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Magneto Convection in Tilted Square Cavity with Differentially Thermally Active Vertical Walls

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Abstract: Problem statement: The purpose of this study is to study numerically the magnetoconvection in a tilted square cavity with differentially thermally active vertical walls. The two vertical side walls of tilted enclosure are differentially thermally active, each consisting of alternating equal sized hot and cold surface elements facing each other in an opposed manner while the horizontal top and bottom walls are adiabatic. **Approach:** Alternating Direction Implicit (ADI) and Successive over Relaxation methods are applied to solve the reformulated vorticity-stream function equations. **Results:** The results are obtained for a wide range of non dimensional parameters, such as Grashof number from 5000-50000, Hartmann number from 0-10 and angles of inclinations from 0-180° of the enclosure in the counterclockwise direction. The detailed flow structure and the associated heat transfer inside the cavity are presented. **Conclusion/Recommendations:** The average Nusselt number increases with increase in Grashof number but decreases with increase in Hartmann number and behaves in a non-linear fashion with angles of inclination. Further extension of this study could include the effect of aspect ratio, for various Prandtl number and also a study on partially open square cavities with adiabatic walls and a partial opening.

Key words: Numerical analysis, inclination angles, vertical walls, finite difference scheme, relaxation method, external parameters, angles of inclination, grashof number, uniform temperature, stream function, Alternating Direction Implicit (ADI)

INTRODUCTION

Natural convection in rectangular enclosures has numerous engineering applications, among which are electronic packages, solar collectors, thermal design of buildings, storage systems, cooling of nuclear reactors. A subdivision of the natural convection problem in a rectangular cavity is the case where one wall is partially/fully heated and the opposite wall is partially/fully cooled while the other two walls are kept adiabatic. This cavity configuration is of special interest in many engineering applications, such as solar receivers, solar passive design and cooling of electronic equipment.

Azwadi *et al*. (2010) studied the natural convection heat transfer inside an inclined enclosure and observed that the natural convection increased as the inclination angle increased until it reached the critical angle after which the natural convection started to decrease. Munir *et al*. (2011) investigated the natural convection in an inclined square cavity and found that the vortex formation, size and flow characteristics were significantly affected by the magnitude of inclination angles.

Magnetoconvection in square enclosures has numerous engineering applications. Some of them are solidification process to weaken the buoyancy-driven fluctuations, to modify the interface shape and the rates of solidification in the manufacturing process of semi-conductor crystals. Mehmet and Buyuk (2006) studied the steady laminar natural convection flow in the presence of a magnetic field in an inclined enclosure heated from one side and cooled from the adjacent side and observed that the counterclockwise inclination affected the temperature field significantly. Kandaswamy *et al*. (2008) investigated the natural convection in a square cavity filled with an electrically conducting fluid with partially active vertical walls, for nine different combinations of active locations in the presence of external magnetic field parallel to the gravity and observed that the heat transfer is maximum for the middle-middle thermally active locations while it is poor for the top-bottom thermally active locations.

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From the literature survey, it is observed that the analysis of buoyancy driven flow in an inclined square cavity with vertical walls each consisting of alternating equal sized heated and cooled portions facing each other in the presence of magnetic field has not received much attention. Hence, in the present study, an attempt is made to investigate the influence of buoyancy and external magnetic forces in an inclined square enclosure filled with a perfectly electrically conducting fluid. The objective of the present study is to investigate the effect of magnetic field on natural convection in the enclosure for a wide range of Grashof number, Hartmann number at different angles of inclinations.

MATERIALS AND METHODS

In the present study a two dimensional inclined square cavity of length l as shown in Fig. 1a is considered. Initially at time t = 0, the fluid is assumed to be motionless and at a uniform temperature θ_c. The heated portion is kept at a temperature θ_h and the cooled portion at temperature θ_c along the vertical walls with $\theta_h > \theta_c$ in an opposed manner. Other two walls are maintained at adiabatic condition. The heated portion along the left vertical wall is making an angle δ with the horizontal direction. At δ = 90°, the heated and cooled portions are along the vertical walls. It is further assumed that all other thermodynamic properties are independent of temperature and that compressibility and dissipation effects are negligible. The flow within the enclosure is laminar and gravitational acceleration acts parallel to the vertical walls. Let u, v denote the velocity components in the x and y directions respectively. An external magnetic field is assumed to be applied parallel to gravity and the induced magnetic field is neglected.

