) & \text{if} \quad \varepsilon \in B - A, \\ F(\varepsilon) \cup G(\varepsilon) & \text{if} \quad \varepsilon \in A \cap B. \end{cases}.$$

Definition 9: (Ali *et al*., 2009). The extended intersection of two soft sets (F, A) and (G, B) over a common universe U is the soft set (H, C) where $C = A \cup B$, and $\forall \varepsilon \in C$:

$$H(\varepsilon) = \begin{cases} F(\varepsilon) & \text{if} \quad \varepsilon \in A - B \\ G(\varepsilon) & \text{if} \quad \varepsilon \in B - A \\ F(\varepsilon) \cap G(\varepsilon) & \text{if} \quad \varepsilon \in A \cap B \end{cases}$$

Note: The definition of intersection of two soft sets given by Maji *et al*. (2003) was not correct because they defined H (ε) = F (ε) or G (ε). This was pointed out by Ali *et al*. (2009). In fact also gave the definition of restricted intersection.

Definition 10: If (F, A) and (G, B), are two soft sets, then (F, A) and (G, B), denoted by (F, A) ∧ (G, B), is defined by:

$$(F, A) \wedge (G, B) = (H, A \times B)$$

where $H(\alpha, \beta) = F(\alpha) \cap G(\beta) \forall (\alpha, \beta) \in A \times B$.

Definition 11: If (F, A) and (G, B) are two soft sets, then (F, A) OR (G, B) denoted by (F, A) ∧ (G, B), is defined by:

$$(F, A) \wedge (G, B) = (H, A \times B)$$

where $H(\alpha, \beta) = F(\alpha) \cup G(\beta) \forall (\alpha, \beta) \in A \times B$.

Multiparameterized soft set: We introduce the definition of a multiparameterized soft set and its basic operations, namely complement, union, intersection and and OR. We give examples for these concepts. Basic properties of the operations are derived.

Let U be a universe set and E_i be a set of parameters for $i \in I$ such that $\bigcap_{i \in I} E_i = \varnothing$. Let P (U) denote the power set of $U, E = \bigcup_{i \in I} E_i, P(E)$ denotes the power set of E and $A \subset P(E)$.

Definition 12: A pair (F, A) is called a multiparameterized soft set over U where F is a mapping given by F: A → P (U). In other words, a multiparameterized soft set over U is a parameterized family of subsets of the universe U for $\varepsilon \in A$, F (ε) may be considered as the set of ε-approximate sets of the multiparameterized soft set (F, A).

Example 2: Let us consider a multiparameterized soft set (F, E) which describes the ''attractiveness of houses in the state of Selangor, Malaysia'' that Mr. X is considering for purchase. Suppose that there are ten houses in the universe:

$$U = \{h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8, h_9, h_{10}\}$$

Under consideration and that $E_i = \{E_1, E_2, E_3\}$ is a set of decision parameters. Let E_1 be a set of cost parameters given by $E_1 = \{e_{1,1}$ = expensive, $e_{1,2}$ =

cheap, $e_{1,3}$ very expensive, $e_{1,4}$ = very cheap} E_2 is a set of location parameters given by E_2 = {$e_{2,1}$ = Subang Jaya, $e_{2,2}$ = Klang, $e_{2,3}$ = Ampang, $e_{2,4}$ = Shah Alam, $e_{2,5}$ = Kajang} and E_3 is a set of color parameters given by E_3 = {$e_{3,1}$ = green, $e_{3,2}$ = dark green, $e_{3,3}$ = blue, $e_{3,5}$ = red}. Let $E = \bigcup_i E_i$ and $A \subseteq P(E)$ such that:

$$A = \{a_1 = \{e_{1,1}\},\ a_2 = \{e_{1,1},\ e_{2,1}\},\ a_3 = \{e_{1,1},\ e_{2,1},\ e_{3,1}\},\ a_4 = \{e_{1,2}\}, a_5 = \{e_{1,2},\ e_{2,1},\ e_{3,3}\},\ a_6 = \{e_{1,4},\ e_{2,4}\},\ a_7 = \{e_{1,2},\ e_{3,2}\}\}.$$

