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Modified Ratio-Type Estimators of the Mean using Extreme Ranked Set Sampling

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Abstract: Modified ratio-type estimators of the population mean were suggested using the first or third quartiles of the auxiliary variable based on Simple Random Sampling (SRS) and Extreme Ranked Set Sampling (ERSS) methods. It was found that, the ERSS was more efficient than the SRS based on the same sample size. Also, the ERSS involving the first quartile was superior to the third quartile.

Key words: Auxiliary variable, ratio estimator; simple random sampling, extreme ranked set sampling

INTRODUCTION

Let the bivariate random variable (X,Y) has $F(x,y)$ with means μ_X, μ_Y, variances σ_Y^2, σ_X^2 and correlation coefficient ρ.

Assume that the population mean μ_X of the auxiliary variable X is known to estimate the population mean μ_Y of the study variable Y. The classical ratio estimator of the population mean μ_Y using SRS is defined as

$$\hat{\mu}_{YSRS} = \mu_X\left(\frac{\bar{Y}_{SRS}}{\bar{X}_{SRS}}\right) \quad (1)$$

Where, $\bar{X}_{SRS} = \frac{1}{m}\sum_{i=1}^{m} X_i$, $\bar{Y}_{SRS} = \frac{1}{m}\sum_{i=1}^{m} Y_i$, $\mathrm{Var}(\bar{Y}_{SRS}) = \frac{\sigma_Y^2}{m}$ and $\mathrm{Var}(\bar{X}_{SRS}) = \frac{\sigma_X^2}{m}$ with mean square error (MSE) given by:

$$\mathrm{MSE}(\hat{\mu}_{YSRS}) \cong \frac{1-f}{m}\left(\sigma_Y^2 + R^2\sigma_X^2 - 2R\sigma_{XY}\right) \quad (2)$$

where, $f = \frac{m}{M}$; M is the population size; m is the sample size; σ_{XY} is the covariance between X and Y; $\sigma_{XY} = E\left((X_{SRS}-\mu_X)(Y_{SRS}-\mu_Y)\right)$ and $R = \frac{\mu_Y}{\mu_X}$ is the population ratio. For more details about ratio estimators see Cochran[3]. For more details about modified ratio estimators see Kadilar and Cingi[6].

The Ranked Set Sampling (RSS) was suggested by McIntyre[7] for estimating the population mean of pasture and forage yields. He showed that the RSS produce an unbiased estimator of the population mean with variance less than the one obtained using SRS with equal sample size. Takahasi and Wakimoto[10] provided the necessary mathematical theory of RSS. Samawi and Muttlak[9] proposed the used of RSS to estimate the population ratio. Samawi *et al.*[9]. suggested using extreme ranked set sampling for estimating the population mean. Jemain and Al-Omari[4] considered multistage median ranked set samples for estimating the population mean. Al-Omari and Jaber[5] suggested percentile double ranked set sampling for estimating the population mean. Al-Nasser[1] investigated L ranked set sampling in estimating the population mean.

MATERIALS AND METHODS

Let (X_1,Y_1), (X_2,Y_2),…,(X_m,Y_m) be a bivariate random sample of size m from F(x,y). Let (X_{11},Y_{11}), (X_{12},Y_{12}), ..., (X_{1m},Y_{1m}), (X_{21},Y_{21}), (X_{22},Y_{22}), ..., (X_{2m},Y_{2m}), …, (X_{m1},Y_{m1}), (X_{m2},Y_{m2}),… , (X_{mm},Y_{mm}) be m independent bivariate random samples each of size m.

