- A unit in the normal mode must pass through the partial failure mode
- A unit which is replaced or repaired in total failure go directly to the normal mode without passing through the partial failure mode
- Unit failure and repair rates are constants
- Failure rates and repair rates follow exponential distribution
- A repaired unit works as a good as new
- The system is down when both units are non-operative

Mean Time to System Failure (MTSF): In this study, the Mean Time to System Failure (MTSF) for the proposed system evaluated using the above-mentioned set of assumptions and method of linear first order differential equations. If we let P(t) denote the probability row vector at time t, the initial conditions for this problem are:

$$P(0) = [P_0(0)P_1(0)P_2(0)P_3(0)P_4(0)P_5(0)] = [1\ 0\ 0\ 0\ 0\ 0]$$

By employing the method of linear first order differential equations For Fig. 1 and we can obtain the following differential equations:

$$\begin{aligned}
&P_0'(t) = -\lambda_1 P_0(t) + \beta_1 P_1(t) + \mu_1 P_2(t) + \delta P_5(t)\\
&P_1'(t) = -(\alpha_1 + \beta_1)P_1(t) + \lambda_1 P_0(t)\\
&P_2'(t) = -(\lambda_2 + \mu_1)P_2(t) + \alpha_1 P_1(t) + \beta_2 P_3(t) + \mu_2 P_4(t)\\
&P_3'(t) = -(\alpha_2 + \beta_2)P_3(t) + \lambda_2 P_2(t)\\
&P_4'(t) = -(\mu_2 P_4(t) + h_2 P_2(t) + \alpha_2 P_3(t)\\
&P_5'(t) = -\delta P_5(t) + \gamma P_0(t)
\end{aligned} \tag{1}$$

The above system of differential equations can be written in the matrix form as:

$$P^* = Q \times P$$

Where:

$$Q = \begin{bmatrix}
-\lambda_1 + \gamma & \beta_1 & \mu_i & 0 & 0 & \delta\\
\lambda_1 & -(\alpha_1 + \beta_1) & 0 & 0 & 0 & 0\\
0 & \alpha_1 & -(\lambda_2 + \mu_1) & \beta_2 & \mu_2 & 0\\
0 & 0 & \lambda_2 & -(\alpha_2 + \beta_2) & 0 & 0\\
0 & 0 & 0 & \alpha_2 & -\mu_2 & 0\\
\gamma & 0 & 0 & 0 & 0 & -\delta
\end{bmatrix}$$

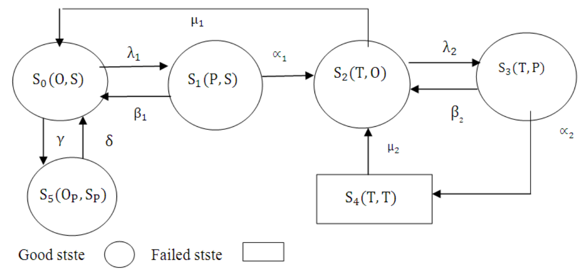

Fig. 1: State of the system

To calculate the MTSF we take the transpose matrix of Q and delete the rows and columns for the absorbing state, the new matrix is called A. the expected time to reach an absorbing state is calculated from:

$$MTSF = P(0)(-A^{-1})\begin{bmatrix}1\\1\\1\\1\\1\end{bmatrix} \tag{2}$$

Where:

$$A = \begin{bmatrix}
-(\lambda_1 + \lambda) & \lambda_1 & 0 & 0 & \gamma\\
\beta_1 & -(\alpha_1 + \beta_1) & \alpha_1 & 0 & 0\\
\mu_i & 0 & -(\lambda_2 + \mu_1) & \lambda_2 & 0\\
0 & 0 & \beta_2 & -(\alpha_2 + \beta_2) & 0\\
\delta & 0 & 0 & 0 & -\delta
\end{bmatrix}$$