Suppose that F (α_1) = {h_2, h_3, h_6, h_7, h_{10}}, F (α_2) = {h_6, h_7}, F (α_3) = $\varnothing$ F (α_4) = {h_1, h_8, h_9}, F (α_5) = {h_8}, F (α_6) = $\varnothing$ and F (α_7) = {h_9} Then we can view the multiparameterized soft set (F, A) as consisting of the following collection of approximations:

$$(F,A) = \{(a_1,\ \{h_2,h_3,h_6,h_7,h_{10}\}),\ (a_2,\ \{h_6,h_7\}),\ (a_3,\ \varnothing),(a_4,\ \{h_1,h_8,h_9\}),\ (a_5,\ \{h_8\}),\ (a_6,\ \varnothing),\ (a_7,\ \{h_9\})\}.$$

Each approximation has two parts: a predicate and an approximate value set.

Definition 13: For two multiparameterized soft sets (F, A) and (G, B) over U, (F, A) is called a multiparameterized soft subset of (G, B) if:

- $A \subset B$ and
- $\forall \varepsilon \subset A$, F ($\varepsilon$) and G ($\varepsilon$) are identical approximations

This relationship is denoted by $(F,A) \tilde{\subset} (G,B)$. In this case (G, B) is called a multiparameterized soft superset of (F, A) denoted by $(G,B) \tilde{\supset} (F,A)$.

Definition 13: Two multiparameterized soft sets (F, A) and (G, B) over a common universe U are said to be multiparameterized soft equal if (F, A) is a multiparameterized soft subset of (G, B) and (G, B) is a multiparameterized soft subset of (F, A).

Example 3: Let A = {α_1, α_4, α_7} and B = {α_1, α_2, α_3, α_4, α_5, α_7} $\subset$ P (E) clearly, $A \subset B$. Let (F, A) and (G, B) be two multiparameterized soft sets over the same universe U = {h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8, h_9, h_{10},} such that (F, A) = {(α_1, {h_2, h_6, h_{10}}), (α_4 {h_1, h_8}), (α_7, {h_9})} and:

$$(G,B) = \{(a_1,\{h_2,h_3,h_6,h_7,h_{10}\}),\ (a_2,\{h_6,h_7\}),\ (a_3,\varnothing),\ (a_4,\{h_1,h_8,h_9\}),(a_5,\{h_8\}),\ (a_6,\varnothing),\ (a_7,\{h_9\})\}$$

Therefore $(F,A) \tilde{\subset} (G,B)$

Definition 14: Let P (E) be a power set of param- eters. The not set of P (E) denoted by $\neg P(E)$, is defined by $\neg P(E) = \{\neg a_1, \neg a_2, \neg a_3, ..., \neg a_n\}$ where $\neg a_i$ = not $\alpha_i, \forall_i$.

Example 4: Consider the example as presented in Example 2. The:

$$\neg A = \{\neg a_1 = \{\neg e_{1,1}\}, \neg a_2 = \{\neg e_{1,1}, \neg e_{2,1}\},\ \neg a_3 = \{\neg e_{1,1}, \neg e_{2,1}, \neg e_{3,1}\},\ \neg a_4 = \{\neg e_{1,2}\},\ \neg a_5 = \{\neg e_{1,2}, \neg e_{2,1}, \neg e_{3,3}\},\ \neg a_6 = \{\neg e_{1,4}, \neg e_{2,4}\},\ \neg a_7 = \{\neg e_{1,2}, \neg e_{3,2}\}\}.$$

Definition 15: The complement of amultipara- meterized soft set (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, \neg A)$ where $F^c : \neg A \rightarrow P(U)$ is mapping given by $F^c(\alpha) = U - F(\neg\alpha), \forall \alpha \subset \neg A$.

