

Dissecting Two Approaches to Energy Prices

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Abstract: Problem statement: This research tested the viability of Geometric Brownian Motion as a stochastic model of oil prices. **Approach:** Using autoregressions and unit root tests, we determined that oil prices tend not to exhibit the Markov Property and thus GBM may be a problematic model. **Results:** Instead, oil prices seem to be mean reverting over the long run, possibly following an Ornstein-Uhlenbeck process. **Conclusion/Recommendations:** To determine whether or not OPEC was the cause of mean reversion, we repeated the tests after controlling for quotas, only to find the same results did not apply over the short run.

Key words: Markov property, commodity market, Ornstein-Uhlenbeck (OU), energy prices, quota periods, stochastic processes

INTRODUCTION

Petroleum constitutes more than 40 percent of the world's energy consumption, making it the single largest source of energy used on the planet (International Energy Agency Key World Energy Statistics, 2006). As a consequence, fluctuations in the price of crude oil can have significant ramifications for the world's economic activity.

Unlike markets for stocks and bonds, commodity markets typically include homogenous products. Many scholars have argued that this is one reason cartels can manipulate the price of oil (John, 2009; Kaufmann *et al.*, 2008). The Organization of the Petroleum Exporting Countries (OPEC) is a group of twelve oil exporting countries that collectively hold 79 percent of the world's crude oil reserves and 44 percent of the world's crude oil production.

Some however argue that OPEC lacks solidarity and is unable to enforce the policies needed to maintain control over oil prices (Bassam, 2007; Reynolds and Pippenger, 2010). Ultimately, the degree of OPEC's power remains an unresolved question, but statistical analysis of market data may yet provide additional insight. While economists and political scientists weigh the merits of these competing points of view, many mathematical models have been developed to help characterize the behavior of these dynamic markets.

Since the chaotic and random nature of financial markets make deterministic models of oil markets untenable, most scholars have turned to stochastic analysis as an alternative. Specifically for the price of

oil, two stochastic processes have emerged as leading models: Geometric Brownian Motion (GBM) and the Ornstein-Uhlenbeck (OU) process. Although many variations exist, these two core stochastic processes define the current discussion regarding the nature of the world's oil market.

This essay seeks to provide insight into the extent of OPEC's market power. If OPEC were to regularly manipulate oil prices, we might expect empirical data to reveal symptoms of such manipulation. If oil price changes were truly random as implied by the GBM model, it could be evidence of OPEC's inability to exert influence on world markets. Alternatively, if data revealed strong patterns as implied by the OU process, it could indicate a unique market structure with price-fixing behavior.

MATERIALS AND METHODS

Daily crude oil spot prices were obtained from January 1, 1986 to September 21, 2010 for a total of 6236 observations available at the United States Energy Information Administration's website (United States Energy Information Administration, 2011). The starting date is important, as 1985 marked the end of OPEC's direct price control mechanisms and the beginning of the current system of OPEC's production quotas. Furthermore, we obtained data on OPEC production quotas from OPEC's statistics webpage, including the dates that new quotas were instituted as well as the amount that each quota specified for each country (Organization of the Petroleum Exporting Countries).

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In order to evaluate GBM as a stochastic model of oil spot prices, we ask the following two questions:

- Are daily changes in oil price log-normally distributed?
- Are changes in the price of oil independently distributed from previous price changes?

Although we are examining continuous time stochastic processes, our empirical analysis will be limited to discrete stochastic processes, seeing as we only are able to test data in discrete time. We use the notion that we can test one-unit increments rather than every point along the process and still get solid results. For example instead of comparing dP_t and dP_{t-1} we compare $P_t - P_{t-1}$ and $P_{t-1} - P_{t-2}$.

RESULTS

Testing data for lognormality: Kurtosis Analysis. We employed various methods to test for a lognormal distribution. Our first step was to divide each day's price by the previous day's price, giving us a ratio (P_t/P_{t-1}). Rather than directly test for lognormality, we take an indirect approach by taking the natural logarithm of these ratios and obtaining daily log price changes. These log price changes can then be evaluated against a normal distribution to test if the unmodified price changes are log-normally distributed. The mean of the log-data change was 0:000168, with a standard deviation of 0:0263. By subtracting the mean and dividing by the standard deviation, we standardize the data, allowing us to use the standard normal distribution for comparison.

After evaluating the data, we isolate two arguments that log-changes in oil prices do not follow a normal distribution. The first is that the data exhibits strong leptokurtosis. Leptokurtosis is characterized by fatter tails than would be expected in a normal distribution. With regards to oil prices, this means that wide daily fluctuations in oil prices are more likely to occur than would be predicted by GBM.

The second piece of evidence against the log normal distribution is more direct. We evaluated data in MATLAB's Kolmogorov Smirnov test, to test the null hypothesis that the log-data was normally distributed. The test rejected the null hypothesis at $\alpha = 0:001$, the lowest available for the Kolmogorov Smirnov test.