The steady state mean Time to System Failure (MTSF)is given by:

$$MTSF = \left(\frac{a_1 + a_2}{a_3}\right)$$

Where:

$$\begin{aligned}
&a_1 = (\gamma + \delta)(\alpha^2\lambda^2 - \alpha^2\mu^1 + \beta^2\mu^1)(\alpha^1 + \beta^1)\\
&a_2 = \delta\lambda^1(\alpha^1(\alpha^2 + \lambda^2) + \alpha^2(\lambda^2 + \mu^1) + \beta^2(\alpha^1 + \mu^1))\\
&a_3 = \delta(\alpha_1\alpha_2\lambda_1\lambda_2)
\end{aligned} \tag{3}$$

Availability analysis: The initial conditions for this problem are the same as for the reliability case.

P(0) = [1, 0, 0, 0, 0, 0, 0, 0, 0],the differential equations form can be expressed as:

$$\begin{bmatrix} P_0^* \\ P_1^* \\ P_2^* \\ P_3^* \\ P_4^* \\ P_5^* \end{bmatrix} = \begin{bmatrix} -(\lambda_1+\lambda) & \beta_1 & \mu_i & 0 & 0 & \delta \\ \lambda_1 & -(\alpha_1+\beta_1) & 0 & 0 & 0 & 0 \\ 0 & \alpha_1 & -(\lambda_2+\mu_1) & \beta_2 & \mu_2 & 0 \\ 0 & 0 & \lambda_2 & -(\alpha_2+\beta_2) & 0 & 0 \\ 0 & 0 & 0 & \alpha_2 & -\mu_2 & 0 \\ \gamma & 0 & 0 & 0 & 0 & -\delta \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{bmatrix}$$

In the steady state, the derivatives of the state probabilities become zero, i.e.:

$$QP(\infty) = 0 \tag{4}$$

Then the steady state probabilities can be calculated as follows:

$$A(\infty) = P_0(\infty)+P_1(\infty)+P_2(\infty)+P_7(\infty) \tag{5}$$

Then the matrix form became:

$$\begin{bmatrix} -(\lambda_1+\lambda) & \beta_1 & \mu_i & 0 & 0 & \delta \\ \lambda_1 & -(\alpha_1+\beta_1) & 0 & 0 & 0 & 0 \\ 0 & \alpha_1 & -(\lambda_2+\mu_1) & \beta_2 & \mu_2 & 0 \\ 0 & 0 & \lambda_2 & -(\alpha_2+\beta_2) & 0 & 0 \\ 0 & 0 & 0 & \alpha_2 & -\mu_2 & 0 \\ \gamma & 0 & 0 & 0 & 0 & -\delta \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

to obtain $P_0(\infty)+P_1(\infty)+P_2(\infty)+P_7(\infty)$ we solve the Eq. 4 by using following normalizing condition:

$$P_0(\infty)+P_1(\infty)+P_2(\infty)+P_3(\infty)+P_4(\infty) = 1 \tag{6}$$

We substitute the equation (6) in any one of the redundant rows in equation to (4) yield:

$$\begin{bmatrix} -(\lambda_1+\lambda) & \beta_1 & \mu_i & 0 & 0 & \delta \\ \lambda_1 & -(\alpha_1+\beta_1) & 0 & 0 & 0 & 0 \\ 0 & \alpha_1 & -(\lambda_2+\mu_1) & \beta_2 & \mu_2 & 0 \\ 0 & 0 & \lambda_2 & -(\alpha_2+\beta_2) & 0 & 0 \\ 0 & 0 & 0 & \alpha_2 & -\mu_2 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

The steady state availability $A(\infty)$ is given by:

$A(\infty) = P_0+P_1+P_2+P_3+P_5$, or

$$A(\infty) = 1-P_4 = 1-\frac{N}{D} \tag{7}$$

Where:
$N = \delta\alpha_1\alpha_2\lambda_1\lambda_2$
$D = (\gamma+\delta)\mu_1\mu_2(\alpha_1+\beta_1)(\alpha_2+\beta_2)+\delta(\mu_2\mu_1\lambda_1(\alpha_2+\beta_2)+ \mu_2\lambda_1\alpha_1(\beta_2+\lambda_2)+ \lambda_1\alpha_2\alpha_1(\lambda_2+\mu_2))$

