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Moments of Order Statistics from Nonidentically Distributed Three Parameters Beta typeI and Erlang Truncated Exponential Variables

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Abstract: Problem statement: Moments of order statistics of independent non-identically distributed (INID) random variables is not an easy subject to deal with for continuous distributions. One is forced to use messy algebraic calculations (whether one uses permanents or not). This was the motivation behind this study. In this study the moments of order statistics arising from independent nonidentically distributed three parameters Beta type I distribution and Erlang Truncated Exponential distribution were derived. **Approach:** We employed an easier technique established by Barakat & Abdelkader will be referred to as (BAT). **Results:** The mean, the second moment and the variance of the median and the smallest order statistics for the first distribution were given for different values of the shape parameter and different sample sizes. **Conclusion:** The results can be used to make some inferences and used the BAT technique to derive moments of order statistics arising from independent nonidentically distributed for any other continuous distribution with distribution function (cdf) in the form: F(x) = 1-λ (x).

Key words: Moments, non-identically distributed order statistics, permanents, three parameters beta type I distribution, erlang truncated exponential distribution

INTRODUCTION

The moments of order statistics (os) of Independent Non-Identically Distributed (INID) random variables (rvs) have been established in literature in two directions. The first direction was initiated by (Balakrishnan, 1994). It requires a basic relation between the probability density function (pdf) and the cumulative distribution function (cdf). This technique is referred to as Differential Equation Technique (DET). It enables one to compute all the singleand product moments of all order statistics in a simple recursive manner and the derivation of the moments depends mainly on integration by parts. The second technique was established by Barakat and Abdelkader(2003) will be referred to as (BAT). It is an easier manner to evaluate the moments of INID os but can be applied to distributions with cdf in the form: F(x) = 1-λ(x) or by using the survival function of the distribution under study. BAT cannot be used to evaluate the product moments. In this study it was our interest to use the BAT technique to get the moments of os arising from INID distributed three parameters Beta type I (Johnson *et al*., 1994) and Erlang Truncated Exponential (El-Alosey, 2007) and (Mohsin,2009) random variables .

The subject of os from INID rvs is discussed widely in the literature (David, 1981; Bapat and Beg, 1989; Davidand Nagaraja, 2003). (Barakat, 2002). found the limit behavior of bivariate order statistics from INID distributed random variables. (Güngor, Gokhan and Bulut 2005) expressed the multivariate order statistics by marginal Ordering of INID random vectors under discontinuous df`s.

Applications of the previous two methods are also found in literature for several continuous distributions. The DET was used by (Balakrishnan, 1994) to derive recurrence relations satisfied by single and product moments of os from INID rvsfor the Exponential and right truncated distributions. (Childs and Balakrishnan, 2006) applied DET to derive the moments of os from INID rvs for Logistic random variables. The first application of BAT was by (Barakat and Abdelkader, 2000) to weibull distribution and then the method was generalized by (Barakat and Abdelkader, 2003) and was

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applied to Erlang, Positive Exponential, Pareto and Laplace distributions. Abdelkader(2004) and (Abdelkader, 2008) used a closed expression for the survival function of Gamma and Beta distributions to compute the moments of os from INID rvs using BAT. Jamjoom(2006) applied it to Burr XII rvs.

Let $X_1, X_2,\ldots,X_n$ be independent random variables having cdfs $F_1(x), F_2(x),\ldots, F_n(x)$ and pdfs, $f_1(x), f_2(x),\ldots, f_n(x)$ respectively. Let $X_{1:n}\le, X_{2:n}\le\ldots\le X_{n:n}$ denote the os obtained by arranging the n X_i's in increasing order of magnitude. Then the pdf of the rth os. $X_{r:n}(1\le r\le n\le)$ can be written as:

$$f_{r:n}(x)=\frac{1}{(r-1)!(n-r)!}\sum_p\prod_{\alpha=1}^{r-1}F_{i_\alpha}(x)f_{i_r}(x)\prod_{c=r+1}^{n}\{1-F_{i_c}(x)\} \quad (1)$$

Where, $\sum_p$ denotes the summation over all n permutations$[(i_1, i_2,\ldots,i_n)$ of $(1, 2,\ldots,n)$. (Bapat and Beg, 1989) put the previous pdf of the rth os $X_{r:n}$ in the form of permanent as:

$$f_{r:n}(x)=\frac{1}{(r-1)!(n-r)!}\mathrm{per}\left[\underbrace{F(x)}_{r-1}\ \underbrace{f(x)}_{1}\ \underbrace{\{1-F(x)\}}_{n-r}\right] \quad (2)$$

To derive the moments of os from INID rvs arising from this distribution we need the following theorem which is established by (Barakat and Abdelkader, 2003).

