

## Fuzzy Parametric Deduction for Material Removal Rate Optimization

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**Abstract: Problem statement:** A general optimization scheme without equipment operations for CNC (computer numerical control) finish turning is deemed to be necessarily developed. **Approach:** In this study, four parameters (cutting depth, feed rate, speed, tool nose runoff) with three levels (low, medium, high) were considered to optimize the Material Removal Rate (MRR) based on  $L^9(3^4)$  orthogonal array. Twenty-seven fuzzy control rules using trapezoid membership function with respective to seventeen linguistic grades for material removal rate were additionally constructed. Considering thirty input and eighty output intervals, the defuzzification using center of gravity was moreover completed. Through the Taguchi experiment, the optimum fuzzy deduction parameters could then be received. **Results:** The confirmation experiment for optimum deduction parameters was furthermore computed within the parameter ranges on an ECOCA-3807 CNC lathe. It is shown that the material removal rate from the fuzzy deduction optimization parameters was significantly advanced comparing to that from the benchmark. **Conclusions:** This study not only proposed a parametric deduction optimization scheme using orthogonal array, but also contributed the satisfactory fuzzy approach to the material removal rates for CNC turning with profound insight.

**Key words:** Computer Numerical Control (CNC), Taguchi method, fuzzy deduction optimization, Material Removal Rate (MRR), CNC turning, deduction parameters, fuzzy linguistic, fuzzy control rules, control factors, design parameters, process parameters

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### INTRODUCTION

Machining operations have been the core of the manufacturing industry since the industrial revolution (Rao, 2006). The existing optimization researches for Computer Numerical Controlled (CNC) turning were either simulated within particular manufacturing circumstances (Sukontanakarn and Parnichkun, 2009; Lin *et al.*, 2001; Haber *et al.*, 2004; Meng *et al.*, 2000) or achieved through numerous frequent equipment operations (Kopac, 2002; Tosun and Ozler, 2004). Nevertheless, these are regarded as computing simulations and the applicability to real world industry is still uncertain. Therefore, a general optimization scheme without equipment operations is deemed to be necessarily developed.

The machining process on a CNC lathe is programmed by speed, feed rate and cutting depth, which are frequently determined based on the job shop experiences. However, the machine performance and the product characteristics are not guaranteed to be acceptable. Therefore, the optimum turning conditions have to be accomplished. It is mentioned that the tool nose run-off will affect the performance of the machining process (Lan, 2010). Therefore, the tool nose run-off is also selected as one of the control factors in this study.

Material removal rate is commonly considered as a major manufacturing goal (Tosun and Ozler, 2004) for turning operations in many of the existing researches. Taguchi method, an experimental design method, has been widely applied to many industries. It can not only optimize quality characteristics through the setting of design parameters, but also reduce the sensitivity of the system performance to sources of variation (Huh *et al.*, 2003; Nalbant *et al.*, 2007; Ismail *et al.*, 2009; Kim *et al.*, 2003). The Taguchi method adopts a set of orthogonal arrays to investigate the effect of parameters on specific quality characteristics to decide the optimum parameter combination. These kinds of arrays use a small number of experimental runs to analyze the quality effects of parameters as well as the optimum combination of parameters.

To achieve the deduction optimization, it is necessary to first describe the dynamic behavior of the system to be controlled. Because of the number, complexity and unclear, vague nature of the variables of the dynamic systems that may influence the decision maker's decision, fuzzy set theory is the most suitable solution (Zadeh, 1965; Zimmerman, 2001). Fuzzy linguistic models permit the translation of verbal expressions into numerical ones (Gungor and Arıkan, 2007). Therefore, the input output

relationship of the process can be described by the collection of fuzzy control rules involving linguistic variables rather than a complicated dynamic mathematical model.