The conservation equations for an unsteady laminar two dimensional flow of fluid in the presence of a magnetic field under Boussinesq approximation are as given below in Eq. 1-4:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$\begin{aligned}&\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\\&-g\beta\sin\delta(\theta-\theta_c)-\frac{1}{\rho}\frac{\partial p'}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)\\&-\frac{\sigma}{\rho}\left(B_0^2 u\cos^2\delta+B_0^2 v\sin\delta\cos\delta\right)\end{aligned} \quad (2)$$

$$\begin{aligned}&\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=\\&g\beta\cos\delta(\theta-\theta_c)-\frac{1}{\rho}\frac{\partial p'}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)\\&-\frac{\sigma}{\rho}\left(B_0^2 u\sin\delta\cos\delta+B_0^2 v\sin^2\delta\right)\end{aligned} \quad (3)$$

$$\frac{\partial\theta}{\partial t}+u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}=\frac{K}{\rho C_P}\left(\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right) \quad (4)$$

The initial and boundary conditions are considered as in Eq. 5:

$$\begin{aligned}&t=0;\ 0\le x\le\ell;\ 0\le y\le\ell;\ u=v=0;\ \theta=\theta_c\\&t>0\\&x=0;\ u=v=0;\ \frac{\partial\theta}{\partial x}=0;\\&x=\ell;\ u=v=0;\ \frac{\partial\theta}{\partial x}=0\\&y=0;\ u=v=0;\ \theta=\begin{cases}\theta_h & 0\le x\le\ell/2\\ \theta_c & \ell/2<x\le\ell\end{cases}\\&y=\ell;\ u=v=0;\ \theta=\begin{cases}\theta_c & 0\le x\le\ell/2\\ \theta_h & \ell/2<x\le\ell\end{cases}\end{aligned} \quad (5)$$

Introducing the following dimensionless variables as defined below:

$$\begin{aligned}&X=\frac{x}{\ell},\ Y=\frac{y}{\ell},\ U=\frac{u\ell}{\nu},\ V=\frac{v\ell}{\nu},\ P=\frac{p'\ell^2}{\rho\nu^2}\\&T=\frac{\theta-\theta_c}{\theta_h-\theta_c},\ \tau=\frac{t\nu}{\ell^2},\\&Gr=\frac{g\beta(\theta_h-\theta_c)\ell^3}{\nu^2},\ Pr=\frac{\nu}{\alpha},\ Ha=B_0\ell\sqrt{\frac{\sigma}{\mu}}\end{aligned} \quad (6)$$

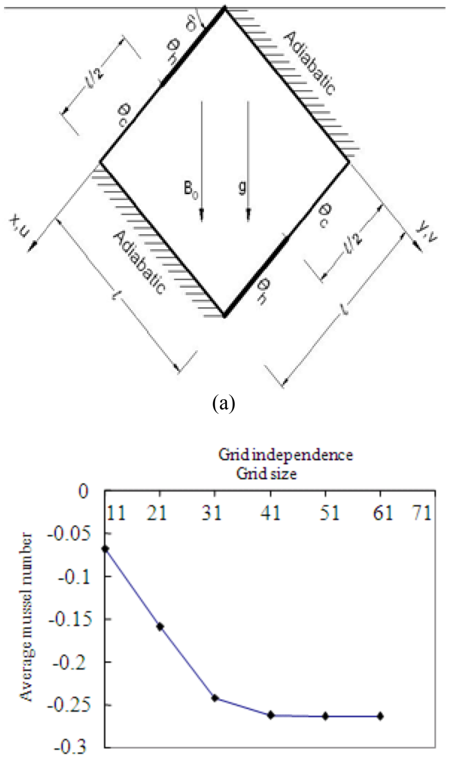
The above Eq. 1-4 get modified as:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \quad (7)$$