Busy period analysis: The initial conditions for this problem are the same as for the reliability case: The differential equations form can be expressed as availability case. Then the steady state busy period $B\infty$ is given by:

$$\beta(\infty) = 1-(P_0(\infty)+P_7(\infty)) = 1-\frac{M}{D} \tag{8}$$

where, $M = \mu_1\mu_2(\gamma+\delta)(\alpha_2+\beta_2)(\alpha_1+\beta_1)$.

The expected frequency of preventive maintenance: The initial conditions for this problem are the same as for the reliability case. Then the steady state, the expected frequency of preventive maintenance per unit time $K\infty$ is given by:

$$K(\infty) = P_5(\infty) = \mu_1\mu_2\gamma(\alpha_2+\beta_2)(\alpha_1+\beta_1)/D) \tag{9}$$

Cost analysis: The expected total profit per unit time incurred to the system in the steady-state is given by:

Profit = Total revenue-total cost

$$PF = R\times A(\infty)-C_1\times B(\infty)-C_2\times K(\infty) \quad (10)$$

Where:

PF = The profit incurred to the system
R = The revenue per unit up-time of the system
C_1 = The cost per unit time which the system is under repair
C_2 = The cost per preventive maintenance

Special case: When the preventive maintenance is not available,

The mean time to system failure is given by:

$$MTSF = \left(\frac{b_1 + b_2}{b_3}\right) \quad (11)$$

Where:

$$b_1 = (\alpha_2\lambda_2 + \alpha_2\mu_1 + \beta_2\mu_1)(\alpha_1 + \beta_1)$$
$$b_2 = \lambda_1(\alpha_1(\alpha_2 + \lambda_2) + \alpha_2(\lambda_2 + \mu_1) + \beta_2(\alpha_1 + \mu_1))$$
$$b_3 = \alpha_1\alpha_2\lambda_1\lambda_2$$

The steady state availability of the system is given by:

$$A(\infty) = P_0+P_1+P_2+P_3 \text{ or}$$

$$A_1(\infty) = 1 - P_4 = 1 - \frac{N_1}{D_1} \quad (12)$$

Where:

$$N_1 = \alpha_1\alpha_2\lambda_1\lambda_2$$
$$D_1 = \mu_1\mu_2(\alpha_1+\beta_1)(\alpha_2+\beta_2)+\mu_2\mu_1\lambda_1(\alpha_2+\beta_2)+ \mu_2\lambda_1\alpha_1(\beta_2+\lambda_2)+\lambda_1\alpha_2\alpha_1(\lambda_2+\mu_2)$$

The steady state busy period of the system is given by:

$$\beta_1(\infty) = 1 - (P_0 + P_5) = 1 - \frac{M_1}{D_1} \quad (13)$$

where, $M_1 = \mu_1\mu_2(\alpha_1+\beta_1)(\alpha_2+\beta_2)$.

The expected total profit incurred to the system in the steady-state is given by:

$$PF = R\times A_1(\infty)-C_1\times B_1(\infty) \quad (14)$$

MATERIALS AND METHODS

Many authors have studied two-unit cold standby system with two types of operation and repair. The question was raised whether the preventive maintenance increases the reliability of the system.

In this study the MTTF, availability and cost analysis of a two-dissimilar-unit repairable redundant system with three states and preventive maintenance were discussed to show the system with preventive maintenance increase the reliability of the system.

We analyze the system by using Kolmogorov's forward equations method. After the model is developed a particular case study is discussed to validate the theoretical results. Next, some numerical computations are derived to show the effect of preventive maintenance on the reliability of the system.