With all the viewpoints above, this study considers four parameters (cutting depth, feed rate, speed, tool nose runoff) with three levels (low, medium, high) to optimize the MRR in CNC finish turning. The fuzzy control rules using trapezoid membership function with respect to seventeen linguistic grades for MRR are additionally constructed. The defuzzification is then quantified using center of gravity and introduced as the S/N (signal-to-noise) ratio to Taguchi experiment and thus the optimum deduction parameters can then be received. This study definitely proposes a fuzzy deduction optimization approach and satisfactory fuzzy linguistic deduction technique for improving material removal rate in CNC turning with profound insight.

### MATERIALS AND METHODS

In this study, the variable quantification and deduction optimization for CNC turning operations are proposed using fuzzy set theory and Taguchi method respectively. They are described as below.

**Fuzzy set theory:** Let  $x$  be an universe of discourse,  $\tilde{A}$  is a fuzzy subset of  $x$  if for all  $x \in X$ , there is a number  $\mu_{\tilde{A}}(x) \in [0,1]$  assigned to represent the membership of  $x$  to  $\tilde{A}$  and  $\mu_{\tilde{A}}(x)$  is called the membership function of  $\tilde{A}$ . A trapezoid fuzzy number  $\tilde{A}$  can be defined by a trip-let  $(a, b, c, d)$  (Fig. 1) (Kaufmann and Gupta, 1991). The membership function is defined as:

$$\mu_{\tilde{A}}(x : a, b, c, d) = \begin{cases} \frac{x-a}{b-a} & a < x \leq b \\ 1 & b < x \leq c \\ \frac{x-d}{c-d} & c < x \leq d \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

In this study, the three most important parameters for the MRR are primarily concluded through literature review. Additionally, twenty-seven fuzzy control rules using trapezoid membership function with respect to seventeen linguistic grades will be constructed following IF-THEN rules.

To eliminate the computation, thirty input and eighty output intervals are considered to prepare the defuzzification. Through Cartesian product, the degree of membership for both input and output can thus be attained as:

$$R = \text{Input} * \text{Output} \quad (2)$$

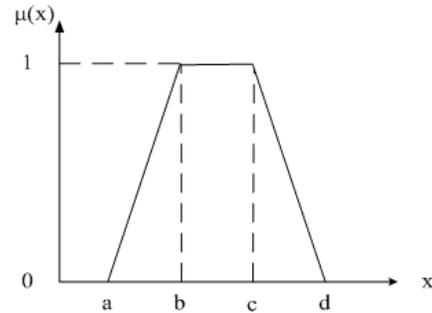


Fig. 1: Trapezoid fuzzy numbers

Here, “Input” describes the parameter, Output represents the attribute and R denotes the fuzzy relation between the parameter and attribute.

The “OR” rules are then utilized for combining rules for maximum degree of membership as:

$$\mu_{R1} + \mu_{R2} = \max \{ \mu_{R1}, \mu_{R2} \} \quad (3)$$

where, 1-R2 symbolize for the two rules.

In this study, the average value using center of gravity is determined to represent the fuzzy set as:

$$F(x_i) = \frac{\sum_i x_i * \mu_{\tilde{A}}(x_i)}{\sum_i \mu_{\tilde{A}}(x_i)} \quad (4)$$

where,  $F(x_i)$  is the final rating of activity,  $\mu_{\tilde{A}}(x_i)$  describes the membership function of fuzzy set  $\tilde{A}$ .

**Taguchi method:** The Taguchi method is a robust design method technique (Palanikumar, 2008; Ross, 1998), which provides a simple way to design an efficient and cost effective experiment. In order to efficiently reduce the numbers of conventional experimental tasks, the orthogonal array (Lan and Wang, 2009a; Wei *et al.*, 2002) by using design parameters (control factors) in column and standard quantities (levels) in row is proposed and further adopted. The performance measure, signal-to-noise ratio (S/N) (Park *et al.*, 2005) proposed by Taguchi is used to obtain the optimal parameter combinations. The larger S/N means the relation to the quality will become better. The lower quality characteristic will be regarded as a better result when considering the smaller-the-best quality. The related S/N ratio is defined as:

