

## Application of Sivasubramanian Kalimuthu Hypothesis to Triangles

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**Abstract: Problem statement:** The interior angles sum of a number of Euclidean triangles was transformed into quadratic equations. The analysis of those quadratic equations yielded the following proposition: There exists Euclidean triangle whose interior angle sum is a straight angle. **Application:** In this study, the researchers introduced a new hypothesis for quadratic equations and derived an entirely new result. **Results:** The result of the study was controversial, but mathematically consistent. **Conclusion/Recommendations:** The researchers politely requested the research community to establish the quadratic equation's hypothesis.

**Key words:** Quadratic equations, tachyon physics, Euclid, elements, postulates, triangles and angles

### INTRODUCTION

For two thousand years, many attempts were made to prove the parallel postulate using Euclid's first four postulates. The main reason that such a proof was highly sought after was that the fifth postulate isn't so evident unlike the other postulates. If the order that the postulates were listed in the Elements is significant, it indicates that Euclid included this postulate lastly when he realised he could not prove it or prove it without it.

Ibn Al-Haytham (Alhazen) (965-1039), an Iraqi mathematician, made the first attempt at proving the parallel postulate using a proof by contradiction, where he introduced the concept of projection and transformation into geometry. He formulated Lambert's quadrilateral, which was later named after Johann Lambert. Abramovich Shenfeld names the "Ibn al-Haytham Lambert quadrilateral" and his attempted proof also shows similarities to Playfair's axiom.

Omar Khayyám (1050-1123) made the first attempt at formulating a non-Euclidean postulate as an alternative to the parallel postulate and he was the first to consider the cases of elliptical geometry and hyperbolic geometry, though he excluded the latter. The Khayyám-Sadr al-Din quadrilateral was also first mentioned by Omar Khayyám in the late 11th century in Book I of Expositions of the Difficulties in the Postulates of Euclid. Unlike many commentators on Euclid before and after him (including Giovanni Saccheri and Girolamo Saccheri), Khayyám was not trying to prove the parallel postulate as such but to derive it from an equivalent postulate: "Two convergent straight lines

intersect and it is impossible for two convergent straight lines to diverge in the direction in which they converge. He realised that three possibilities arose from omitting Euclid's Fifth Postulate, if two perpendiculars to one line do not cross another line, judicious choice of the last can mean that the internal angles where it meets the two perpendiculars are equal (it is then parallel to the first line). If these equal internal angles are right angles, we get Euclid's Fifth; otherwise, they must be either acute or obtuse. He persuaded himself that the acute and obtuse cases lead to contradiction, but had made a tacit assumption equivalent to the fifth to get there.

Nasir al-Din al-Tusi (1201-1274), in his *Al-risala al-shafiya'an al-shakk fi'l-khutut al-mutawaziya* (Discussion Which Removes Doubt about Parallel Lines) (1250), wrote detailed critiques of the parallel postulate and on Khayyám's attempted proof a century earlier. Nasir al-Din attempted to derive a proof by contradiction of the parallel postulate. He was also one of the first to consider the cases of elliptical geometry and hyperbolic geometry, though he ruled out both of them.

Euclidean, elliptical and hyperbolic geometry. The Parallel Postulate is satisfied only for models of Euclidean geometry.

Nasir al-Din's son, Sadr al-Din (sometimes known as "Pseudo-Tusi"), wrote a book on the subject in 1298, based on Nasir al-Din's later thoughts, which presented one of the earliest arguments for a non-Euclidean hypothesis equivalent to the parallel postulate. "He essentially revised both the Euclidean system of axioms and postulates and the proofs of many propositions from

the elements. His research was published in Rome in 1594 and was studied by European geometers. This study marked the starting point for Saccheri's work on the subject.

Giordano Vitale (1633-1711), in his book *Euclide restituo* (1680, 1686), used the Khayyam-Saccheri quadrilateral to prove that if three points are equidistant on the base AB and the summit CD, then AB and CD are everywhere equidistant. Girolamo Saccheri (1667-1733) pursued the same line of reasoning more thoroughly, correctly obtaining absurdity from the obtuse case (proceeding, like Euclid, from the implicit assumption that lines can be extended indefinitely and have infinite length), but failing to debunk the acute case (although he managed to wrongly persuade himself that he had).

Where Khayyám and Saccheri had attempted to prove Euclid's fifth by disproving the only possible alternatives, the nineteenth century finally saw mathematicians exploring those alternatives and discovering the logically consistent geometries which result. In 1829, Nikolai Ivanovich Lobachevsky published an account of acute geometry in an obscure Russian journal (later re-published in 1840 in German). In 1831, János Bolyai included, in a book by his father, an appendix describing acute geometry, which doubtlessly, he had developed independently of Lobachevsky. Carl Friedrich Gauss had studied the problem before that, but he did not publish any of his results. However, upon hearing of Bolyai's results in a letter from Bolyai's father, Farkas Bolyai, he wrote:

"If I commenced by saying that I am unable to praise this study, you would certainly be surprised for a moment. But I cannot say otherwise. To praise it would be to praise myself. In the whole course of the work, the path taken by your son, the results to which he is led, coincide almost entirely with my meditations, which have occupied my mind pretty much for the last thirty or thirty-five years."