Theorem 1: Let $X_1, X_2,\ldots,X_n$ be independent nonidentically distributed rvs. The kth moment of all order statistics, , for $1\le r\le n$ and $k = 1, 2,\ldots$ is given by:

$$\mu_{r:n}^{(k)}=\sum_{j=n-r+1}^{n}(-1)^{j-(n-r+1)}\binom{j-1}{n-r}I_j(k) \quad (3)$$

Where:

$$I_j(k)=\sum_{1\le i_1<i_2<i_j\le n}\ldots\sum k\int_0^\infty x^{k-1}\prod_{t=1}^{j}G_{i_t}(x)dx,\ j=12,\ldots,n \quad (4)$$

$G_{it}(x) = 1-F_{it}(x)$ with $(i, i_2\ldots, i_n)$ is a permutation of $(1, 2, \ldots, n)$ for which $i_1<i_2<,\ldots, <i_n$.

Proof: The proof of this theorem can be found in Barakat and Abdelkader (2003).

MATERIALS AND METHODS

We consider the three parameters Bet type I distribution (Johnson *et al*., 1994) with cdf:

$$F(x)=1-\left[\frac{\delta-x}{\delta-w}\right]^{p},\ w\le x\le\delta \quad (5)$$

and Erlang truncated exponential (El-Alosey, 2007) and (Mohsin, 2009) with cdf:

$$F(x)=1-e^{-\beta x}(1-e^{-\lambda}),0\le x\le\infty,\beta,\lambda>0 \quad (6)$$

Now, we consider the case when the variables X_i 's be INID rvs, then the above cdfs can be written as:

$$F_i(x)=1-\left[\frac{\delta-x}{\delta-w}\right]^{p_i},\ w\le x\le\delta,\ p_i>0,\delta,w>0 \quad (7)$$

$$F_i(x)=1-e^{-\beta x}(1-e^{-\lambda i}),0\le x\le\infty,\beta,\lambda_i>0 \quad (8)$$

Applications of theorem 1 for the above distributions will be stated as theorem 2 nd 3.

Moments of os from INID three parameters beta type I rvs:

Theorem 2: For any real numbers δ, w > 0, $1\le r\le n$ and k = 1, 2, …:

$$I_j(k)=k\sum_{1\le i_1<i_2<i_j\le n}\ldots\sum\frac{\delta^{k+\sum_{t=1}^{j}P_{i_t}}}{(\delta-w)^{\sum_{t=1}^{j}P_{i_t}}}\beta_{\frac{\delta-w}{\delta}}\left(\sum_{t=1}^{j}P_{i_t}+1,k\right) \quad (9)$$

Proof: On applying theorem 1 and using (4), we get:

$$I_j(k)=k\sum_{1\le i_1<i_2<i_j\le n}\ldots\sum k\int_w^\delta x^{K-1}\left[\frac{\delta-x}{\delta-w}\right]^{\sum_{t=1}^{j}P_{i_t}}dx$$

Substituting $y=\frac{\delta-x}{\delta-w}$ the above equation reduces to:

$$I_j(k)=k\sum_{1\le i_1<i_2<\ldots<i_j\le n}\ldots\sum(\delta-w)(\delta)^{K-1}\int_0^1\left[1-\frac{\delta-w}{\delta}y\right]^{k-1}y^{\sum_{t=1}^{j}P_{i_t}}dy$$