$$S/N - 10 \left( \log \sum_{i=1}^n \frac{y_i^2}{n} \right) (dB) \quad (5)$$

Where:

n = Number of experiments for each experimental set

$y_i$  = expresses the quality characteristic at the i-th experiment

On the contrary, the larger quality characteristic will have better result t when considering the larger-the-best quality, therefore, by taking the inverse of quality characteristic into Eq. 5, the related S/N ratio can also be deduced and shown in Eq. 6:

$$S/N - 10 \left( \log \sum_{i=1}^n \frac{1/y_i^2}{n} \right) \text{dB} \quad (6)$$

In this study, the defuzzification result for MRR is introduced to the Taguchi experiment as the S/N ratio. Therefore, it is judged as the quality of smaller-the-best. In addition to the S/N ratio, a statistical Analysis Of Variance (ANOVA) (Wu and Chyu, 2002) can be employed to indicate the impact of process parameters. In this way, the optimal levels of process parameters can be estimated.

**Research design:** Material Removal Rate (MRR) is considered the major quality in this study. Four parameters with three levels are selected to optimize the MRR in finish turning based on the  $L_9(3^4)$  orthogonal array. Additionally, twenty-seven fuzzy control rules with respective to seventeen linguistic grades for the MRR are constructed. Considering thirty input and eighty output intervals, the defuzzification using center of gravity is thus completed for the Taguchi experiment and thus the optimum general deduction parameters can then be received.

**Construction of orthogonal array:** In this study, the four turning parameters (A-speed, B-cutting depth, C-feed rate and D-tool nose runoff) (Lan and Wang, 2009b) with three different levels (low, medium and high) (Table 1) are constructed for the deduction optimization of machining operation. In Table 1, the three levels of speed, cutting depth and feed rate are considered according to the machining handbook suggested by the tool manufacturer. The tool nose runoff is positioned by using different shims located under the tool shank in the tool holder. The orthogonal array is then selected to perform the nine sets of deduction experiments.

The material removal rate can be expressed as  $MRR = 1000fdv$ . As the experimental results (Sukontanakarn and Parnichkun, 2009) the surface speed (v) has the least effect to the MRR. Therefore, the depth of cut (d) and feed rate (f) are considered major parameters for MRR. The seven linguistic grades for MRR are determined as Table 2.

**Defuzzification:** In this study, the three parameter levels are selected based on the Taguchi experimental method, therefore, each triangle membership function is related to the peak point of its fuzzy area. Considering thirty input and eighty output intervals, the defuzzification of seventeen linguistic grades using center of gravity can then be completed.

Table 1: Orthogonal array

Parameter Experiment	A (speed)	B (cutting depth)	C (feed rate)	D (tool nose runoff)
1	Low	Low	Low	Low
2	Low	Medium	Medium	Medium
3	Low	High	High	High
4	Medium	Low	Medium	High
5	Medium	Medium	High	Low
6	Medium	High	Low	Medium
7	High	Low	High	Medium
8	High	Medium	Low	High
9	High	High	Medium	Low

Table 2: Fuzzy rules for material removal rate

Rules	Feed rate	Cutting depth	Speed	Deduction
1	Low	Low	Low	smallest
2	Low	Low	Medium	super small
3	Low	Low	High	small small
4	Low	Medium	Low	super small
5	Low	Medium	Medium	large
6	Low	Medium	High	much smaller
7	Low	High	Low	smaller
8	Low	High	Medium	a little smaller
9	Low	High	High	a little larger
10	Medium	Low	Low	small small
11	Medium	Low	Medium	smaller
12	Medium	Low	High	a little smaller
13	Medium	Medium	Low	much smaller
14	Medium	Medium	Medium	medium
15	Medium	Medium	High	much larger
16	Medium	High	Low	a little larger
17	Medium	High	Medium	larger
18	Medium	High	High	large large
19	High	Low	Low	a little smaller
20	High	Low	Medium	a little larger
21	High	Low	High	larger
22	High	Medium	Low	much larger
23	High	Medium	Medium	large
24	High	Medium	High	super large
25	High	High	Low	large large
26	High	High	Medium	extreme large
27	High	High	High	largest