Example 5: Consider the example as presented in Example 2. Then we have:

$$(F,\ A)^c = \{(\neg a_1, \{F(\neg a_1)\}), (\neg a_2, \{F(\neg a_2)\}),\ (\neg a_3, \{F(\neg a_3)\}), (\neg a_4, \{F(\neg a_4)\}),\ (\neg a_5, \{F(\neg a_5)\}), (\neg a_6, \{F(\neg a_6)\}),\ (\neg a_7, \{F(\neg a_7)\})\}.$$

Definition 16: A multiparameterized soft set (F, A) over U is said to be a nullmultipara- meterized soft set denoted by Φ if $\forall \varepsilon \in A$, F ($\varepsilon$) = $\varnothing$ (null-set).

Example 6: Suppose that there are ten houses in the universe U, where U is the set of red and expensive houses under consideration in Kajang. Let E_1, E_2, E_3 be sets of cost, locations and colours parameters respectively where E_1 = {$e_{1,1}$ = Expensive, $e_{1,2}$ = cheap, $e_{1.3}$ = very expensive, $e_{1.4}$ = very cheap}, E_2 = {$e_{2,1}$ = Subang Jaya, $e_{2,2}$ = Klang, $e_{2,3}$ = Ampang, $e_{2,4}$ = Shah

Alam, $e_{2,5}$ = Kajang} E_3 = {$e_{3,1}$ = Green, $e_{3,2}$ = dark green, $e_{3,3}$ = blue, $e_{3,4}$ = light blue. Let $E = \bigcup_i E_i$ and A $\subseteq$ P (E) such that:

$$A = \{a_1 = \{e_{3,1}\},\ a_2 = \{e_{1,1},\ e_{2,1},\ e_{3,3}\},\ a_3 = \{e_{1,1},\ e_{2,1},\ e_{3,1}\},\ a_4 = \{e_{1,2},\ e_{3,2}\},\ a_5 = \{e_{1,2},\ e_{2,1},\ e_{3,3}\},\ a_6 = \{e_{1,4},\ e_{3,1}\},\ a_7 = \{e_{1,2},\ e_{3,2}\}\}.$$

The multiparameterized soft set (F, A) is the collection of approximations as given below:

$$F(\alpha_1) = \varnothing,\ F(\alpha_2) = \varnothing, \ldots,\ F(\alpha_7) = \varnothing$$

Thus (F, A) is a null multiparameterized soft set.

Definition 17: A multiparameterized soft set (F, A) over U is said to be an absolute multipar- ameterized soft set denoted by $\tilde{A}$, if $\forall \varepsilon \in A, F(\varepsilon) = U$.

Example 7: Suppose that there are ten houses in the universe U where U is the set of red and expensive houses under consideration in Kajang. Let E_1 be a set of cost parameters such that E_1 = {$e_{1,1}$ = expensive, $e_{1,2}$ = cheap, $e_{1,3}$ very expensive, $e_{1,4}$ = very cheap} E_2 is a set of location parameters such that E_2 = {$e_{2,1}$ = Subang Jaya, $e_{2,2}$ = Klang, $e_{2,3}$ = Ampang, $e_{2,4}$ = Shah Alam, $e_{2,5}$ = Kajang} and E_3 is a set of colour parameters such that E_3 = {$e_{3,1}$ = green, $e_{3,2}$ = dark green, $e_{3,3}$ = blue, $e_{3,4}$ light blue, $e_{3,5}$ = red} Let $E = \bigcup_i E_i$ and A $\subseteq$ P (E), such that:

$$A = \{a_1 = \{e_{1,1}\},\ a_2 = \{e_{1,1},\ e_{2,5}\},\ a_3 = \{e_{1,1},\ e_{2,5},\ e_{3,5}\},\ a_4 = \{e_{2,5},\ e_{3,5}\},\ a_5 = \{e_{1,1},\ e_{3,5}\},\ a_6 = \{e_{3,1}\},\ a_7 = \{e_{2,5}\}\}.$$

The multiparameterized soft set (F, A) is the collection of approximations as given below:

$$F(a_1) = U, F(a_2) = U, \ldots, F(a_7) = U$$

Then (F, A) is an absolute multiparameterized soft set.

