Journal of Mathematics and Statistics 7 (1): 45-50, 2011
ISSN 1549-3644
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# New Family of Exact Soliton Solutions for the Nonlinear Three-Wave Interaction Equations

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**Abstract: Problem statement:** To obtain new exact traveling wave soliton solutions for the three-wave interaction equation in a dispersive medium and a non zero phase mismatch. **Approach:** The tanh method is usually used to find a traveling wave analytic soliton solutions for one nonlinear wave and evolution partial differential equation. Here, we generalize this method to solve a system of nonlinear evolution partial differential equations, then we use this generalization to find new family of exact traveling wave soliton solutions for the nonlinear three-wave interaction equation. **Results:** We were able to generalize the tanh method and apply this generalization to the (TWI) system of (PDE's). We derive a system of algebraic Eq. 28-32 and introduced some interested sets of solutions for this system, these sets of solutions leads us to write explicit analytic new family of soliton solutions for the three-wave interaction equation. **Conclusion:** The generalization of the tanh method is proved its efficiency in obtaining exact solutions for nonlinear evolution partial differential equations. This method also can be used similarly to obtain exact solutions for another interested nonlinear evolution system of partial differential equations.

**Key words:** Soliton solutions, tanh method, three-wave interaction, non zero phase mismatch

## INTRODUCTION

Obtaining an exact solution for a nonlinear equation is considered an interesting problem for mathematicians, so, what if we have a system of nonlinear equations? As an example on those systems is the nonlinear Three-Wave Interaction (TWI) system of Partial Differential Equations (PDE's), which represents a mathematical model for three interacting optics waves? This system describes many physical phenomena, such as, the resonant quadratic nonlinear interaction of three optics waves (Ibragimov and Struthers, 1997), the second harmonic generation process which produces the first coherent or laser light source (Rushchitskii, 1996; Kumar *et al*., 2008) and the study of the model in $x^2$ materials (Chen *et al*., 2004). Many analytic solutions for the (TWI) system were found, such as, the solution of this system when it includes the phase mismatch ($\Delta k$) (Ibragimov *et al*., 2001), the solution of this system when it includes the second order dispersion (Werner and Drummond, 1994; Menyuk *et al*., 1994; Tahar, 2007), the solution of this system when it doesn't include the second order dispersion by the Inverse Scattering Transform method (IST) (Ibragimov *et al*., 1998; Batiha, 2007) and the exact soliton solution found by some ansatz introduced by (Huang, 2000).

In this study we introduce a direct generalization of the tanh method (Wazwaz, 2004) to solve a nonlinear evolution system of (PDE's), then we apply this generalization and get new families of soliton solutions for the (TWI) system of (PDE's) which includes nonzero quadratic dispersion coefficients $\{g_1, g_2, g_3\}$ and a nonzero phase mismatch ($\Delta k$) in Eq. 14. In all of the obtained solutions we mentioned that replacing the tanh function with the coth function will keep our solutions valid.

## MATERIALS AND METHODS

This method is used to solve many nonlinear wave and evolution equations, such as, the soliton solutions found for many forms of the fifth order Kdv equation (Wazwaz, 2007). In using this method, we are looking for a traveling wave solution u(x, y, z, t) to a given nonlinear (PDE) in the form:

$$H(u, u_t, u_x, u_y, y_z, u_{xx}, u_{yy}, u_{zz}, ...) = 0 \quad (1)$$

If we assume that:

$$\theta = c_1 x + c_2 y + c_3 z - \lambda t \quad (2)$$

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where, $c_i$, $\lambda$ are unknown constants to be found, then Eq. 1 will be transformed to an Ordinary Differential Equation (ODE) in the form:

$$F(U, U', U'', \dots) = 0 \tag{3}$$

where, $U(\theta) = u(x, y, z, t)$ and the derivatives appear in (3) are with respect to $\theta$. If we introduce the variable $Y(\theta) = \tanh(\theta)$ then the derivatives appear in (3) become:

$$\begin{aligned} \frac{dU}{d\theta} &= (1 - Y^2)U'(Y) \\ \frac{d^2U}{d\theta^2} &= (-1 + Y^2)(2YU'(Y) + (-1 + Y^2)U''(Y)) \end{aligned} \tag{4}$$

where similar formula for higher derivatives can be obtained as well. The tanh method now admits solutions of the form:

$$U(\theta) = u(x, y, z, t) = S(Y) = \sum_{i=0}^{n} a_i Y(\theta)^i \tag{5}$$

where, n is an unknown positive integer, which can be determined from the resulted equation by balancing the linear and nonlinear terms of the highest orders. After determining n we substitute Eq. 5 in Eq. 3. Doing this, will give us an algebraic coefficients of some powers of $Y(\theta)$, making these coefficients zeros, will produce a system of algebraic equations in $a_i$ and $c_i$, then we solve this system and substitute the result in (2) and (5), to get a solution $u(x, y, z, t)$ for (1).

**Generalization of the tanh method:** This is a direct generalization for the above tanh method. Here, we suppose that we have a system of nonlinear evolution (PDE's) in the form:

$$\vec{H}(t, \vec{X}, u_r(t, \vec{X}), u_{rt}(t, \vec{X}), u_{rx_i}(t, \vec{X}), u_{rx_i x_j}(t, \vec{X}), \dots, u_{rx_i x_j \dots x_m}(t, \vec{X})) = \vec{0} \tag{6}$$

where, $\vec{X} = [x_1 x_2 \dots x_n], u_r(t, \vec{X}), 1 \le r \le n$, are the required solutions and:

$$u_{rx_i x_j \dots x_m}(t, \vec{X}) = \frac{\partial^m u_r(t, \vec{X})}{\partial x_i \partial x_j \dots x_m} \tag{7}$$

If we assume that:

$$\theta = c_1 x_1 + c_2 x_2 + \dots + c_n x_n - \lambda t \tag{8}$$

where, $c_r$, $\lambda$ are unknown constants to be found, then the partial derivatives in (7) becomes:

$$\frac{\partial^m u_r(t, \vec{X})}{\partial x_i \partial x_j \cdots \partial x_m} = c_i c_j \dots c_m \frac{d^m u_r(\theta)}{d\theta^m} \tag{9}$$

if we substitute Eq. 9 in Eq. 6, then Eq. 6 will be transformed to a system of (ODE's) in the form:

$$\vec{F}(u_r(\theta), u_r'(\theta), u_r''(\theta), \dots) = \vec{0} \tag{10}$$

where the derivatives appear in Eq. 10 are with respect to $\theta$. If we assume that:

$$Y(\theta) = \tanh(\theta) \tag{11}$$

then the derivatives appear in (10) are given by:

$$\begin{aligned} \frac{du_r}{d\theta} &= (1 - Y^2)u_r'(Y) \\ \frac{d^2u_r}{d\theta^2} &= (-1 + Y^2)(2Yu_r'(Y) + (-1 + Y^2)u_r''(Y)) \end{aligned} \tag{12}$$

where, several formula for $\frac{d^3u_r}{d\theta^3}, \frac{d^4u_r}{d\theta^4}, \dots$ can be easily obtained as well. The generalized tanh method now admits solutions of the form:

$$u_r(\theta) = u_r(t, x_1, x_2, , x_n) = S_r(Y) = \sum_{i=0}^{n_r} A_{ri} Y(\theta)^i \tag{13}$$

Where:
$n_r$ = Positive integers depend on r
$A_{ri}$ = Constant coefficients (usually $A_{ri}$ are complex numbers) yet to be determined

From the resulted system of (ODE) we determine $n_r$ by balancing the highest nonlinear terms with the highest orders of the given system. After determining $n_r$ we substitute Eq. 13 in Eq. 10, to get a system of equations, where in each individual equation in this system, there will be some algebraic coefficients contains $A_{ri}$ and $c_i$, multiplied by some powers of $Y(\theta)$, setting these coefficients with zero, will give us a system of algebraic equations, doing the same for the rest of the equations and collecting all these algebraic equations, we will get a universal system of algebraic equations in $A_{ri}$ and $c_r$, then we try to find a solution for this universal system, if we are able to find such a solution we substitute it in Eq. 8 and 13, then the result is an explicit analytic formula for the required solution $u_r(\theta) = u_r(t, x_1, x_2, \dots, x_n)$ for the given system of (PDE's) in (6).