$$\begin{aligned}&\frac{\partial U}{\partial\tau}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-GrT\sin\delta-\frac{\partial P}{\partial X}+\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\\&\qquad-Ha^2(U\cos^2\delta+V\sin\delta\cos\delta)\end{aligned} \quad (8)$$

$$\begin{aligned}&\frac{\partial V}{\partial\tau}+U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=GrT\cos\delta-\frac{\partial P}{\partial Y}+\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\\&\qquad-Ha^2(U\sin\delta\cos\delta+V\sin^2\delta)\end{aligned} \quad (9)$$

$$\frac{\partial T}{\partial\tau}+U\frac{\partial T}{\partial X}+V\frac{\partial T}{\partial Y}=\frac{1}{Pr}\left(\frac{\partial^2 T}{\partial X^2}+\frac{\partial^2 T}{\partial Y^2}\right) \quad (10)$$



(a)

(b)

Fig. 1: (a) Schematic diagram of the physical system (b) Average Nusselt number for different grid sizes at Gr = 50000, Ha = 5 and δ = 150°

Table 1: Comparison of average Nusselt numbers for different Hartmann numbers at Gr = 20000

Gr	Ha	$\overline{N_u}$	$\overline{N_u}$ *
20000	0	2.5188	2.1340
	10	2.2234	2.0233
	50	1.0856	1.0219
	100	1.0110	1.0019

$\overline{N_u}$ = Results of Rudraiah *et al.* (1995) $\overline{N_u}$ * = Present study

The initial and boundary conditions are given in the non dimensional form as:

$$\tau = 0, 0 \le X \le 1, 0 \le Y \le 1,\ U = V = 0, T = 0$$
$$\tau > 0$$
$$X = 0; U = V = 0; \frac{\partial T}{\partial X} = 0;$$
$$Y = 0; U = V = 0; T = \begin{cases} 1 & 0 \le x \le 0.5 \\ 0 & 0.5 < x \le 1 \end{cases} \quad (11)$$
$$X = 1; U = V = 0; \frac{\partial T}{\partial X} = 0;$$
$$Y = 1; U = V = 0; T = \begin{cases} 0 & 0 \le x \le 0.5 \\ 1 & 0.5 < x \le 1 \end{cases}$$

Introducing the vorticity ς and the stream function ψ, the equations governing the problem can be written as:

$$\nabla^2 \psi = -\varsigma \quad (12)$$

$$U = \frac{\partial \psi}{\partial Y}, V = -\frac{\partial \psi}{\partial X} \quad (13)$$

$$\frac{\partial \varsigma}{\partial \tau} + U\frac{\partial \varsigma}{\partial X} + V\frac{\partial \varsigma}{\partial Y} = Gr\left(\sin\delta\frac{\partial T}{\partial Y} + \cos\delta\frac{\partial T}{\partial X}\right) + \nabla^2\varsigma + Ha^2\left(\cos^2\delta\frac{\partial U}{\partial Y} - \sin^2\delta\frac{\partial V}{\partial X}\right) - Ha^2\sin\delta\cos\delta\left(\frac{\partial U}{\partial X} - \frac{\partial V}{\partial Y}\right) \quad (14)$$

$$\frac{\partial T}{\partial \tau} + U\frac{\partial T}{\partial X} + V\frac{\partial T}{\partial Y} = \frac{1}{Pr}\nabla^2 T \quad (15)$$

The above Eq. 12-15 are called as the stream function, velocity, vorticity and temperature equations, respectively. An approximation to their solution will be obtained at a finite number of grid points at discrete time $\tau = n\Delta\tau$ where n is an integer. It is assumed that all quantities are known at a time $n\Delta\tau$. An implicit alternating direction technique based on suitable finite difference approximations of the vorticity and temperature equations is employed to advance the fields of vorticity and temperature at the interior grid points across a time step $n\Delta\tau$ to the new level $(n+2)\Delta\tau$. The method of Successive over Relaxation is then employed in conjunction with the newly computed vorticity to solve the stream function equation. The new fields of U and V are obtained from space centered finite-difference approximations of the velocity equations. This computational cycle is then repeated for each of the next time steps until a steady state situation is obtained, when the convergence criteria, $\left|\frac{\varphi_{n+1}(i,j) - \varphi_n(i,j)}{\varphi_{n+1}(i,j)}\right| \le \varepsilon$ for temperature, vorticity and stream function have been satisfied. In the above expression, n is any time level, ε is of order 10^{-5} and φ represents T, ς and ψ. The numerical solutions presented in this study were acquired from a 41×41 grid system and with a time increment of order 10^{-3}. Further increase in the number of grids produced essentially the same results as seen in Fig. 1b.