By substituting $z=\frac{\delta-x}{\delta}y$ we get:

$$I_j(k) = k\sum_{1\le i_1<i_2<\ldots<i_j\le n}\ldots\sum \frac{\delta^{k+\sum_{t=1}^{j}P_{i_t}}}{(\delta-w)^{\sum_{t=1}^{j}P_{i_t}}}\int_0^{\frac{\delta-w}{\delta}}[1-z]^{k-1} z^{\sum_{t=1}^{j}P_{i_t}+1-1}dz$$

$$\therefore I_j(k) = k\sum_{1\le i_1<i_2<\ldots<i_j\le n}\ldots\sum \frac{\delta^{k+\sum_{t=1}^{j}P_{i_t}}}{(\delta-w)^{\sum_{t=1}^{j}P_{i_t}}}\beta_{\frac{\delta-w}{\delta}}(\sum_{t=1}^{j}P_{i_t}+1,k)$$

where, $\beta_z(a,b)$ is the incomplete beta function defined by:

$$\int_0^z x^{a-1}(1-x)^{b-1}dx = \beta_z(a,b)$$

Moments of os from INID erlang truncated exponential rvs:

Theorem 3: For $1\le r\le n$and k = 1,2, …:

$$I_j(k) = k\sum_{1\le i_1<i_2<\ldots<i_j\le n}\ldots\sum \frac{\Gamma(k)}{(\beta)^k\left[j-\sum_{t=1}^{j}e^{-\lambda_{i_t}}\right]^k} \tag{10}$$

Proof: On applying theorem 1 and using (8), we get:

$$I_j(k) = k\sum_{1\le i_1<i_2<\ldots<i_j\le n}\ldots\sum k\int_0^\infty x^{k-1}\prod_{t=1}^{j}e^{-\beta x(1-e^{-\lambda_{i_t}})}dx,$$

$$= \sum_{1\le i_1<i_2<\ldots<i_j\le n}\ldots\sum k\int_0^\infty x^{k-1}e^{-\beta x\sum_{t=1}^{j}(e^{-\lambda_{i_t}})}dx$$

On integrating term by term using the integral:

$$\int_0^\infty x^{k-1}e^{ax}dx = \frac{\Gamma(k)}{a^k}$$

We get:

$$I_j(k) = k\sum_{1\le i_1<i_2<\ldots<i_j\le n}\ldots\sum \frac{\Gamma(k)}{(\beta)^k\left[\sum_{t=1}^{j}(1-e^{-\lambda_{i_t}})\right]^k}$$

$$= k\sum_{1\le i_1<i_2<\ldots<i_j\le n}\ldots\sum \frac{\Gamma(k)}{(\beta)^k\left[j-\sum_{t=1}^{j}e^{-\lambda_{i_t}}\right]^k}$$

and the proof is completed.

RESULTS

Result 1: Substituting (9) in (3) the kth moments of the rth os from INID three parameters beta type I can be finally written as:

$$\mu_{r:n}^{(k)} = \sum_{j=n-r+1}^{n}(-1)^{j-(n-r+1)}\binom{j-1}{n-r}k\sum_{1\le i_1<i_2<\ldots<i_j\le n}\ldots \sum \frac{\delta^{k+\sum_{t=1}^{j}P_{i_t}}}{(\delta-w)^{\sum_{t=1}^{j}P_{i_t}}}\times\ \beta_{\frac{\delta-w}{\delta}}(\sum_{t=1}^{j}P_{i_t}+1,k) \tag{11}$$

Remark 1: The kth moment of the largest os $X_{n:n}$ from INID three Beta type I rvs can be written as:

$$\mu_{n:n}^{(k)} = \sum_{j=1}^{n}(-1)^{j-1}I_j(k) \tag{12}$$

where, I_j (k) is defined in (9).

Remark 2: The kth moment of the first os $X_{1:n}$ from INID three parameters Beta type I rvs can be written as:

$$\mu_{1:n}^{(k)} = I_n(k) \tag{13}$$

Where:

$$I_n(k) = k\frac{\delta^{k+\sum_{i=1}^{n}P_i}}{(\delta-w)^{\sum_{i=1}^{n}P_i}}\beta_{\frac{\delta-w}{\delta}}\left(\sum_{i=1}^{n}P_i+1,k\right),\delta,w>0 \tag{14}$$

Remark 3: IID case The Independent Identically Distributed (IID) case can be deduced from theorem (2). Ij(k) can be written as:

$$I_j(k) = k\binom{n}{j}\frac{\delta^{k+j\,p_{i_t}}}{(\delta-w)^{jp_{i_t}}}\beta_{\frac{\delta-w}{\delta}}\left(jp_{i_t}+1,k\right) \tag{15}$$

where, βz (a, b) is the incomplete beta function defined before.