The resulting geometries were later developed by Lobachevsky, Riemann and Poincaré into hyperbolic geometry (the acute case) and spherical geometry (the obtuse case) independent of the parallel postulate. Euclid's fifth axiom was finally demonstrated to be independent of the other axioms by Eugenio Beltrami.

**Construction:** In the Euclidean construction as shown in Fig. 1.

Draw a triangle ABC. Choose points D and E on the base BC such that BD = DE = EC. Join A and D; Join A and E.

Let x, y and z denote the sum of the interior angles of triangles ABD, ADE and AEC respectively. Let a, b and c respectively refer to the sum of the interior angles in triangles ABE, ADC and ABC.

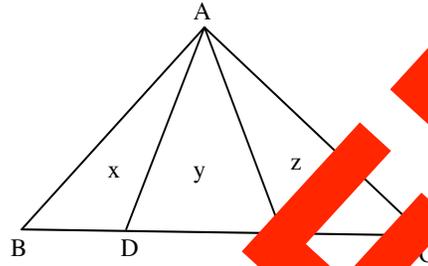


Fig. 1: Euclidean

Angles:

$$\angle ADB + \angle ADE = \angle ADE + \angle AEC = 180^\circ = v \text{ (say)} \quad (1)$$

Using (1)

$$x + y = v \quad (2)$$

$$y + z = v \quad (3)$$

$$x + b = v \quad (4)$$

Squaring (2):

$$x^2 + 2xy + y^2 = v^2 \quad (2a)$$

Squaring (4):

$$x^2 + 2xb + b^2 = v^2 \quad (4a)$$

From (2a):

$$x^2 + 2xy + y^2 - v^2 - a^2 - 2va = 0 \quad (2b)$$

From (4a):

$$x^2 + 2xb + b^2 - v^2 - c^2 - 2vc = 0 \quad (4b)$$

If  $\alpha$  and  $\beta$  are the roots of (2b), then according to the laws of quadratic equations,

$$\alpha + \beta = -\frac{B}{A} \quad (5)$$

and

$$\alpha\beta = \frac{C}{A} \quad (5a)$$

(5)+(5a) shows:

$$\alpha + \beta + \alpha\beta = \frac{C - B}{A} \quad (6)$$

Applying (6) in (2b):

$$\alpha + \beta + \alpha \beta = -2y - v^2 - a^2 - 2va \tag{7}$$

Assuming (6) in (4b):

$$\alpha + \beta + \alpha \beta = -2b - v^2 - c^2 - 2vc \tag{8}$$

Applying Sivasubramanian and Kalimuthu hypothesis in (7) and (8):

$$\begin{aligned} 2y + v^2 + a^2 + 2va &= 2b + v^2 + c^2 + 2vc \\ \text{i.e., } 2b - 2y + c^2 - a^2 + 2v(c - a) &= 0 \\ 2b - 2y + (c + a)(c - a) + 2v(c - a) &= 0 \\ \text{i.e., } 2b - 2y + (c - a)(c + a + 2v) &= 0 \end{aligned} \tag{9}$$

$$(4)-(2) \text{ gives } b - y = c - a \tag{10}$$

$$\text{Putting (10) in (9), } 2(c - a) + (c - a)(c + a + 2v) = 0$$

$$\text{i.e., } (c - a)(2v + 2 + c + a) = 0$$

Since the second factor can NOT be equal to zero,  $c - a = 0$ :

$$\text{i.e., } c = a \tag{11}$$

Putting (11) in (10):

$$y = b \tag{12}$$

Applying (12) in (3):

$$z = v = 180^\circ \text{ [by 1]} \tag{13}$$

From (13) we get that the sum of the interior angles of triangle:

$$\text{AEC is a straight angle} \tag{14}$$

### MATERIALS AND METHODS

Addition, subtraction, multiplication and division are the four fundamental operations of number theory. Multiplication is the shortest form of addition. And division is the shortest way of subtraction. So, these four operations are reduced to only two operations viz. addition and subtraction. By applying the "addition" operation of number theory, the triangle properties were transformed into linear algebraic equations and the linear algebraic equations were converted into quadratic

equations. The laws of set theory may be applied in future investigations.

### RESULTS

Although the result (14), i.e., the sum of interior angles of the Euclidean triangle  $\text{AEC} = 180^\circ$ , a right angle is controversial, this result is mathematically consistent. This result can be easily extended to both Lobachevsky, Riemann triangles.

### DISCUSSION

Let us recall that the famous French mathematician Legendre showed that if the sum of the interior angles of any triangle is equal to two right angles, the parallel postulate is true. Since we have proved (14) without assuming the fifth Euclidean postulate, Eq. 14 proves the parallel postulate<sup>[1-3]</sup>. But the mere existence of consistent models of Non-Euclidean geometries demonstrate the Euclidean cannot be deduced from Euclid's postulate IV. But our result is consistent. There is something new in the measure of mathematics. Further studies will unlock this mystery.

### CONCLUSION

Equation 14 reveals that there is something hidden in the measures of mathematics. Only further studies will unlock this mystery. Future probe may give rise to a new field of geometry.

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