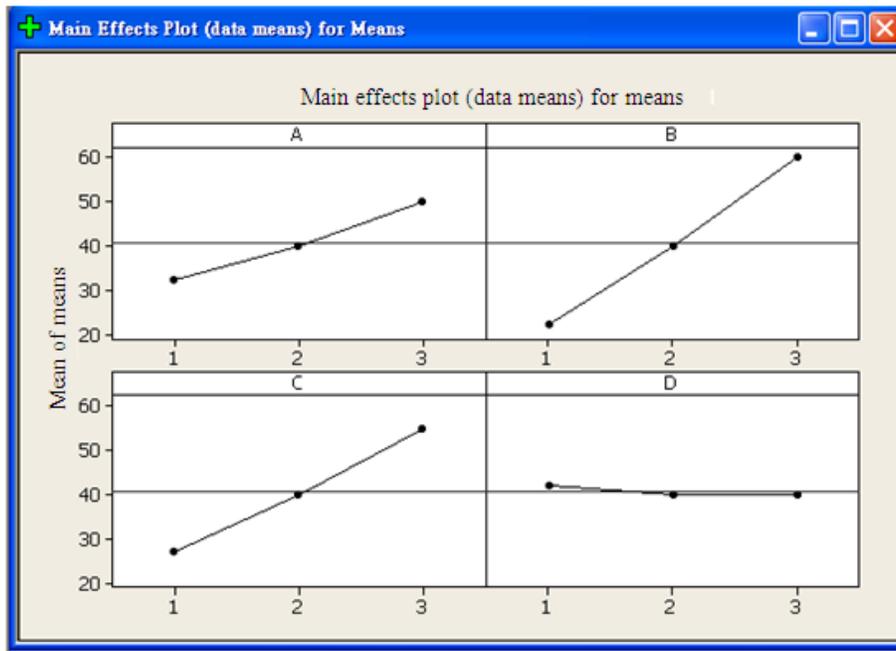


Fig. 2 : Plot of main effects

Table 3: Trapezoid ranges of three linguistic levels for parameters

Linguistic level	Low	Medium	High
Range	[0,12.5,15]	[0,2.5,27.5,30]	[15,17.5,30]

Table 4: Trapezoid ranges of seventeen linguistic levels for attributes

Linguistic level	Smallest	Extreme small	Super small	Small small	Small
Range	[0,2.5,5]	[0,2.5,7.5,10]	[5, 7.5,12.5,15]	[10,12.5,17.5,20]	[15,17.5,22.5,25]
Smaller	Much smaller	A little smaller	Medium	A little larger	Much larger
[20,22.5,27.5,30]	[25,27.5,32.5,35]	[30,32.5,37.5,40]	[35,37.5,42.5,45]	[40,42.5,47.5,50]	[45,47.5,52.5,55]
Larger	Large	Large large	Super large	Extreme large	Largest
[50,52.5,57.5,60]	[55,57.5,62.5,65]	[60,62.5,67.5,70]	[65,67.5,72.5,75]	[70,72.5,77.5,80]	[75,77.5,80]

Table 5: Quantified results for linguistic results

Linguistic level	Smallest	Extreme small	Super small	Small small	Small	Smaller	Much smaller	A little smaller
Defuzzification	1.666667	5	10	15	20	25	30	35
Medium	A little larger	Much larger	Larger	Large	Large large	Super large	Extreme large	Largest
40	45	50	55	60	65	70	75	78.33333

The degree of membership for input (parameter) and output (attribute) can be described as shown in Table 3-4 respectively. Utilizing the average value of the fuzzy set to represent the entire set, we then have the quantified result for the fuzzy item of seventeen linguistic grades as shown in Table 5.