Definition 18: The union of two multiparamet- erized soft sets (F, A) and (G, B) over a common universe U is the multiparameterized soft set (H, C) where $C = A \cup B$, and $\forall \varepsilon \in C$:

$$H(\varepsilon) = \begin{cases} F(\varepsilon) & \text{if} \quad \varepsilon \in A - B, \\ G(\varepsilon) & \text{if} \quad \varepsilon \in B - A, \\ F(\varepsilon) \cup G(\varepsilon) & \text{if} \quad \varepsilon \in A \cap B. \end{cases}$$

Example 8: Consider the example as presented in Example 3.1. Let (F, A) and (G, B) be two multiparameterized soft sets over the same universe U = {$h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8, h_9, h_{10}$,} such that:

$$A = \{a_1 = \{e_{1,1}\},\ a_2 = \{e_{1,1},\ e_{2,1}\},\ a_3 = \{e_{1,1},\ e_{2,1},\ e_{3,1}\},\ a_4 = \{e_{1,2}\},\ a_5 = \{e_{1,2},\ e_{2,1},\ e_{3,3}\},\ a_6 = \{e_{1,4},\ e_{2,4}\},\ a_7 = \{e_{1,2},\ e_{3,2}\}\},$$

$$B = \{b_1 = \{e_{1,1},\ e_{2,1}\},\ b_2 = \{e_{1,3}\},\ b_3 = \{e_{1,4},\ e_{2,4}\},\ b_4 = \{e_{1,1},\ e_{2,2},\ e_{3,3}\},\ b_5 = \{e_{1,2},\ e_{3,2}\}\}.$$

Suppose that F (α_1) = U = {h_1, h_3, h_6, h_7, h_{10}}, F (α_2) = {h_6, h_7}, F (α_3) =$\varnothing$, F (α_4) = {h_1, h_8, h_9,}, F (α_5) = {h_8}, F (α_6) =$\varnothing$, F (α_7) = {h_9} and G (b_1) = {h_2, h_4, h_6, h_8,}, G (b_2) = {h_{10}}, G (b_3) =$\varnothing$, G (b_4) = {h_4, h_6} and G (b_5) = {h_8, h_9}. Then:

$$C = A \cup B = \{c_1 = \{e_{1,1}\},\ c_2 = \{e_{1,1},\ e_{2,1}\},\ c_3 = \{e_{1,1},\ e_{2,1},\ e_{3,1}\},\ c_4 = \{e_{1,2}\},\ c_5 = \{e_{1,2},\ e_{2,1},\ e_{3,3}\},\ c_6 = \{e_{1,4},\ e_{2,4}\},\ c_7 = \{e_{1,2},\ e_{3,2}\},\ c_8 = \{e_{1,3}\},\ c_9 = \{e_{1,1},\ e_{2,2},\ e_{3,3}\}\}.$$

Hence the union of the two multiparameterized soft sets (F, A) and (G, B) over a common universe U is:

$$(H,C) = \{(c_1,\{h_2,h_3,h_6,h_7,h_{10}\}),\ (c_2,\{h_2,h_4,h_6,h_7,h_8\}),\ (c_3,\varnothing),\ (c_4,\{h_1,h_8,h_9\}),\ (c_5,\{h_8\}),\ (c_6,\varnothing),\ (c_7,\{h_8,h_9\}),\ (c_8,\{h_{10}\}),\ (c_9,\{h_4,h_6\})\}.$$

Definition 19: The intersection of two multipara- meterized soft sets (F, A) and (G, B) over a common universe U is the soft set (H, C) where $C = A \cup B$, and $\forall \varepsilon \in C$:

$$H(\varepsilon)=\begin{cases}F(\varepsilon) & \text{if } \varepsilon\in A-B,\\ G(\varepsilon) & \text{if } \varepsilon\in B-A,\\ F(\varepsilon)\cap G(\varepsilon) & \text{if } \varepsilon\in A\cap B.\end{cases}$$