Result 2: Substituting (10) in (3) the kth moments of the rth os from INID Erlang truncated xponential can be finally written as:

$$\mu_{r:n}^{(k)} = \sum_{j=n-r+1}^{n}(-1)^{j-(n-r+1)}\binom{j-1}{n-r}k\sum_{1\le i_1<i_2<\ldots<i_j\le n}\ldots \sum \frac{\Gamma(k)}{(\beta)^k\left[j-\sum_{t=1}^{j}e^{-\lambda_{i_t}}\right]^k} \tag{16}$$

Remark 1: The kth moment of the largest os$X_{n:n}$ from INID Erlang truncated exponential rvs can be written as:

$$\mu_{n:n}^{(k)} = \sum_{j=1}^{n} (-1)^{j-1} I_j(k) \quad (17)$$

where, Ij (k) is defined in (10)

Remark 2: The k_{th} moment of the first os $X_{1:n}$from INID Erlang truncated exponential rvs. can be written as:

$$\mu_{1:n}^{(k)} = I_n(k) \quad (18)$$

Where:

$$I_n(k) = \frac{k\Gamma(k)}{(\beta)^K [n - \sum_{i=1}^{n} e^{-\lambda_i}]^k} \quad (19)$$

Remark 3: The Independent Identically Distributed (IID) case can be deduced from theorem (2) . $I_j(k)$ is written as:

$$I_j(k) = \binom{n}{j} \frac{k\Gamma(k)}{j^{k-1} (\beta)^K [1 - e^{-\lambda_i}]^k} \quad (20)$$

Numerical calculations for three parameter beta type I: Simple programs written by Mathematica 7 were used to calculate the moments given in tables 1, 2, 3, 4, 5.

Example (1): Multiple outliers sample:
Let:
n =3,
X_1~ Beta typ I(δ = 1, w =0.5, P_1 = 0.1), X_2~Beta typ I (δ = 1, w = 0.5, p_2= (0,(0.5)2)), X_3~ Beta typ I(δ = 1, w = 0.5, p_3 = 0,(0.5)2).

Then from (3) the kth of the median $X_{2:3}$ is given by:

$$\mu_{2:3}^{(k)} = \sum_{j=2}^{3} (-1)^{j-2} \binom{j-1}{1} I_j(k) \quad (21)$$
$$= I_2(k) - 2I_3(k)$$

From (9):

$$I_2(k) = k \sum_{1 \le i_1 < i_2 < \ldots \le 3} \ldots \sum \frac{\delta^{k + \sum_{t=1}^{2} P_{i_t}}}{(\delta - w)^{\sum_{t=1}^{j} P_{i_t}}} \beta_{\frac{\delta-w}{\delta}} \left(\sum_{t=1}^{2} P_{i_t} + 1, k\right)$$
$$= \frac{\delta^{k+p_1+p_2}}{(\delta-w)^{p_1+p_2}} \beta_{\frac{\delta-w}{\delta}} (p_1 + p_2 + 1, k) + \frac{\delta^{k+p_1+p_2}}{(\delta-w)^{p_1+p_3}} \beta_{\frac{\delta-w}{\delta}} (p_1 + p_3 + 1, k) + \frac{\delta^{k+p_2+p3}}{(\delta-w)^{p_2+p_3}} \beta_{\frac{\delta-w}{\delta}} (p_2 + p_3 + 1, k) \quad (22)$$

$$I_3(k) = k \sum_{1 \le i_1 < i_2 < i_3 .. \le 3} \ldots \sum \frac{\delta^{k + \sum_{t=1}^{3} P_{i_t}}}{(\delta - w)^{\sum_{t=1}^{3} P_{i_t}}} \beta_{\frac{\delta-w}{\delta}} \left(\sum_{t=1}^{3} P_{i_t} + 1, k\right) \quad (23)$$
$$= \frac{\delta^{k+p_1+p_2+p_3}}{(\delta-w)^{p_1+p_2+p_3}} \beta_{\frac{\delta-w}{\delta}} (p_1 + p_2 + p_3 + 1, k)$$

Substituting (22) and (23) in (21), $\mu_{2:3}^{(k)}$ is then can be obtained.