**Basic equations:** We will apply the generalized tanh method to the following nonlinear evolution (TWI) system of (PDE's) given by (Ibragimov *et al*., 1998; Faiedh *et al*., 2006):

$$\begin{aligned} &i(\partial_z A_1 + \tfrac{1}{v_1}\partial_t A_1) - g_1\partial_{t,t}A_1 = -\sigma A_3 A_2^* e^{i\Delta kz} \\ &i(\partial_z A_2 + \tfrac{1}{v_2}\partial_t A_2) - g_2\partial_{t,t}A_2 = -\sigma \tfrac{\omega_2}{\omega_1} A_3 A_1^* e^{i\Delta kz} \\ &i(\partial_z A_3 + \tfrac{1}{v_3}\partial_t A_3) - g_3\partial_{t,t}A_3 = -\sigma \tfrac{\omega_3}{\omega_1} A_1 A_2^* e^{-i\Delta kz} \end{aligned} \quad (14)$$

The system in (14) describes three interacting plane waves traveling in the positive z direction in a nonlinear dispersive medium, with the following associated electric fields $E_j(z, t)$ given by:

$$\begin{aligned} &E_j(z,t) = A_j(z,t)e^{i(\omega j = K_{jz})} \\ &+A_j^*(z,t)e^{-i(\omega_j - K_{j^z})}, \quad j = 1, 2 \text{ and } 3 \end{aligned} \quad (15)$$

Where:

| | | |
|---|---|---|
| $A_j(z, t)$ | = | The slowly varying complex-amplitude envelopes of the three waves |
| $\omega_j$ | = | The center frequencies |
| $K_j$ | = | The wave numbers given by $K_j = \frac{n_j\omega_j}{c}$ |
| c | = | The speed of light |
| $n_j$ | = | The refractive indexes |
| $v_j$ | = | The group velocities of the three waves which are in general different from each others |
| $g_j$ | = | The second-order non zero dispersion coefficients |
| σ | = | The nonlinear coupling constant given by $\sigma \approx \frac{2\pi x_{nl}\omega_1^2}{k_1 c^2}$ |
| $X_{nl}$ | = | The nonlinear dielectric susceptibility |
| Δk | = | The phase velocity mismatch given by $\Delta k = K_3 - K_2 - K_1$ |
| * | = | For the complex conjugate, $i^2 = -1$ |

If we assume that:

$$\begin{aligned} &A_{1,2} \to Q_{1,2}, \quad A_3 \to -Q_3^*, g_{1,2}, \quad g_3 \to -G_3, \\ &\sigma \to -p_1, \quad \sigma\tfrac{\omega_2}{\omega_1} \to -p_2, \quad \sigma\tfrac{\omega_3}{\omega_1} \to -p_3 \end{aligned} \quad (16)$$

then the system in (14) will be transformed to the following symmetric form:

$$\begin{aligned} &i(\partial_z Q_1 + \tfrac{1}{v_1}\partial_t Q_1) - G_1\partial_{t,t}Q_1 = -p_1 Q_2^* Q_3^* e^{i\Delta kz} \\ &i(\partial_z Q_2 + \tfrac{1}{v_2}\partial_t Q_2) - G_2\partial_{t,t}Q_2 = -p_2 Q_1^* Q_3^* e^{i\Delta kz} \\ &i(\partial_z Q_3 + \tfrac{1}{v_3}\partial_t Q_3) - G_3\partial_{t,t}Q_3 = -p_3 Q_1^* Q_2^* e^{i\Delta kz} \end{aligned} \quad (17)$$

if we also let (Ibragimov *et al*., 1998):

$$S = (t - \frac{1}{v_1}z), \ \eta = z, \ Q_r = Q_{r0}q_r(S, \eta), \ r = 1, 2, 3 \quad (18)$$