Example 9: Let (F, A) and (G, B) be two multi-parameterized soft sets over the same universe U as given in Example 8. Then:

$$\begin{aligned}C=A\cup B=\{&c_1=\{e_{1,1}\},\ c_2=\{e_{1,1},\ e_{2,1}\},\\ &c_3=\{e_{1,1},\ e_{2,1},\ e_{3,1}\},\ c_4=\{e_{1,2}\},\\ &c_5=\{e_{1,2},\ e_{2,1},\ e_{3,3}\},\\ &c_6=\{e_{1,4},\ e_{2,4}\},\ c_7=\{e_{1,2},\ e_{3,2}\},\\ &c_8=\{e_{1,3}\},\ c_9=\{e_{1,1},\ e_{2,2},\ e_{3,3}\}\}.\end{aligned}$$

Hence the intersection of (F, A) and (G, B) over U is:

$$\begin{aligned}(O,C)=\{&(c_1,\ \{h_2,h_3,h_6,h_7,h_{10}\}),\\ &(c_2,\ \{h_6\}),\ (c_3,\ \varnothing),\ (c_4,\ \{h_1,h_8,h_9\}),\\ &(c_5,\ \{h_8\}),\ (c_6,\ \varnothing),\ (c_7,\ \{h_9\}),\\ &(c_8,\ \{h_{10}\}),\ (c_9,\ \{h_4,h_6\})\}.\end{aligned}$$

We now derive the following properties of union and intersection of multiparameterized soft sets.

Proposition 1: If (F, A), (G, B) and (H, C) are three multiparameterized soft sets over U then:

- $(F,A)\cup(G,B)=(G,B)\cup(F,A)$
- $(F,A)\cap(G,B)=(G,B)\cap(F,A)$
- $(F,A)\cup((G,B)\cup(H,C))=((F,A)\cup(G,B))\cup(H,C)$
- $((F,A)\cap(G,B))\cap(H,C)\ ((F,A)\cap(G,B))\cap(H,C)$
- $((F,A)\cup(G,B))\cap((F,A)\cup(H,C))$
  $((F,A)\cup(G,B))\cap((F,A)\cup(H,C))$
- $((F,A)\cap(G,B))\cup((F,A)\cap(H,C))$
  $((F,A)\cap(G,B))\cup((F,A)\cap(H,C))$

Proof: Straightforward from definitions 18 and 19.

Definition 20: If (F, A) and (G, B) are two multiparameterized soft sets, then "(F, A) AND (G, B)" denoted by (F, A) $\wedge$ (G, B) is defined by, (F, A) $\wedge$ (G, B) = (H, A×B) where $H(\alpha,\beta)=F(\alpha)\cap G(\beta)\ \forall\ (\alpha,\beta)\in A\times B$.

Example 10: Let us consider multiparameterized soft sets (F, A) and (G, B) which describes the ''attractiveness of houses in the state of Selangor, Malaysia'' that Mr. X and Mr. Y considering for purchase respectively over the same universe U = {h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8, h_9, h_{10},} such that A = {α_1 = {$e_{1,1}$}, α_2 = {$e_{1,1}$, $e_{2,1}$}, α_3 ={$e_{1,1}$, $e_{2,1}$, $e_{3,1}$} B = {b_1 = {$e_{1,1}$, $e_{2,1}$}, b_2 = {$e_{1,3}$}, b_3 = {$e_{1,4}$, $e_{2,4}$}}.

Suppose that F (α_1) = {h_2, h_3, h_6, h_7, h_{10}}, F (α_2) = {h_6, h_7} and F (α_3) =∅ and G (b_1) = {h_2, h_4, h_6, h_8}, G (b_2) = {h_{10}} and G (b_3) =∅, Then:

$$\begin{aligned}A\times B=\{&(a_1,b_1),\ (a_1,b_2),\ (a_1,b_3),\ (a_2,b_1),\ (a_2,b_2),\\ &(a_2,b_3),(a_3,b_1),(a_3,b_2),(a_3,b_3)\}\end{aligned}$$