After considering these two factors, it seems unrealistic that oil price changes are well modeled by a Lognormal distribution. This provides substantial reason to believe that Geometric Brownian motion is a

poor fit for modeling oil prices. Although the distribution of oil price changes fails to meet the Lognormal distribution, this does not give us any inherent reason to believe that oil prices are mean reverting, or would follow an Ornstein-Uhlenbeck process. Further analysis is required to confirm or reject this hypothesis.

Testing data for increment independence: Autoregression analysis: The next important aspect of Geometric Brownian motion that we seek to test is the independence of increments. Unlike the log normality analysis, determining whether or not oil price changes are independent of previous changes is important in not only determining if Geometric Brownian motion is a poor model, but also if the Ornstein-Uhlenbeck process is viable.

First of all, since we only have discrete time data, we must discuss the discrete time analog to the Ornstein-Uhlenbeck process. Rather than using a differential equation, we must use a difference equation.

An autoregressive model of order p describes the difference equation (1):

$$X_t = c + \sum_{i=1}^p \phi_i X_{t-i} + \varepsilon_t \quad (1)$$

Where:

X_t = The price change at time t

c = A constant intercept

ε_t = Our white noise error term

Like all difference equations, we argue that X_t is a function of X_{t-i} and ϕ_i is the coefficient term for the i 'th autoregressor. Since this equation is linear, we can test it empirically by running a series of ordinary least squared linear regressions.

Assuming oil prices follow Eq. 1, we estimate the parameters by taking each day's ratio of prices and evaluating them with five ordinary least squared autoregressions, ranging from first order to fifth. The results are in Table 1, where:

- Asterisk denotes statistically significant t-statistics at $\alpha = 0:05$
- Autoregression based on 6231 observations

The results are striking. The sheer number of statistically significant coefficients indicates that price changes in period $t-1$ and $t-2$ will likely determine the behavior of a price change in period t . Furthermore, the magnitudes of these t-statistics are almost all over 3:2,

Table 1: Autoregression Results

Autoregression	Variable	Value	t-Statistic
First order	c	0.000785	0.584
	ϕ_1	-0.041100	-3.246*
Second order	c	0.008270	0.616
	ϕ_1	-0.043200	-3.417*
	ϕ_2	-0.052500	-4.145*
Third order	c	0.008080	0.602
	ϕ_1	-0.042100	-3.321*
	ϕ_2	-0.051500	-4.068*
	ϕ_3	0.021900	1.727

indicating significance at the 99:93 percent confidence interval at least. It seems to be a pretty statistically robust pattern that previous price changes impact future ones.

Not only does the autoregression put the last nail into the coffin of Geometric Brownian motion, but it also provides pretty substantial evidence that oil prices do indeed follow an Ornstein-Uhlenbeck process. Note that all of the statistically significant coefficients are negative. This means that if there is a large upward swing in oil prices, the next change is likely to be going down. Since price fluctuations are likely to be followed by movements in the opposite direction, one can see that the arguments for mean-reversion may hold some water.

The second order autoregression provided the most ironclad t-statistics, so we will use the following difference equation as a function of oil price changes:

$$X_t = c + \phi_1 X_{t-1} + \phi_2 X_{t-2} + \varepsilon_t$$

Testing data for mean reversion: Characteristic polynomial root analysis: The final test to determine whether or not oil prices are mean reverting is to determine whether or not all the roots of the data's characteristic polynomial have a modulus less than one. If this criterion is fulfilled, we know that the general solution to the homogenous difference equation will result in geometric decay and thus a mean reverting process.

Since we do not need an initial condition to test for equilibrium stability, we examine our second order linear homogenous difference Eq. 2:

$$X_t = \phi_1 X_t + \phi_2 X_{t-2} \tag{2}$$

By following (Luenberger, 1979), we look for all solutions of the form:

$$X_t = c\lambda^t$$

Substituting and simplifying, we can find the characteristic polynomial. By finding the roots of the

characteristic polynomial we can determine the stability of the system:

$$\lambda^2 - \phi_1\lambda - \phi_2 = 0$$

These steps assume that $c\lambda^t \neq 0$, but if $c\lambda^t = 0$ then we would get $X_t = 0$ for all t. Since we wish to consider other solutions to 2, we assume both $c \neq 0$ and $\lambda \neq 0$, allowing us to divide by $c\lambda^t$.

With our oil price change data, we use the parameters estimated in the second order autoregression $\phi_1 = -0.0432$ and $\phi_2 = -0.0525$ in the characteristic polynomial in order to solve for λ . Using the quadratic formula, we easily obtain $\lambda = -0.0216 \pm 0.228i$. Since both of these roots have a modulus less than one, we can conclude that the difference equation has a stable equilibrium (i.e., the process is mean reverting).