RESULTS

If we put $\lambda_2 = 0.02$, $\alpha_1 = 0.03$, $\alpha_2 = 0.04$, $\beta_1 = 0.05$, $\beta_2 = 0.06$, $\gamma = 0.02$, $\delta = 0.08$, $\mu_1 = 0.02$, $\mu_2 = 0.03$ in Eq. 3, 7, 8, 10 and Eq. 11-14 we get the following:

- Table 1 shows relation between failure rate (λ_1) and the MTSF of the system (with and without preventive maintenance)
- Table 2 shows relation between failure rate (λ_1) and availability of the system (with and without preventive maintenance)

Table 1: Relation between failure rate (λ_1) and the MTSF (with and without PM)

λ_1	MTSF of the system with preventive maintenance	MTSF of the system without preventive maintenance
1.00	1433.30	1200.00
0.02	850.00	733.33
0.03	655.56	577.78
0.04	558.33	500.00
0.05	500.00	453.33
0.06	461.11	422.22
0.07	433.33	400.00
0.08	412.50	383.33
0.09	396.30	370.37
0.10	383.33	360.00

Table 2: Relation between failure rate (λ_1) and availability (with and without PM)

λ_1	Availability of the system with preventive maintenance	Availability of the system without preventive maintenance
1.00	0.96970	0.96429
0.02	0.95122	0.94444
0.03	0.93878	0.93182
0.04	0.92982	0.92308
0.05	0.92308	0.91667
0.06	0.91781	0.91176
0.07	0.91358	0.90789
0.08	0.91011	0.90476
0.09	0.90722	0.90217
0.10	0.90476	0.90000

Table 3: Relation between failure rate (λ_1) and busy period (with and without PM)

λ_1	Busy period of the system with preventive maintenance	Busy period of the system without preventive maintenance
1.00	0.24242	0.28571
0.02	0.39024	0.44444
0.03	0.48980	0.54545
0.04	0.56140	0.61538
0.05	0.61538	0.66667
0.06	0.65753	0.70588
0.07	0.69136	0.73684
0.08	0.71910	0.76190
0.09	0.74227	0.78261
0.10	0.76190	0.80000

Table 4: Relation between failure rate (λ_1) and the profit (with and without PM)

λ_1	The profit of the system with preventive maintenance	The profit of the system without preventive maintenance
1.00	939.40	935.72
0.02	907.32	900.00
0.03	885.72	877.28
0.04	870.17	861.54
0.05	858.47	850.00
0.06	849.32	841.17
0.07	841.97	834.21
0.08	835.95	828.57
0.09	830.93	823.91
0.10	826.67	820.00

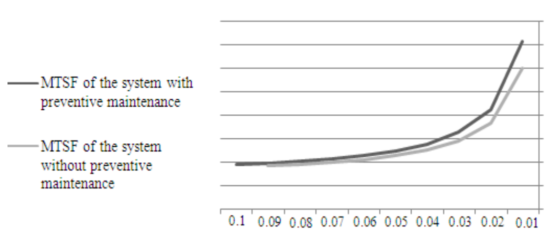

Fig. 2: Relation between the failure rate (λ_1) and the MTSF

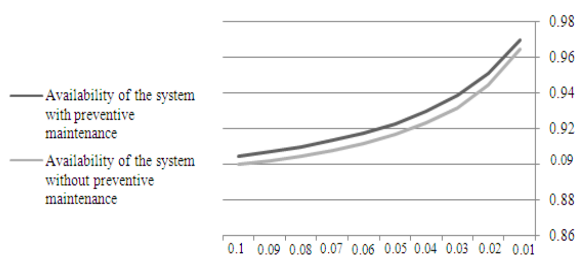

Fig. 3: Relation between the failure rate (λ_1) and the availability

- Table 3 shows relation between failure rate (λ_1) and busy period of the system (with and without preventive maintenance)
- Table 4 shows relation between failure rate (λ_1) and the profit of the system (with and without preventive maintenance)