Where:

$$Q_{10} = \frac{-i}{\sqrt{-p_2 p_3}}, Q_{20} = \frac{i}{\sqrt{-p_1 p_3}}, Q_{30} = \frac{1}{\sqrt{-p_1 p_2}} \quad (19)$$

then the system in (17) will be rescaled to become:

$$\begin{aligned} &i(\frac{\partial q_1}{\partial\eta} - \gamma_1\frac{\partial q_1}{\partial S}) + \alpha_1\frac{\partial^2 q_1}{\partial S^2} = q_2^* q_3^* e^{i\Delta k\eta} \\ &i(\frac{\partial q_2}{\partial\eta} - \gamma_2\frac{\partial q_2}{\partial S}) + \alpha_2\frac{\partial^2 q_2}{\partial S^2} = q_1^* q_3^* e^{i\Delta k\eta} \\ &i(\frac{\partial q_3}{\partial\eta} - \gamma_3\frac{\partial q_3}{\partial S}) + \alpha_1\frac{\partial^2 q_3}{\partial S^2} = q_1^* q_2^* e^{i\Delta k\eta} \end{aligned} \quad (20)$$

where, $\alpha_{1,2} = -g_{1,2}$, $\alpha_3 = g_3$, $\gamma_1 = 0$, is added for purposes of obtaining symmetric solutions later, while $\gamma_2$ and $\gamma_3$ which are called the temporal walk-off parameters are given by the formula:

$$\gamma_r = \left(\frac{1}{v_1} - \frac{1}{v_r}\right), r = 2,3$$

**Applying the generalized tanh method:** To apply the generalized tanh method, Eq. 6 is now our system given in (20). To transform the system of (PDE's) in (20) to a system of (ODE's) as in equation (10), we use the following assumptions suggested in (Huang, 2000):

$$q_r(S,\eta) = u_r(\theta)e^{i\theta_r} \quad (21)$$

$$\theta(S, \eta) = \Omega S - K\eta, \ \theta_r(S, \eta) = K_r\eta - \Omega_r S, \ r = 1, 2, 3 \quad (22)$$

where, Ω, K, $K_r$, $\Omega_r$ are unknown real constants to be determined. Using Eq. 21 and 22, the system in (20) will be transformed to the following system of (ODE's):

$$\Phi_1 u_1(\theta) + u_2^*(\theta)u_3^*(\theta) + iX_1 u_1^{'}(\theta) - \tau_1 u_1^{''}(\theta) = 0$$
$$\Phi_2 u_2(\theta) + u_1^*(\theta)u_3^*(\theta) + iX_2 u_2^{'}(\theta) - \tau_2 u_2^{''}(\theta) = 0 \quad (23)$$
$$\Phi_3 u_3(\theta) + u_1^*(\theta)u_2^*(\theta) + iX_3 u_3^{'}(\theta) - \tau_3 u_3^{''}(\theta) = 0$$

Where:

$$\Phi_i = (K_i + \Omega_i(\gamma_i + \alpha_i\Omega_i)),\ X_i = (K + \Omega(\gamma_i + 2\alpha_i\Omega_i))$$
$$\tau_i = \alpha_i\Omega^2,\ i = 1, 2, 3,\ \ K_3 = -(K_1 + K_2 - \Delta k), \quad (24)$$
$$\Omega_3 = -(\Omega_1 + \Omega_2)$$

Notice that from Eq. 24, if later we able to find $\Phi_i$ and $X_i$, this will give us 6 real values for them, then we can easily use these values to find $\Omega_1$, $\Omega_2$, $K_1$, $K_2$, K, $\Omega$. This means that $\tau_i$ which are related by $\tau_1 = \frac{\alpha_1}{\alpha_2}\tau_2 = \frac{\alpha_1}{\alpha_3}\tau_3$ are actually known once we determine $\Omega$. What left now is to determine $n_r$ given in Eq. 13, this can be done by substituting Eq. 12 in Eq. 23, we get:

$$\Phi_1 S_1(Y) + S_2^*(Y)S_3^*(Y) - (-1 + Y^2)((2Y_{\tau 1} + iX_1)$$
$$S_1^{'}(Y) + (-1\_+ Y^2)\tau_1 S_1^{''}(Y)) = 0$$
$$\Phi_2 S_2(Y) + S_1^*(Y)S_3^*(Y) - (-1 + Y^2)((2Y_{\tau 2} + iX_2)$$
$$S_2^{'}(Y) + (-1\_+ Y^2)\tau_2 S_2^{''}(Y)) = 0 \quad (25)$$
$$\Phi_3 S_3(Y) + S_1^*(Y)S_2^*(Y) - (-1 + Y^2)((2Y_{\tau 3} + iX_3)$$
$$S_3^{'}(Y) + (-1\_+ Y^2)\tau_3 S_3^{''}(Y)) = 0$$

The maximum powers appear in Eq. 25 are $2n_r$ and $n_r + 2$, those powers come from the terms $S_i^*(Y)S_j^*(Y)$ and $Y^3 S_r^{'}(Y), Y^4 S_r^{''}(Y)$. If we make these powers equal, then we get $n_1 = n_2 = n_3 = 2$, so Eq. 13 becomes:

$$u_r(\theta)A_r + B_r\tanh(\theta) + C_r\tanh^2(\theta), r = 1,2,3 \quad (26)$$

where, $A_r$, $B_r$, $C_r$ are generally complex numbers yet to be determined. If we substitute Eq. 26 in 23 and use the identity:

$$\tanh^n(\theta) = \tanh^{n-2}(\theta)(1 - \operatorname{sech}^2(\theta)) \quad (27)$$

then we get a system of equations consists of the following algebraic coefficients of some powers of $Y(\theta)$, these coefficients are:

**The constant coefficients:**

$$(A_i + C_i)\Phi_i + B_j^*B_k^* + (A_j^* + C_j^*)(A_k^* + C_k^*) = 0 \quad (28)$$

**The coefficients of tanh(θ):**

$$B_i\Phi_i + B_j^*(A_k^* + C_k^*) + B_k^*(A_j^* + C_j^*) = 0 \quad (29)$$

**The coefficients of sech$^2$(θ):**

$$(4\tau_i - \Phi_i)C_i + iB_iX_i - B_j^*B_k^* - A_k^*C_j^* - (A_j^* + 2C_j^*)C_k^* = 0 \quad (30)$$

**The coefficients of sech$^2$(θ)tanh(θ):**

$$2B_i\tau_i + 2iC_iX_i - B_j^*C_k^* - B_k^*C_j^* = 0 \quad (31)$$

**The coefficients of sech$^4$(θ):**

$$-6C_i\tau_i + C_j^*C_k^* = 0 \quad (32)$$

where, {i, j, k} take the values {1, 2, 3},{2, 1, 3} and {3, 1, 2}. The system (28-32) is a nonlinear system of 15 equations in 15 unknowns, namely $A_r$, $B_r$, $C_r$, $\Phi_r$ and $X_r$.

## RESULTS

We were able to find the following interested sets of solutions for the system in (28-32), however, finding another interested sets of solutions for this system is still an open problem. In all of these founded sets some of the unknowns were arbitrary constants, which mean that the system has infinitely many similar solutions.

**Set 1:** Let r = 1, 2 and 3, choose arbitrary real constants $\Phi_r>0$ and $X_r$, choose $\{\delta_1, \delta_2, \delta_3\}$, such that $\delta_1 + \delta_2 + \delta_3 = 2n\pi$, n is an integer and:

$$B_r = C_r = 0,\ A_1 = -\sqrt{\Phi_2\Phi_3 e^{i\delta 1}}$$
$$A_2 = -\sqrt{\Phi_1\Phi_3 e^{i\delta 2}},\ \ A_3 = -\sqrt{\Phi_1\Phi_2 e^{i\delta 3}} \quad (33)$$

**Set 2:** Let r = 1, 2 and 3, {i, j, k} have the values {1, 2, 3}, {2, 1, 3} and {3, 2, 1}, $\tau_r$ are arbitrary real numbers, choose $\{\delta_1, \delta_2, \delta_3\}$ such that $\delta_1 + \delta_2 + \delta_3 = 2n\pi$, n is an integer and:

$$B_r = 0,\ X_r = 0, C_i = 6\sqrt{\tau_j\tau_k e^{i\delta_i}},\ \ A_r = -C_r, \Phi_r = 4\tau_r \quad (34)$$

Notice that, since $\tau_r = \alpha_r\Omega^2$ are arbitrary real numbers, this means, $\Omega$ is actually chosen to be arbitrary real number.