Modified ratio estimator of the population mean under SRS: Let q_1 be the first quartile and q_3 the third quartile of the auxiliary variable X. The modified ratio

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estimator of the population mean based on SRS using the first and third quartiles, respectively are given by

$$\hat{\mu}_{YSRS1} = \bar{Y}_{SRS}\left(\frac{\mu_X + q_1}{\bar{X}_{SRS} + q_1}\right) \text{ and}$$

$$\hat{\mu}_{YSRS3} = \bar{Y}_{SRS}\left(\frac{\mu_X + q_3}{\bar{X}_{SRS} + q_3}\right) \quad (3)$$

where $\bar{X}_{SRS}$ and $\bar{Y}_{SRS}$ are the sample means of X and Y, respectively. Using Taylor series method $\hat{\mu}_{YSRS}$ can be approximated as:

$$\begin{aligned}\hat{\mu}_{YSRSh} &\cong \mu_Y - K_h\left(\bar{X}_{SRS} - \mu_X\right) + \left(\bar{Y}_{SRS} - \mu_Y\right) + K_h D_h \\ &\quad \left(\bar{X}_{SRS} - \mu_X\right)^2 - D_h\left(\bar{X}_{SRS} - \mu_X\right)\left(\bar{Y}_{SRS} - \mu_Y\right) \\ &\cong \bar{Y}_{SRS} - K_h\left(\bar{X}_{SRS} - \mu_X\right) + K_h D_h\left(\bar{X}_{SRS} - \mu_X\right)^2 \\ &\quad - D_h\left(\bar{X}_{SRS} - \mu_X\right)\left(\bar{Y}_{SRS} - \mu_Y\right)\end{aligned} \quad (4)$$

where $K_h = \frac{\mu_Y}{\mu_X + q_h}$, $D_h = \frac{1}{\mu_X + q_h}$ and $h = 1,3$.

Let us define the following two relations:

$$\sigma_{\bar{X}_{SRS}\bar{Y}_{SRS}} = \beta\frac{\sigma_X^2}{m} \quad (5)$$

and

$$\operatorname{Var}\left(\bar{Y}_{SRS}\right) \cong \beta^2\frac{\sigma_X^2}{m} + \frac{1}{m}\sigma_Y^2\left(1-\rho^2\right) \quad (6)$$

where, $\sigma_{\bar{X}_{SRS}\bar{Y}_{SRS}} = E\left(\left(\bar{X}_{SRS} - \mu_X\right)\left(\bar{Y}_{SRS} - \mu_Y\right)\right)$ and $\beta = \rho\frac{\sigma_Y}{\sigma_X}$.

Lemma 1: Using Taylor series expansion for the first degree of approximation we have:

- $\hat{\mu}_{YSRSh} \cong \bar{Y}_{SRS} - K_h\left(\bar{X}_{SRS} - \mu_X\right)$
- $\operatorname{Bias}\left(\hat{\mu}_{YSRSh}\right) \cong 0$
- $\operatorname{Var}\left(\hat{\mu}_{YSRSh}\right) \cong \left(K_h - \beta\right)^2\frac{\sigma_X^2}{m} + \left(1-\rho^2\right)\frac{\sigma_Y^2}{m}$

Proof:

- The proof is directly from Eq. 4 after deleting the last two terms
- To prove 2

$$\begin{aligned}\operatorname{Bias}\left(\hat{\mu}_{YSRSh}\right) &= E\left(\hat{\mu}_{YSRSh}\right) - \mu_Y \\ &\cong E\left(\bar{Y}_{SRS} - K_h\left(\bar{X}_{SRS} - \mu_X\right)\right) - \mu_Y \\ &= E\left(\bar{Y}_{SRS}\right) - K_h E\left(\bar{X}_{SRS}\right) + K_h\mu_X - \mu_Y \\ &= \mu_Y - K_h\mu_X + K_h\mu_X - \mu_Y = 0\end{aligned}$$

To prove 3, from (1) the variance is:

$$\begin{aligned}\operatorname{Var}\left(\hat{\mu}_{YSRSh}\right) &\cong \operatorname{Var}\begin{pmatrix}\mu_Y - K_h\left(\bar{X}_{SRS} - \mu_X\right) \\ +\left(\bar{Y}_{SRS} - \mu_Y\right)\end{pmatrix} \\ &= 0 + K_h^2\operatorname{Var}\left(\bar{X}_{SRS}\right) + \operatorname{Var}\left(\bar{Y}_{SRS}\right) - 2K_h\operatorname{Cov}\left(\bar{X}_{SRS}, \bar{Y}_{SRS}\right) \\ &= \frac{\sigma_Y^2}{m} + K_h^2\frac{\sigma_X^2}{m} - 2K_h\sigma_{\bar{X}_{SRS}\bar{Y}_{SRS}} \\ &= \beta^2\frac{\sigma_X^2}{m} + \frac{1}{m}\sigma_Y^2\left(1-\rho^2\right) + K_h^2\frac{\sigma_X^2}{m} - 2K_h\beta\frac{\sigma_X^2}{m}\end{aligned}$$

from 5 and 6.