Table 1 represents the mean, the second moment and the variance of the medianof the sample size n = 3 arising from three parameter Beta type 1 distribution. These computations are done when (δ = 1, w = 0.5, p_1= 0.1, p_2 =(0,(0.5)2)).

Example (2): Single outlier sample: When w = 0 ,δ =1, the three parameter Beta type 1 distribution reduces to F(x) = 1−1[1−x]^p, 0≤x≤1 and Eq (9) can be written as:

$$I_j(k) = k \sum_{1 \le i_1 < i_2 \ldots < i_j \le n} \ldots \sum \beta \left(\sum_{t=1}^{j} P_{i_t} + 1, k\right) \quad (24)$$

Let:
X_1, X_2,…, X_9~Beta typ I(δ = 1, w =0, P = (0,(0.5)2)), X_n~Beta typ I (δ = 1, w = 0.5, p_{10}= (0,(0.5)2))10, 20, 50, 100, 10000)

From (13, 14) the kth moment of the smallest os when the sample size n = 10 is:

$$\mu_{1:n}^{(k)} = I_n(k)$$

$$\mu_{1:10}^{(k)} = I_{10}(k)$$

$$= k \sum_{1\le i_1 < i_2 < \ldots < i_{10} \le 10} \ldots \sum \beta\left(\sum_{t=1}^{10} P_{i_t} + 1, k\right) \qquad (25)$$

$$= k\beta\left(\sum_{t=1}^{9}(1) + p_{10} + 1, k\right)$$

$$= k\beta\,(10 + p_{10}, k)$$

Table 2 and 3 represent the mean, the second moment and the variance of thesmallest os of the sample size n =10 arising from three parameter Beta type 1distributions. These computations are done when:

(δ = 1, w = 0, p = (0,(0.5)2)), p_{10} = (0,(0.5)2))10, 20, 50, 100, 10000)

Theorem 2 of Abdelkader (2008) with α = 1 is the same as Eq. 25.

Example 3: Single outlier sample:

Let:
$X_1, X_2, \ldots, X_9$~Beta typ I(δ = 1, w = 0.5, P = (0,(0.5)2)), X_n~Beta typ I (δ = 1, w = 0.5, p_{10}= (0,(0.5)2))10, 20, 50, 100, 10000)

From (13, 14) the kth moment of the smallest os when the sample size n = 10 is:

$$\mu_{1:10}^{(k)} = I_{10}(k)$$

$$= k \sum_{1\le i_1 < i_2 < \ldots < i_{10} \le 10} \ldots \sum \frac{1}{0.5^{\sum_{t=1}^{10} P_{10}}} \beta_{0.5}\left(\sum_{t=1}^{10} P_{i_t} + 1, k\right)$$

$$= \frac{k}{(0.5)^{\sum_{t=1}^{9} P_{i_t} + p_{10}}} \beta_{0.5}\left(\sum_{t=1}^{9}(1) + p_{10} + 1, k\right) \qquad (26)$$

$$= \frac{k}{(0.5)^{9+p_{10}}} \beta_{0.5}(10 + p_{10}, k)$$

Table 1: n = 3, (δ =1, w = 5P1 = 0.1, P2 = (0(0.5)2),P3 = 0,(0.5)2)