Prior to the present calculations, as a partial verification of the computational procedure, the results of average Nusselt number at different Grashof numbers and Hartmann numbers were estimated using the FORTRAN code developed and compared with the solutions given by Rudraiah *et al.* (1995) as seen in

Table 1 in which they have studied the natural convection in a square enclosure and are found to be in good agreement.

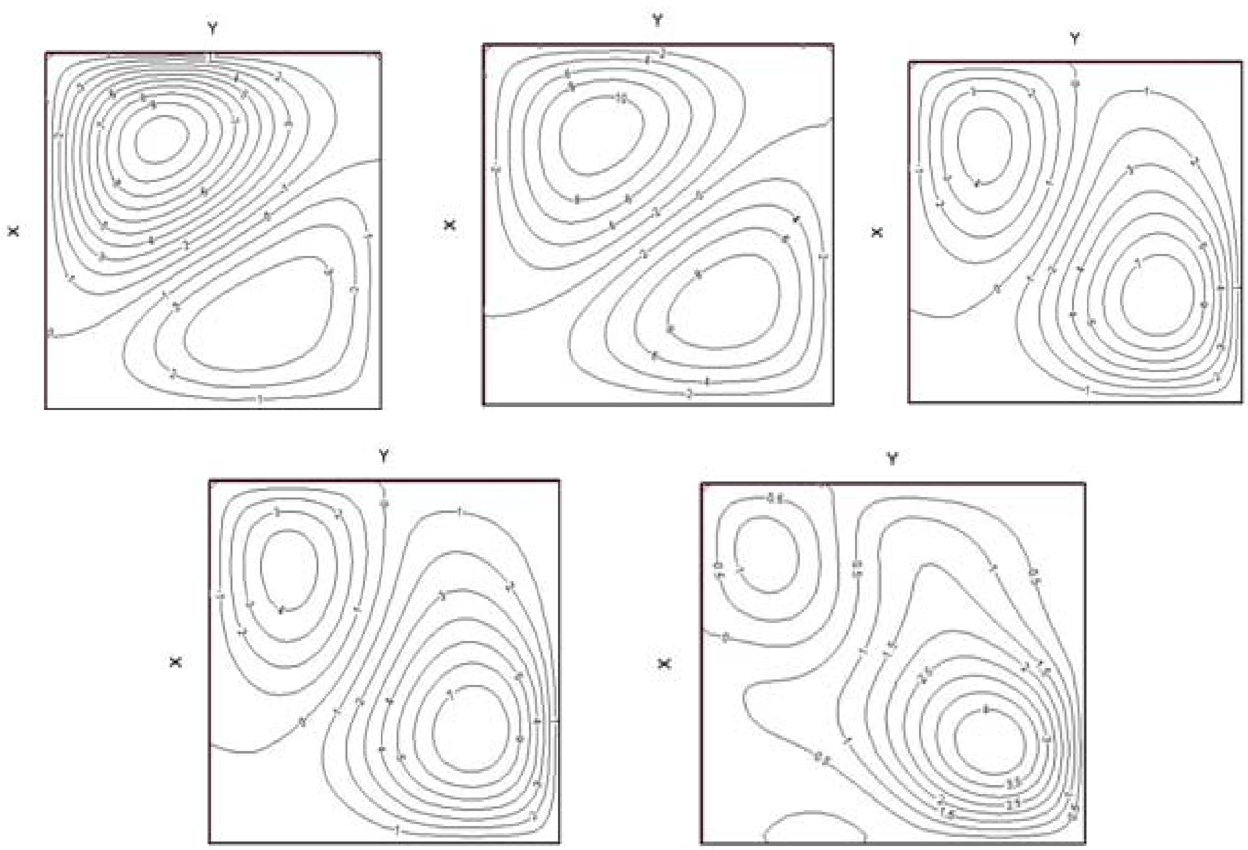
RESULTS

Heat transfer in an inclined square cavity for differentially thermally active locations, inclination angles, Grashof numbers and Hartmann numbers is studied numerically in the presence of a magnetic field. The computations are carried out for Grashof numbers (Gr) in the range from 500-50000, Hartmann number (Ha) from 1-10, angles of inclination δ from 30-180° and at Prandtl number (Pr) = 0.733. The mid height vertical velocity profiles at different Grashof numbers, Hartmann numbers and inclination angles are presented.

DISCUSSION

When Gr = 50000, Ha = 5 and all angles of inclination from 30-180°, it is observed in Fig. 2a, two counter rotating cells appear in the enclosure. Each thermally active location generates an identical upward buoyancy force and therefore dual cell flow is obtained with fluid flowing down the middle of the enclosure. The dual cell structure prohibits direct convective heat transfer between the active locations. Each cell behaves like an independent one preventing warm (cool) fluid from the hot (cold) location mixing with cool (warm) fluid from the cold (hot) location. The higher values of the streamlines indicate stronger rotation due to the higher value of Grashof number. The effect of stronger circulation is also displayed by the isotherms as seen in Fig. 2b.

It is noted in Fig. 3a, when Gr = 50000, Ha = 10 and at δ = 30°, two counter rotating cells appear with a primary cell rotating in clockwise direction. At δ = 60°, the secondary cell rotating in anti-clockwise direction, grows significantly and suppresses the primary cell. The fluid flow gains strength with ψ_{max} = 17 at δ = 120°. By increasing angle of inclination from 120° in steps of 30°, the change in the direction of the flow pattern is noticed. Strength of the flow increases when Gr increases from 5000-50,000 as seen from the higher values of the streamlines. It is found that in Fig. 3b, the isotherms are greatly influenced by the inclination of the enclosure.