**Set 3:** Let r = 1, 2 and 3, {i, j, k} have the values {1, 2, 3}, {2, 1, 3} and {3, 2, 1}, $\tau_r$ are arbitrary real numbers, choose $\{\delta_1, \delta_2, \delta_3\}$, such that $\delta_1 + \delta_2 + \delta_3 = 2n\pi$, n is an integer and:

$$B_r = 0,\ X_r = 0,\ C_i = 6\sqrt{\tau_j \tau_k e^{i\delta_i}},\ A_r = \frac{-1}{3}C_r,\ \Phi_r = -4\tau_r \quad (35)$$

**Set 4:** Let r = 1, 2 and 3, {i, j, k} have the values {1, 2, 3}, {2, 1, 3} or {3, 2, 1}, choose arbitrary non zero real numbers $b_r$, $A_1$, $A_2$ and:

$$\begin{aligned}
&C_r = \tau_r = 0,\ B_r = ib_r,\ X_i = \frac{b_i b_k}{b_i}, \\
&A_3 = \frac{-A_1A_2b_3 + b_1b_2b_3}{A_2b_1 + A_1b_2} \\
&\Phi_1 = \frac{(A_2^2 + b_2^2)b_3}{A_2b_1 + A_1b_2},\ \Phi_2 = \frac{(A_1^2 + bl^2)b_3}{A_2bl + A_1b_2}, \\
&\Phi_3 = \frac{A_2b_1 + A_1b_2)b_2}{b_3}
\end{aligned} \quad (36)$$

## DISCUSSION

To find an explicit formula for $Q_r(z, t)$ in (17), we do the following steps:

- Substitute the values of $\Phi_r$, $X_r$ in (24) and solve the resulted equations to get $\Omega$, K, $K_r$, $\Omega_r$
- Substitute the values of $\Omega$, K, $K_r$, $\Omega_r$, in (22) to get $\theta(S, \eta)$, $\theta_r(S, \eta)$
- Substitute the values of $A_r$, $B_r$, $C_r$ in (26) to get $u_r(\theta)$
- Substitute the formula for $u_r(\theta)$, $\theta(S, \eta)$, $\theta_r(S, \eta)$, in (21), to get $q_r(S, \eta)$
- Use (19) and $q_r(S, \eta)$ to write an explicit formula for $Q_r(z, t)$ in (18)

Applying the above steps on Set 1, we get the following solution for (17):

$$\begin{aligned}
Q_1(z,t) &= \sqrt{\frac{\Phi_2\Phi_3}{p_2p_3}e^{\frac{i(K_1v_1z + z\Omega_1 + v_1(\delta_1 - t\Omega_1))}{v_1}}} \\
Q_2(z,t) &= \sqrt{\frac{\Phi_1\Phi_3}{p_1p_3}e^{\frac{i(K_2v_1z + z\Omega_2 + v_1(\delta_2 - t\Omega_2))}{v_1}}} \\
Q_3(z,t) &= \sqrt{\frac{\Phi_1\Phi_2}{-p_1p_2}e^{\frac{i(K_3v_1z + z\Omega_3 + v_1(\delta_3 - t\Omega_3))}{v_1}}}
\end{aligned} \quad (37)$$