## RESULTS

By considering the parameter combinations of the nine sets of experiment based on the  $L_9(3^4)$  orthogonal array, the quantified results from fuzzy deduction for the MRR are determined and shown as Table 6.

Introducing the deduction results as the Signal to Noise ratio (S/N) for MRR under larger-the-best expectation, the results of factor responses are calculated and listed in Table 7. The mean effects for S/N ratios are then drawn by MINITAB 14 and shown as Fig. 2. Therefore, the optimum fuzzy deduction multi-attribute turning parameters are found to be A (High), B (High), C (High) and D (Low).

The finishing diameter turning operation of S45C ( $\phi 45 \times 250$  mm) work piece on an ECOCA-3807 CNC lathe is arranged for the experiment. The TOSHIBA WTJNR2020K16 tool holder with MITSUBISHI NX2525 insert is utilized as the cutting tool. The four

turning parameters (speed, cutting depth, feed rate and tool nose runoff) with three different levels (low, medium and high) are experimentally distinguished for the machining operation on the basis of  $L_9(3^4)$  orthogonal array. In Table 8, the three levels of speed, cutting depth and feed rate are identified from the machining handbook suggested by the tool manufacturer. The tool nose runoff is positioned by using different shims located under the tool holder and determined by measuring the tip after face turned the work piece. When the tool nose is set approximately 0.1 mm higher (lower) than the center of the work piece, it is regard as “High (Low)”. When the tool nose is set within  $\pm 0.03$ mm, it is considered as “Medium”.

**DISCUSSION**

To verify the applicability of the optimum result achieved by our proposed deduction optimization technique, the machining operations under both fuzzy deduction optimization parameters and benchmark parameters; A (medium), B (medium), C (medium), D (medium), which are often introduced into the confirmation experiment in many of the studies (Tosun and Ozler, 2004; Lin and Lin, 2006) for comparison to the optimum parameters, are performed on the CNC lathe.

Table 6: Fuzzy deduction results

Attributes experiment	MRR
1	1.6667
2	30
3	65
4	20
5	55
6	45
7	45
8	35
9	70

Table 7: Result of factor responses

Parameter level	A	B	C	D
Low	32.22	22.22	27.22	42.22
Medium	40.00	40.00	40.00	40.00
High	50.00	60.00	55.00	40.00
Delta	17.78	37.78	27.78	2.22
Rank	3.00	1.00	2.00	4.00

Table 8: Parameters and levels

Level parameter	Low (Level 1)	Medium (Level 2)	High (Level 3)
A: speed (m/min)	150.0	200.00	250.0
B: cutting depth (mM)	1.0	2.00	3.0
C: feed rate (mm/rev)	0.2	0.30	0.4
D: tool nose runoff (mm)	-0.1	$\pm 0.03$	0.1

Table 9: Confirmation results

	MRR
Fuzzy Deduction (A3B3C3D1)	$0.0375 \text{ mm}^3 \text{ min}^{-1}$
Benchmark (A2B2C2D2)	$0.012 \text{ mm}^3 \text{ min}^{-1}$

The machined results are concluded and listed in Table 9. From Table 9, it is observed that the MRR under fuzzy deduction parameters is significantly improved by 212.50% from the benchmark parameters. It is shown that our proposed fuzzy deduction optimization technique can really advance the MRR for CNC turning.

**CONCLUSION**

In this study, the parametric deduction scheme was proposed and applied to achieve the optimum CNC finish turning parameters under the considerations of MRR. A confirmation experiment of the optimum deduction parameters was calculated to indicate the effectiveness of the proposed fuzzy deduction optimization method. Through the confirmation test, the experimental results validate the potency that the MRR can be advanced from our parametric deduction optimization technique.

Parametric optimization is a hard-solving issue. This study not only proposes a deduction optimization approach using orthogonal array, but also contributes the satisfactory fuzzy technique for improving the MRR in CNC turning with profound insight. The competition of manufacturing industry will then be economically excited through the proposed development in this study.

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