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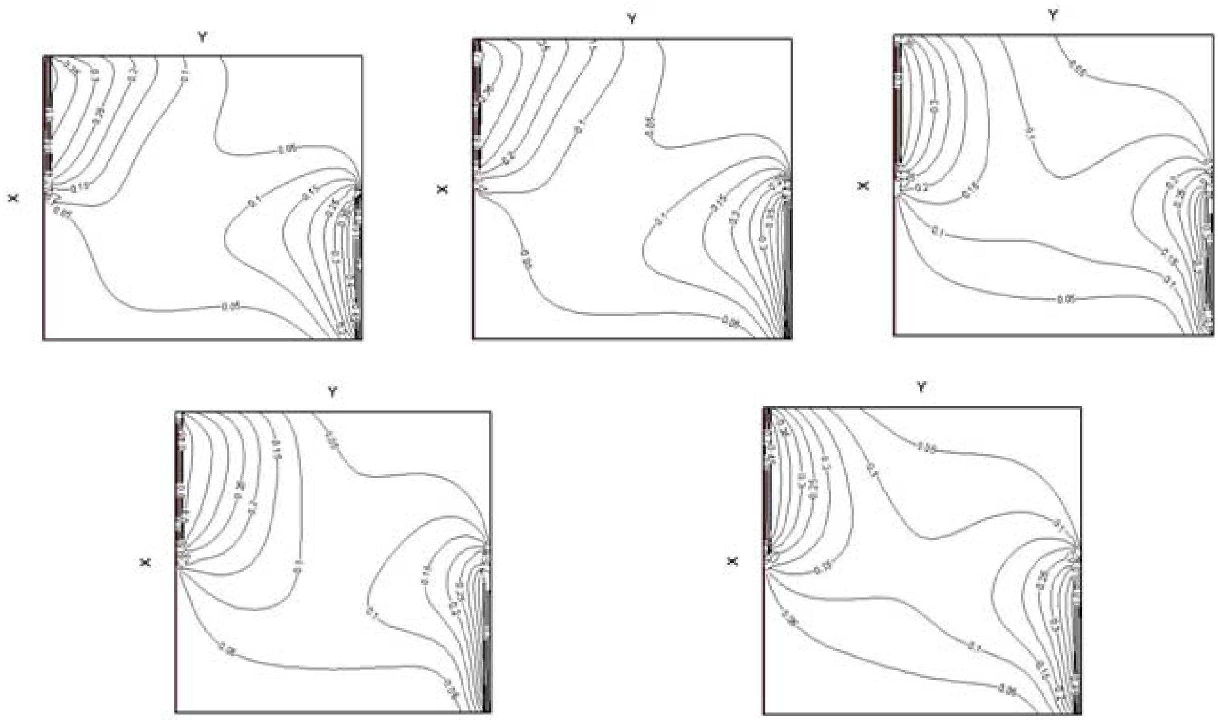
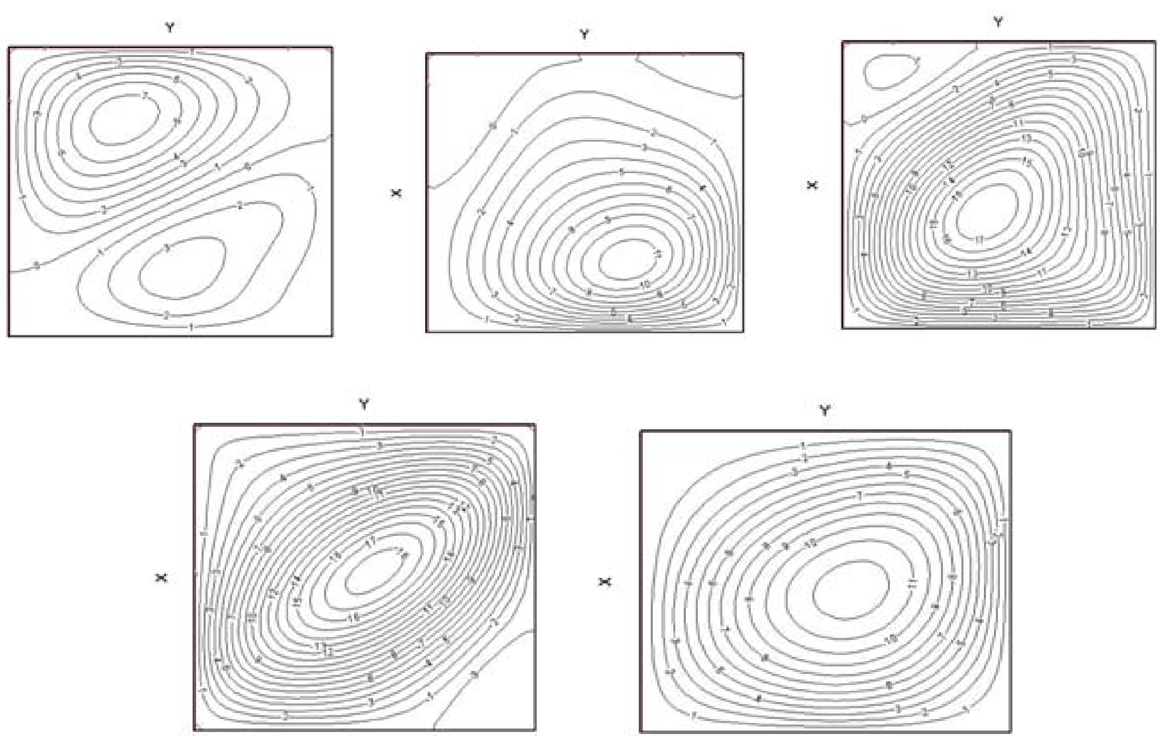


(b)

Fig. 2: (a) and (b) Streamlines and isotherms at Gr=50000, Ha=5 at δ = 30, 60, 90, 120, 150 and 180°