$$\operatorname{Var}\left(\hat{\mu}_{YSRSh}\right) = \left(K_h - \beta\right)^2\frac{\sigma_X^2}{m} + \left(1-\rho^2\right)\frac{\sigma_Y^2}{m}.$$

Modified ratio estimator of the population mean under ERSS: The Extreme Ranked Set Sampling (ERSS) can be described as follows:

Step 1: Randomly select m sets each of m bivariate units from the target population.

Step 2: Rank the units within each set with respect to a variable of interest by visual inspection or any other cost free method.

Step 3: For actual measurement, if the sample size m is even, from the first $\frac{m}{2}$ sets select the lowest ranked unit X together with the associated Y and from the other $\frac{m}{2}$ sets select the largest ranked unit X together with the associated Y. If the sample size is odd, from the first $\frac{m-1}{2}$ sets select the lowest ranked unit X together with the associated Y and from the other $\frac{m-1}{2}$ sets select the largest ranked unit X together with the associated Y and from the remaining set the median ranked unit X together with the associated Y. The procedure can be repeated n times if needed to obtain nm units.

As shown by[9], the ranking on X is more efficient than the ranking on Y; therefore, in this study the ranking is performed on the auxiliary variable X.

Let $\left(X_{i(1)},Y_{i[1]}\right),\left(X_{i(2)},Y_{i[2]}\right),\ldots,\left(X_{i(m)},Y_{i[m]}\right)$ be the order statistics of $X_{i1},X_{i2},...,X_{im}$ and the judgment order of Y_{i1}, $Y_{i2},\ldots,Y_{im}$, $(i=1,2,...,m)$, where ( ) and [ ] refers that the ranking of X is perfect while the ranking of Y has errors. Let $\left(X_{i(1)},Y_{i[1]}\right)$ be the minimum of X together with the associated judgment minimum Y for the ith set $(i=1,2,...,m)$ and $\left(X_{i(m)},Y_{i[m]}\right)$ be the maximum of X together with the associated judgment maximum Y for the ith set $(i=1,2,...,m)$ and let $\left(X_{\frac{m+1}{2}\left(\frac{m+1}{2}\right)},Y_{\frac{m+1}{2}\left[\frac{m+1}{2}\right]}\right)$ be the median of X together with the associated judgment median Y for the set of the rank $i=\frac{m+1}{2}$. The ERSS estimator of the population mean μ_Y from a bivariate sample of size m is given by:

$$\hat{\mu}_{YERSS1}=\bar{Y}_{ERSS}\left(\frac{\mu_X+q_1}{\bar{X}_{ERSS}+q_1}\right)\text{ and }\quad \hat{\mu}_{YERSS3}=\bar{Y}_{ERSS}\left(\frac{\mu_X+q_3}{\bar{X}_{ERSS}+q_3}\right) \tag{7}$$

where q_1 and q_3 are the first and third quartiles of the auxiliary variable X, respectively.

If m is even, then $(X_{1(1)},X_{1[1]})$, $(X_{2(1)},X_{2[1]})$,

$$,...,\left(X_{\frac{m}{2}(1)},Y_{\frac{m}{2}[1]}\right),\left(X_{\frac{m+2}{2}(m)},Y_{\frac{m+2}{2}[m]}\right),\left(X_{\frac{m+4}{2}(m)},Y_{\frac{m+4}{2}[m]}\right),...,$$