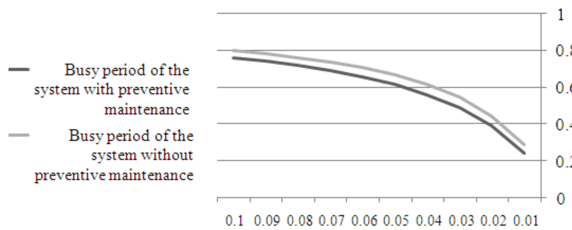

Fig. 4: Relation between the failure rate (λ_1) and the busy period

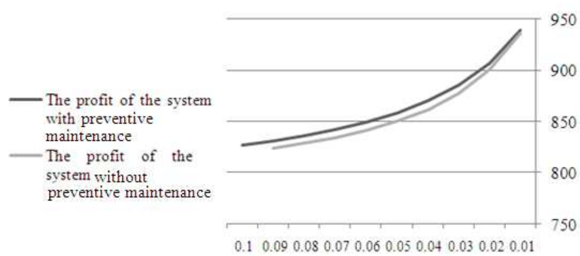

Fig. 5: Relation between the failure rate (λ_1) and the expected total profit

- Figure 2 shows relation between the failure rate (λ_1) and the MTSF
- Figure 3 shows relation between the failure rate (λ_1) and the Availability
- Figure 4 shows relation between the failure rate (λ_1) and the busy period
- Figure 5 shows relation between the failure rate (λ_1) and the expected total profit

DISCUSSION

By comparing the characteristic, MTSF, availability and the profit function with respect to (λ_1) for both systems with and without preventive maintenance graphically, it was observing that:

The increase of failure rate (λ_1) at constant $\lambda_2 = 0.02$, $\alpha_1 = 0.03$, $\alpha_2 = 0.04$, $\beta_1 = 0.05$, $\beta_2 = 0.06$, $\gamma = 0.02$, $\delta = 0.08$, $\mu_1 = 0.02$, $\mu_2 = 0.03$, $R = 1000$, $C_1 = 100$, $C_2 = 10$, the MTSF, availability and the profit function of the system decrease for both systems with and without preventive maintenance.

CONCLUSION

We conclude from the Fig. 1-4 that the system with preventive maintenance is greater than the system without preventive maintenance with respect to the MTSF, availability and the profit function, i.e., the system with preventive maintenance is better than the system without preventive maintenance.

REFERENCES

1. Lasanovsky, 1982. Availability of two-unit cold standby system with degraded state. IEEE, Trans. Reliab. R., 31: 123.
2. Hatogama, Y., 1997. Reliability analysis of three-state system. IEEE Trans. Reliab., R-28: 386-393.
3. Dhillon, B.S., 1980. A system with two kind of three-state elements. IEEE Trans. Reliab., R-29: 345-393.
4. Mokaddis, G.S. and M.L. Tawfek, Stochastic analysis of a two-similar unit Standby redundant system with two types of repair facilities. Microelect. Reliab., 35: 1467-1472. DOI: 10.1016/0026-2714(95)00180-A
5. Gopalan, M.N. and H.E. Nagarwalia, 1985. Cost benefit analysis of a one server two-unit cold stand by system with repair and preventive maintenance. Microelectr. Reliab., 25: 267-269.
6. Said, El., M. Kh. and M.S. El-Sherbeny, 2008. Cost analysis of a two unit cold standby system with preventive maintenance by using the Kolmogorov's forward equations method. Am. J. Applied Sci., 5: 405-410. http://www.encyclopedia.com/doc/1G1-149267629.html
7. Rander, M.C., K. Suresh and K. Ashok, 1994. Cost analysis of two dissimilar cold standby system with preventive maintenance and replacement of standby. Microelect. Reliab., 34: 171-174. http://cat.inist.fr/?aModele=afficheN&cpsidt=3937735
8. Goel, L.R., R. Gapta and S.K. Singh, 1985. Cost analysis of a two-unit cold standby system with two types of operation and repair. Microelect. Reliab., 25: 71-75. http://cat.inist.fr/?aModele=afficheN&cpsidt=9254299