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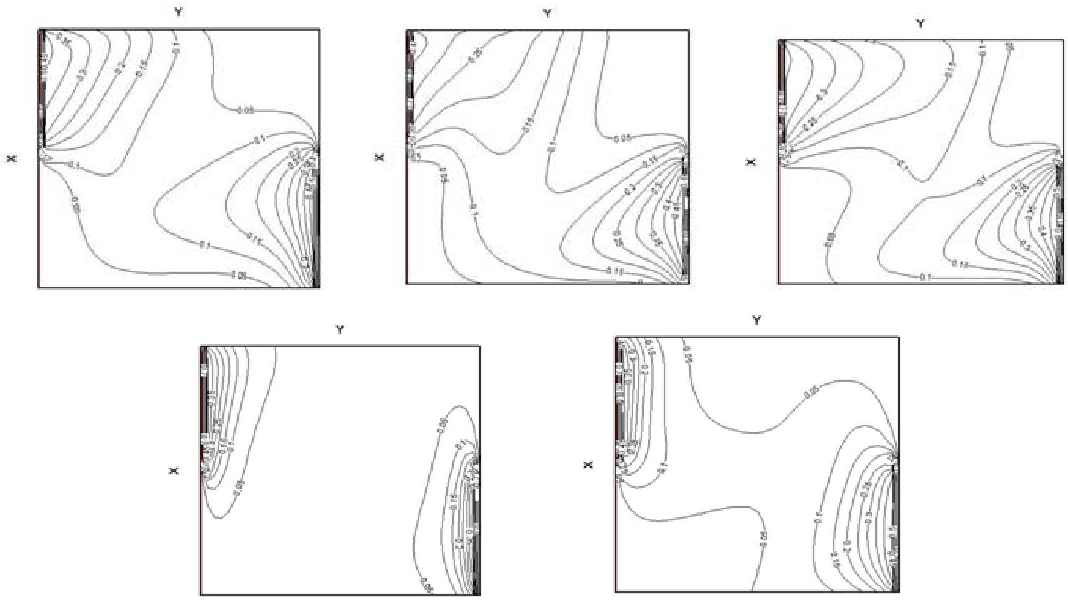
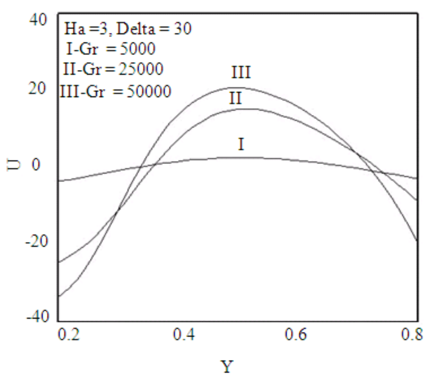


(b)

Fig. 3: (a) and (b) Streamlines and isotherms at Gr = 50000, Ha = 10 at δ = 30, 60, 90, 120, 150 and 180°



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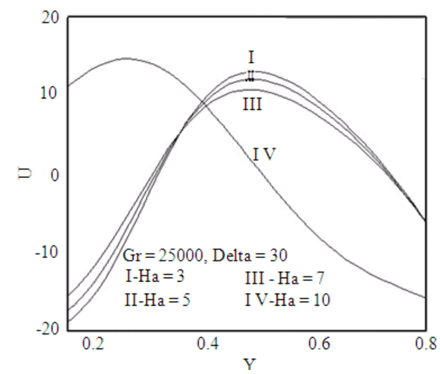

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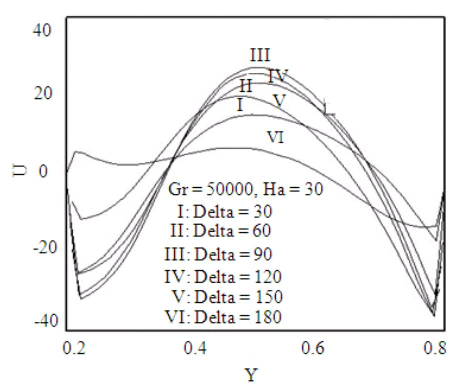

(c)

Fig. 4: (a) Mid height velocity profiles when Ha = 3,δ (DELTA) = 30 at Gr= 5000, 25000. 50000 (b) Mid height velocity profiles when Gr =25000, δ (DELTA) = 30 at Ha = 3, 5, 7, 10 (c) Mid height velocity profiles when Gr =50000, Ha = 3 at δ (DELTA) =30, 60, 90, 120, 150 and 180°