$(X_{m(m)},X_{m[m]})$, denote the measured ERSSE, where

$$\bar{X}_{ERSSE}=\frac{1}{m}\left(\sum_{i=1}^{\frac{m}{2}}X_{i(1)}+\sum_{i=\frac{m+2}{2}}^{m}X_{i(m)}\right)\text{ with}$$

$$\mathrm{Var}\left(\bar{X}_{ERSSE}\right)=\frac{1}{2m}\left(\sigma^2_{X(1)}+\sigma^2_{X(m)}\right)$$

and

$$\bar{Y}_{ERSSE}=\frac{1}{m}\left(\sum_{i=1}^{\frac{m}{2}}Y_{i[1]}+\sum_{i=\frac{m+2}{2}}^{m}Y_{i[m]}\right)\text{ with}$$

$$\mathrm{Var}\left(\bar{Y}_{ERSSE}\right)=\frac{1}{2m}\left(\sigma^2_{Y[1]}+\sigma^2_{Y[m]}\right)$$

If m is odd, then $(X_{1(1)},X_{1[1]})$, $(X_{2(1)},X_{2[1]})$,…, $\left(X_{\frac{m-1}{2}(1)},Y_{\frac{m-1}{2}[1]}\right)$, $\left(X_{\frac{m+1}{2}\left(\frac{m+1}{2}\right)},Y_{\frac{m+1}{2}\left[\frac{m+1}{2}\right]}\right)$, $\left(X_{\frac{m+3}{2}(m)},Y_{\frac{m+3}{2}[m]}\right)$, …, $\left(X_{m(m)},Y_{m[m]}\right)$ denote the measured ERSSO, where

$$\bar{X}_{ERSSO}=\frac{1}{m}\left(\sum_{i=1}^{\frac{m-1}{2}}X_{i(1)}+X_{\frac{m+1}{2}\left(\frac{m+1}{2}\right)}+\sum_{i=\frac{m+3}{2}}^{m}X_{i(m)}\right)$$

With

$$\mathrm{Var}\left(\bar{X}_{ERSSO}\right)=\frac{1}{m^2}\left(\frac{m-1}{2}\left(\sigma^2_{X(1)}+\sigma^2_{X(m)}\right)+\sigma^2_{X\left(\frac{m+1}{2}\right)}\right)$$

And

$$\bar{Y}_{ERSSO}=\frac{1}{m}\left(\sum_{i=1}^{\frac{m}{2}}Y_{i[1]}+Y_{\frac{m+1}{2}\left[\frac{m+1}{2}\right]}+\sum_{i=\frac{m+3}{2}}^{m}Y_{i[m]}\right)$$

With

$$\mathrm{Var}\left(\bar{Y}_{ERSSO}\right)=\frac{1}{m^2}\left(\frac{m-1}{2}\left(\sigma^2_{Y[1]}+\sigma^2_{Y[m]}\right)+\sigma^2_{Y\left[\frac{m+1}{2}\right]}\right)$$

Either the sample size m is even or odd, we denote to the ERSS estimator of the population mean as $\hat{\mu}_{YERSSh}$. Using Taylor series method the estimators in 7can be approximated as:

$$\begin{aligned}\hat{\mu}_{YERSSh}&\cong\mu_Y-K_h\left(\bar{X}_{ERSS}-\mu_X\right)+\left(\bar{Y}_{ERSS}-\mu_Y\right)+K_hD_h\\&\quad\left(\bar{X}_{ERSS}-\mu_X\right)^2-D_h\left(\bar{X}_{ERSS}-\mu_X\right)\left(\bar{Y}_{ERSS}-\mu_Y\right)\\&\cong\bar{Y}_{ERSS}-K_h\left(\bar{X}_{ERSS}-\mu_X\right)+K_hD_h\left(\bar{X}_{ERSS}-\mu_X\right)^2\\&\quad-D_h\left(\bar{X}_{ERSS}-\mu_X\right)\left(\bar{Y}_{ERSS}-\mu_Y\right)\end{aligned} \tag{8}$$

Let us define the following two relations:

$$\mathrm{Cov}\left(\bar{X}_{ERSS},\bar{Y}_{ERSS}\right)=\begin{cases}\frac{\beta}{2m}\left(\sigma^2_{X(1)}+\sigma^2_{X(m)}\right),\text{m is even}\\ \frac{\beta}{m^2}\left(\frac{m-1}{2}\left(\sigma^2_{X(1)}+\sigma^2_{X(m)}\right)+\sigma^2_{X\left(\frac{m+1}{2}\right)}\right),\text{m is odd}\end{cases} \tag{9}$$