P2/P3	0	0.5	1	1.5	2
μ					
0	0.500000	0.475379	0.466450	0.462238	0.459922
0.5	0.475379	0.398810	0.365980	0.348894	0.338870
1	0.466450	0.365980	0.320276	0.295482	0.280483
1.5	0.462238	0.348894	0.295482	0.265713	0.247318
2	0.459922	0.338870	0.280483	0.247318	0.226502
E(X^2)					
0	0.750000	0.704970	0.689429	0.682412	0.678700
0.5	0.704970	0.568918	0.513282	0.485493	0.469761
1	0.689429	0.513282	0.436875	0.397066	0.373803
1.5	0.682412	0.485493	0.397066	0.349727	0.321458
2	0.678700	0.469761	0.373803	0.321458	0.289703
σ^2					
0.0	0.500000	0.478985	0.471853	0.468748	0.467172
0.5	0.478985	0.409869	0.379341	0.363766	0.354928
1	0.471853	0.379341	0.334298	0.309756	0.295132
1.5	0.468748	0.363766	0.309756	0.279123	0.260292
2	0.467172	0.354928	0.295132	0.260292	0.238400

The mean, the second moment and the variance of the median of the sample size n = 3 in the presence of multiple outliers using Eq 21

Table 2: n = 10, δ = 1, w = 0, $p_1 = p_2 = \ldots = p_9$ = p = (0,(0.5),2), p_{10} = (0,(0.5),2

p/p_{10}	0	0.5	1	1.5	2
μ					
0	1.00000000	0.66666700	0.50000000	0.40000000	0.33333300
0.5	0.18181800	0.16666700	0.15384600	0.14285700	0.13333300
1	0.10000000	0.09523810	0.09090910	0.08695650	0.08333330
1.5	0.06896550	0.06666670	0.06451610	0.06250000	0.06060610
2	0.05263160	0.05128210	0.05000000	0.04878050	0.04761900
E(X^2)					
0	1.00000000	0.53333300	0.33333300	0.22857000	0.16666700
0.5	0.05594410	0.04761900	0.04102560	0.03571430	0.03137250
1	0.01818180	0.01656310	0.01515150	0.01391300	0.01282050
1.5	0.00889878	0.00833333	0.00782014	0.00735294	0.00692641
2	0.00526316	0.00500313	0.00476190	0.00453772	0.00432900
σ^2					
0	0.00000000	0.08888890	0.08333330	0.06857140	0.05555560
0.5	0.02288620	0.01984130	0.01735700	0.01530610	0.01359480
1	0.00818182	0.00749285	0.00688705	0.00635161	0.00587607
1.5	0.00414253	0.00388889	0.00365781	0.00344669	0.00325331
2	0.00249307	0.00237328	0.00226190	0.00215818	0.00206143

The mean, the second moment and the variance of the smallest os of the sample size n = 10 in the presence of single outlier sing Eq. 21

Table (3): n = 10, δ = 1, w = 0, $p_1 = p_2 = \ldots = p_9 = p = (0,(0.5),2)$, $p_{10} = (10,20, 50, 100, 1000)$

p/p_{10}	10	20	50	100	1000
μ					
0	0.0909091	0.047619000	0.019607800	0.009900990	0.000999001
0.5	0.0645161	0.039215700	0.018018000	0.009478670	0.000994530
1	0.0500000	0.033333300	0.016666700	0.009090910	0.000990099
1.5	0.0408163	0.028985500	0.015503900	0.008733620	0.000985707
2	0.0344828	0.025641000	0.014492800	0.008403360	0.000981354
$E(X^2)$					
0	0.01515150	0.004329000	0.000754148	0.000194137	1.900000000
0.5	0.00782014	0.002959670	0.000637806	0.000178003	1.900000000
1	0.00476190	0.002150540	0.000546448	0.000163800	1.900000000
1.5	0.00320128	0.001632990	0.000473401	0.000151232	1.900000000
2	0.00229885	0.001282050	0.000414079	0.000140056	1.900000000
σ^2					
0	0.00688705	0.002061430	0.000369680	0.0000961075	9.900000000
0.5	0.00365781	0.001421800	0.000313157	0.0000881580	9.800000000
1	0.00226190	0.001039430	0.000268670	0.0000811555	9.700000000
1.5	0.00153531	0.000792827	0.000233031	0.0000749554	9.600000000
2	0.00110979	0.000624589	0.000204039	0.0000694395	9.600000000

The mean, the second moment and the variance of the smallest osof the sample size n = 10 in the presence of single outlier Eq. 25