Applying the above steps on Set 2, we get the following solution for (17):

$$\begin{aligned}
Q_1(z,t) &= 6\sqrt{\frac{\tau_2\tau_3}{p_2p_3}e^{\frac{i(zK_1v_1 + z\Omega_1 + v_1(\delta_1 - t\Omega_1))}{v_1}}}\ \mathrm{sech}^2(Kz - t\Omega + \frac{z\Omega}{v_1}) \\
Q_2(z,t) &= -6\sqrt{\frac{\tau_1\tau_3}{p_1p_3}e^{\frac{i(zK_2v_1 + z\Omega_2 + v_1(\delta_2 - t\Omega_2))}{v_1}}}\ \mathrm{sech}^2(Kz - t\Omega + \frac{z\Omega}{v_1}) \\
Q_3(z,t) &= -6\sqrt{\frac{\tau_1\tau_2}{-p_1p_2}e^{\frac{i(zK_3v_1z\Omega_3 + v_1(\delta_3 - t\Omega_3))}{v_1}}}\ \mathrm{sech}^2(Kz - t\Omega + \frac{z\Omega}{v_1})
\end{aligned} \quad (38)$$

Applying the above steps on Set 3, we get the following solution for (17):

$$\begin{aligned}
Q_1(z,t) &= -2\sqrt{\frac{\tau_2\tau_3}{p_2p_3}e^{\frac{i(zK_1v_1 + z\Omega_1 + v_1(\delta_1 - t\Omega_1))}{v_1}}}\ (-1 + 3\tanh^2(Kz - t\Omega + \frac{z\Omega}{v_1})) \\
Q_2(z,t) &= 2\sqrt{\frac{\tau_1\tau_3}{p_1p_3}e^{\frac{i(zK_2v_1 + z\Omega_2 + v_1(\delta_2 - t\Omega_2))}{v_1}}}\ (-1 + 3\tanh^2(Kz - t\Omega + \frac{z\Omega}{v_1})) \\
Q_3(z,t) &= 2\sqrt{\frac{\tau_1\tau_2}{-p_1p_2}e^{\frac{i(zK_3v_1z\Omega_3 + v_1(\delta_3 - t\Omega_3))}{v_1}}}\ (-1 + 3\tanh^2(Kz - t\Omega + \frac{z\Omega}{v_1}))
\end{aligned} \quad (39)$$

Applying the above steps on Set 4, we get the following solution for (17):

$$\begin{aligned}
Q_1(z,t) &= \frac{-1}{\sqrt{-p_2}\sqrt{p_3}}e^{\frac{i(K_1v_1z + (-tv_1 + z)\Omega_1)}{v_1}} \\
&(iA_1 + b_1\tanh(Kz - t\Omega + \tfrac{z\Omega}{v_1})) \\
Q_2(z,t) &= \frac{-1}{\sqrt{-p_1}\sqrt{p_3}}e^{\frac{i(K_2v_1z + (-tv_1 + z)\Omega_2)}{v_1}} \\
&(iA_2 + b_2\tanh(Kz - t\Omega + \tfrac{z\Omega}{v_1})) \\
Q_3(z,t) &= \frac{-1}{\sqrt{p_1}\sqrt{p_2}}e^{\frac{i(K_3v_1z + (-tv_1 + z)\Omega_3)}{v_1}} \\
&(\frac{-A_1A_2b_3 + b_1b_2b_3}{A_2b_1 + A_1b_2} - ib_3 + b_1\tanh(Kz - t\Omega + \tfrac{z\Omega}{v_1}))
\end{aligned} \quad (40)$$

Notice that if instead of the assumption in (26), we assume that:

$$u_r(\theta) = A_r + B_r\coth(\theta) + C_r\coth^2(\theta), \quad r = 1,2,3 \quad (41)$$

then we will obtain the same system as in (28-32), so in the solutions (37-40), we can replace tanh with coth and still get a solution.

## CONCLUSION

We were able to generalize the tanh method, then apply this generalization on the Three-Wave-Interaction

(TWI) system of (PDE's). We were able to construct an algebraic nonlinear system of 15 complex equations and found 4 different sets of solutions for this system, then construct 4 new families of analytic traveling wave soliton solutions for the (TWI) system. We are very positive that there are more sets of solutions for our algebraic system, which means that there are more similar family of analytic solutions for the (TWI) system which can be constructed as well.

## ACKNOWLEDGMENT

The researchers would like to thank Dr. Ma'moon Altheeb from Al-Hussein Bin Talal University for his efforts in reading this study and his valuable suggestions.

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