Analysis on the second order difference equation thus confirms our suspicion that an initial condition that happens to occur far away from the average price will not be a stable equilibrium for the process. Since the magnitude of all characteristic polynomial roots is less than one, the process will decay geometrically back to the mean. In other words, should prices diverge too far from the mean, something will push the prices back to a more normal level. The next question we seek to answer is what exactly this "something" is. It is now that we turn our analysis to OPEC and the possible explanations for mean reversion.

DISCUSSION

A brief discussion regarding OPEC: It is a stated policy of OPEC to try and maintain stable energy prices, despite the absence of explicit price setting since 1985 (Organization of the Petroleum Exporting Countries). Starting at the end of 1985, OPEC abandoned price fixing and instead opted for a system of production quotas that remains in place to this day.

Whether or not these quotas are actually having an impact is still unclear. Regardless of quota policy's actual results, the intentions of OPEC seem pretty clear. Rather than allow oil prices to wander around randomly, OPEC has a vested interest in ensuring oil prices gravitate towards a trendline. In mathematical terms it means that OPEC would prefer that oil prices exhibit mean reversion behavior rather than that of Geometric Brownian motion. We know that oil prices do indeed exhibit the mean reversion property, so the next step is to determine whether or not this behavior is a result of OPEC policy.

Structural breaks from 1993-1996: In order to isolate production quotas from other variables, we examine the data from October 1993 until June 1996. This is the longest uninterrupted period for which OPEC maintained a single quota, lasting 693 days. The question is whether or not the mean reverting characteristic of oil prices is a result of OPEC's production quotas or rather outside of OPEC's influence. By isolating price changes during one uninterrupted production quota, we can test to see whether mean reversion occurs due to OPEC policy. We conduct the same procedure on this data set as we did on the full data set, by first running a series of autoregressions to test for increment independence. All autoregressions provided similar results, but to allow for easier comparison with the whole data set, we show just our results for the second order autoregression in Table 2. Note that the coefficient column refers to the π_j corresponding to the P_i , indicating the influence that a previous day's price change has upon the current day's. Interestingly, there were no significant t-statistics. After running similar regressions on other large quota periods, there were identical results. We can conclude that linear difference equations are insufficient for characterizing oil price fluctuations in the short run and especially over periods of one OPEC quota. However, this hardly proves that prices are not mean reverting, it simply requires that we take a step further in our analysis. Since we do not have a reliable difference equation to examine, we need to conduct a unit root test in a different way to finally determine whether oil prices are mean-reverting over short term periods of one OPEC quota.

Augmented Dickey-Fuller tests: Using the statistical package R, we can run an Augmented Dickey-Fuller test on the raw data to determine whether or not there are any roots in the data that could equal one, implying that the process is not subject to exponential decay but rather is marginally stable. After all, that is our fundamental question. If the price of oil falls dramatically in one day, does OPEC have the power to push it back up in a timely manner?

The answer is no, at least according to five separate Augmented Dickey-Fuller tests which were run on the data over the period of one quota. After segmenting the data, none of the Dickey-Fuller tests were able to reject the null hypothesis that there were unit roots in the data at $\alpha = 0.05$. What this implies is that given some stochastic shock in the price of oil, that shock will remain permanent, at least in the short run. We define the short run in this situation to be the period of two years.

Table 2: Second Order Autoregression for One Quota Period

Variable	Coefficient	t-Statistic
Pt-1	0.0561	1.464
Pt-2	0.0238	0.624
Intercept	0.0026	0.185

CONCLUSION

In order to determine how much influence OPEC has on the world's energy markets, we compared two stochastic models for the price of oil: Geometric Brownian motion and the Ornstein-Uhlenbeck process. Ultimately, Geometric Brownian motion was a poor model for a couple of reasons. First, the distribution of oil prices was far more kurtotic than that of a log normal random variable. Further analysis using a Kolmogorov Smirnov test statistic provided ample evidence that changes in oil prices were far from being log-normally distributed. Autoregressions on price changes also shed doubt on the assumption of independent increments, as statistically significant coefficients were abundant. Deciding upon the Ornstein-Uhlenbeck process was not a hard choice, since the roots of the difference equation's characteristic polynomial were far less than one, guaranteeing the exponential decay of the process to a trendline as time progressed.

We then stated that the mean reversion property of oil prices could potentially be a symptom of OPEC's market power. By splitting up the time series data into periods controlled for quotas, we reran the mean reversion tests and found that the same phenomenon did not hold true over the short run. Analysis on multiple quota periods confirmed this result, further weakening the claim that OPEC's quotas are responsible for mean reversion.

In conclusion, oil prices are mean reverting, but only over long periods of time. Rather than the doing of OPEC, mean reversion is likely the result of a standardized commodity market. Unlike the price of stocks, oil is a tangible product for which people are willing to pay only so much. The nature of a commodity ensures that prices may fluctuate, but very high or low prices are unsustainable.

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