Table 4: n = 10, δ = 1, w = 0.5, $p_1 = p_2 = \ldots = p_9 = p = (0,(0.5),2)$, $p_{10} = (0,(0.5),2$

p/p_{10}	0	0.5	1	1.5	2
μ					
0	0.500000000	0.333333000	0.250000000	0.2000000000	0.166667000
0.5	0.005681820	0.005208330	0.004807690	0.0044642900	0.004166670
1	0.000195313	0.000186012	0.000177557	0.0001698370	0.000162760
1.5	8.400000000	8.100000000	7.000000000	7.6000000000	7.000000000
2	4.000000000	3.900000000	3.000000000	3.7000000000	3.600000000
$E(X^2_{1:10})$					
0	0.750000000	0.466667000	0.333333000	0.2571430000	0.208333000
0.5	0.006555940	0.005952380	0.005448720	0.0050223200	0.004656860
1	0.000213068	0.000202187	0.000192353	0.0001834240	0.000175280
1.5	8.900000000	8.600000000	8.300000000	8.0000000000	7.800000000
2	4.200000000	4.100000000	3.900000000	3.8000000000	3.700000000
σ^2					
0.0	0.500000000	0.355556000	0.270833000	0.2171430000	0.180556000
0.5	0.006523660	0.005925250	0.005425600	0.0050023900	0.004639500
1	0.000213030	0.000202152	0.000192322	0.0001833950	0.000175254
1.5	8.900000000	8.600000000	8.300000000	8.0000000000	7.000000000
2	4.200000000	4.100000000	3.900000000	3.8000000000	3.700000000

The mean, the second moment and the variance of the smallest os of the sample size n = 10 inthe presence of single outlier Eq. 25

Table 5: n = 10, δ = 1, w = 0.5, $p_1 = p_2 = \ldots = p_9 = p = (0,(0.5),2)$, $p_{10} = (10, 20, 50, 100, 1000)$

p/p_{10}	10	20	50	100	1000
μ					
0	0.0454545000	0.0238095000	0.0098039200	0.0049505000	0.0004995000
0.5	0.0020161300	0.0012254900	0.0005630630	0.0002962090	0.0000310791
1	0.0000976563	0.0000651042	0.0000325521	0.0000177557	1.9000000000
1.5	4.9000000000	3.5000000000	1.8000000000	1.0000000000	1.2000000000
2	2.6000000000	1.9000000000	1.1000000000	6.4000000000	7.4000000000
$E(X^2)$					
0	0.049242400	0.0248918000	0.0099924600	0.0049990300	0.0004995000
0.5	0.002138320	0.0012717400	0.0005730290	0.0002989900	0.0000310791
1	0.000102307	0.0000672043	0.0000330857	0.0000179156	1.9000000000
1.5	5.100000000	3.6000000000	1.9000000000	1.0000000000	1.2000000000
2	2.700000000	2.0000000000	1.1000000000	6.4000000000	7.4000000000
σ^2					
0	0.047176300	0.0243249000	0.0098963400	0.0049745200	0.0004997500
0.5	0.002134250	0.0012702300	0.0005727120	0.0002989020	0.0000311090
1	0.000102297	0.0000672001	0.0000330847	0.0000179153	1.0000000000
1.5	5.100000000	3.6000000000	1.9000000000	1.0000000000	1.2000000000
2	2.700000000	2.0000000000	1.1000000000	6.4000000000	7.4000000000

The mean, the second moment and the variance of the smallest os of the sample size n = 10 inthe presence of single outlier Eq.26

Table 4 and 5 represent the mean, the second moment and the variance of the smallest o.s. of the sample size n =10 arising from three parameter Beta type 1distributions. These computations are done when: (δ = 1, w = 0.5, p = (0(0.5), 2), p_{10}= (0,(0.5)2))10, 20, 50, 100, 10000).

DISCUSSION

Using the BAT Technique beautiful and elegant recurrence rlations were obtained for the moments of rder statistics from nonidentically but independently distributed variables for both distributions under study which can be considered an addition to INID. os.

CONCLUSION

BAT technique is strongly recommended for any other continuous distribution with cdf in the form: $F(x) = 1- \lambda(x)$. Comparison between the DET and BAT technique is also recommended for future study.

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