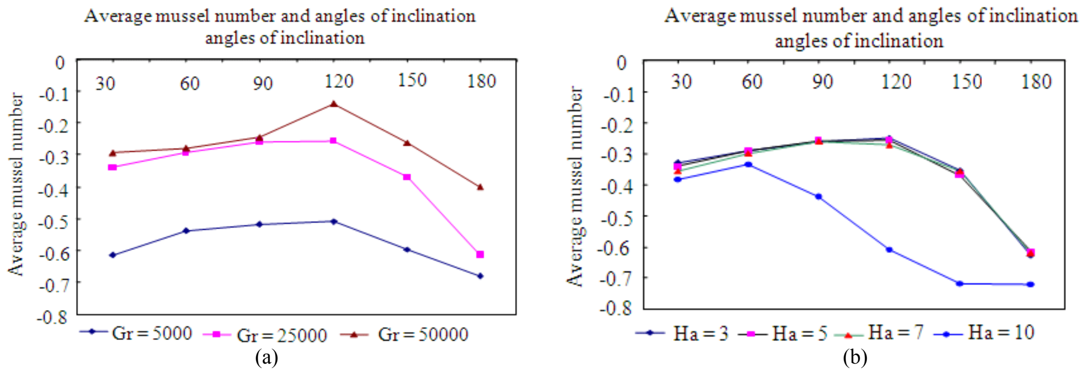

Fig. 5: (a) Average Nusselt number Vs Angles of inclination at Gr = 5000, 25000, 50000 when Ha = 5 (b) Average Nusselt number Vs Angles of inclination at Ha = 3, 5, 7, 10

The mid height velocity profiles at Gr = 5000-25000 and 50000, when Ha = 3 and δ = 30° are presented in Fig. 4a. It is observed that for increase in Grashof number, the mid height velocity increases, as significant increase in the flow strength contribute to higher velocity values. Also as Hartmann number increases, the heat transfer is suppressed and hence the vertical velocity decreases, as seen in Fig. 4b. The effect of inclination angles for δ = 30-180°, when 50000 and for fixed Ha = 3 on the mid height velocity profiles are presented in Fig. 4c. It is observed that the velocity U increases for angles 30-90° and decreases for angles from 120-180°. At δ = 180°, Gr = 50000 and Ha = 3, the vertical velocity decreases appreciably and hence the velocity profile is flattened. The effect of the angles of inclination on the average Nusselt number, $\overline{N_u}$, at Gr = 5000, 25000, 50000 and Ha = 5 are presented in Fig. 5a. The average Nusselt number behaves in a non-linear fashion with angles of inclination. The heat transfer is found to be maximum at δ = 120° and minimum at δ = 180°, irrespective of the Grashof number. It is also observed that the average Nusselt number $\overline{N_u}$ increases considerably with Grashof number since the circulation becomes stronger. The influence of angles of inclination on $\overline{N_u}$ for Hartmann numbers Ha = 3, 5, 7, 10 is illustrated in Fig. 5b. It is observed that the behavior of average heat transfer coefficient is a non-linear function of inclination angles at all values of Hartmann numbers. Angle of inclination has remarkable effect on the average Nusselt number $\overline{N_u}$ at lower values of the Hartmann number Ha = 3, Ha = 5 and the effect is less significant when Hartmann number is increased to Ha = 10. It is also observed that the average Nusselt number $\overline{N_u}$ is a decreasing function of Hartmann number.

CONCLUSION

The present study considers laminar natural convection flow in the presence of a magnetic field in an inclined square enclosure with differentially thermally active vertical walls while the horizontal walls are kept adiabatic. The associated flow characteristics and heat transfer inside the titled enclosure are found to depend strongly upon the strength of the magnetic field and the inclination angles. The mid height vertical velocity increases when Grashof number increases and decreases when the Hartmann number increases. Angle of inclination has remarkable effect on the average Nusselt number $\overline{N_u}$ at lower values of the Hartmann number and the effect is less significant at higher values of Hartmann number. At higher values of the Grashof number, the heat transfer is maximum at δ = 120° and minimum at δ = 180°. The average Nusselt number behaves in a non-linear fashion with angles of inclination.

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