Hence the (F, A) $\wedge$ (G, B) over a common universe U is:

$$\begin{aligned}(H,A\times B)=\{&((a_1,b_1),\{h_2,h_6\}),\ ((a_1,b_2),\{h_{10}\}),\\ &((a_1,b_3),\varnothing),\ ((a_2,b_1),\{h_6\})\\ &((a_2,b_2),\varnothing),\ ((a_2,b_3),\varnothing),\\ &((a_3,b_1),\varnothing),((a_3,b_2),\varnothing),\\ &((a_3,b_3),\varnothing)\}.\end{aligned}$$

Definition 21: If (F, A) and (G, B) are two mult-iparameterized soft sets, then "(F, A) OR (G, B)" denoted by (F, A) $\vee$ (G, B) is defined by:

$$(F, A) \vee (G, B) = (H, A \times B)$$

where $H(\alpha,\beta)=F(\alpha)\cup G(\beta)\forall(\alpha,\beta)\in A\times B$.

Example 11: Let us consider multiparameterized soft sets (F, A) and (G, B) which describes the ''attractiveness of houses in the state of Selangor, Malaysia'' that Mr. X and Mr. Y considering for purchase respectively over the same universe U = {h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8, h_9, h_{10},} such that A = {α_1 = {$e_{1,1}$}, α_2 = {$e_{1,1}$, $e_{,2,1}$}, α_3 = {$e_{1,1}$, $e_{2,1}$, $e_{3,1}$}} B = {$e_{1,1}$, $e_{2,1}$}, b_2 = {$E_{1,3}$}, B_3 = {$E_{1,4}$, $E_{2,4}$}}. Suppose that F (α_1) = {h_2, h_3, h_6, h_7, h_{10}}, F (α_2) = {H_6, H_7} and F (α_3) =∅ and G (b_1) = {h_2, h_4, h_6, h_8}, G (b_2) = {H_{10}} and G (b_3) =∅, Then:

$$\begin{aligned}A\times B=\{&(a_1,b_1),\ (a_1,b_2),\ (a_1,b_3),\ (a_2,b_1),\ (a_2,b_2),\\ &(a_2,b_3),(a_3,b_1),(a_3,b_2),(a_3,b_3)\}\end{aligned}$$

Hence the (F, A) $\vee$ (G, B) over a common universe U is (H, A $\times$ B):

$$\{((a_1,b_1),\{h_2, h_3, h_4, h_6, h_7, h_8, h_{10}\}),$$
$$((a_1,b_2),\{h_2, h_3, h_6, h_7, h_{10}\}),$$
$$((a_1,b_3),\{h_2, h_3, h_6, h_7, h_{10}\}),$$
$$((a_2,b_1),\{h_2, h_4, h_6, h_7, h_8\}),$$
$$((a_2,b_2),\{h_6, h_7, h_{10}\}), ((a_2,b_3),\{h_6, h_7\}),$$
$$((a_3,b_1),\{h_2, h_4, h_6, h_8\}),$$
$$((a_3,b_2),\{h_{10}\}), ((a_3,b_3),\varnothing)\}.$$

Proposition 2: If (F, A) and (G, B) are two multiparameterized soft sets over U then:

- $((F,A)\wedge(G,B))^c = (F,A)^c \vee (G,B)^c$
- $((F,A)\vee(G,B))^c = (F,A)^c \wedge (G,B)^c$

Proof: See Maji *et al*. (2003).

Proposition 3: If (F, A), (G, B) and (H, C) are three multiparameterized soft sets over U, then:

- $(F,A)\wedge((G,B)\wedge(H,C)) = ((F,A)\wedge(G,B))\wedge(H,C),$
- $(F,A)\vee((G,B)\vee(H,C)) = ((F,A)\vee(G,B))\vee(H,C),$
- $(F,A)\vee((G,B)\wedge(H,C)) =$
  $((F,A)\vee(G,B))\wedge((F,A)\vee(H,C)),$
- $(F,A)\wedge((G,B)\vee(H,C)) =$
  $((F,A)\wedge(G,B))\vee((F,A)\wedge(H,C)).$

Proof: Straightforward from definitions 20 and 21.

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