And

$$\mathrm{Var}\left(\bar{Y}_{ERSS}\right)\cong\begin{cases}\frac{\beta^2}{2m}\left(\sigma^2_{X(1)}+\sigma^2_{X(m)}\right)+\frac{\sigma^2_Y}{m}\left(1-\rho^2\right),\text{m is even}\\ \frac{\beta^2}{m^2}\left(\begin{array}{l}\frac{m-1}{2}\left(\sigma^2_{X(1)}+\sigma^2_{X(m)}\right)\\+\sigma^2_{X\left(\frac{m+1}{2}\right)}\end{array}\right)+\frac{\sigma^2_Y}{m}\left(1-\rho^2\right),\text{m is odd}\end{cases} \tag{10}$$

Lemma 2: Using Taylor series expansion for the first degree of approximation we have:

- $\hat{\mu}_{YERSSh} \cong \bar{Y}_{ERSS} - K_h\left(\bar{X}_{ERSS} - \mu_X\right)$
- $\text{Bias}\left(\hat{\mu}_{YERSSh}\right) \cong 0$

$$\text{Var}\left(\hat{\mu}_{YERSSh}\right) \cong \begin{cases} \frac{1}{m}\left(\begin{array}{l}(K_h-\beta)^2\frac{1}{2}\left(\sigma^2_{X(1)}+\sigma^2_{X(m)}\right)\\ +\sigma^2_Y\left(1-\rho^2\right)\end{array}\right), \text{ m is even} \\ \frac{1}{m}\left(\begin{array}{l}\frac{(K_h-\beta)^2}{m}\left(\begin{array}{l}\frac{m-1}{2}\left(\sigma^2_{X(1)}+\sigma^2_{X(m)}\right)\\ +\sigma^2_{X\left(\frac{m+1}{2}\right)}\end{array}\right)\\ +\sigma^2_Y\left(1-\rho^2\right)\end{array}\right), \text{ m is odd}\end{cases}$$

Proof:

- The proof of (1) is directly from Eq. (8) after deleting the last two terms
- To prove 2

$$\begin{aligned}\text{Bias}\left(\hat{\mu}_{YERRSh}\right) &= E\left(\hat{\mu}_{YERRSh}\right) - \mu_Y \\ &\cong E\left(\bar{Y}_{YERRSh} - K_h\left(\bar{X}_{YERRSh} - \mu_X\right)\right) - \mu_Y \\ &= E\left(\bar{Y}_{YERRSh}\right) - K_h E\left(\bar{X}_{YERRSh}\right) + K_h\mu_X - \mu_Y \\ &= \mu_Y - K_h\mu_X + K_h\mu_X - \mu_Y = 0\end{aligned}$$

To prove 3, from (1), if the sample size is even the variance is:

$$\begin{aligned}\text{Var}\left(\hat{\mu}_{YERRSh}\right) &\cong \text{Var}\left(\mu_Y - K_h\left(\bar{X}_{YERRSh} - \mu_X\right) + \left(\bar{Y}_{YERRSh} - \mu_Y\right)\right) \\ &= 0 + K_h^2\text{Var}\left(\bar{X}_{YERRSh}\right) + \text{Var}\left(\bar{Y}_{YERRSh}\right) - \\ &\quad 2K_h\text{Cov}\left(\bar{X}_{YERRSh}, \bar{Y}_{YERRSh}\right) \\ &= \frac{\beta^2}{2m}\left(\sigma^2_{X(1)} + \sigma^2_{X(m)}\right) + \frac{\sigma^2_Y}{m}\left(1-\rho^2\right) + K_h^2\frac{1}{2m} \\ &\quad \left(\sigma^2_{X(1)} + \sigma^2_{X(m)}\right) - 2K_h\frac{\beta}{2m}\left(\sigma^2_{X(1)} + \sigma^2_{X(m)}\right)\end{aligned}$$

from 9 and 10.

$$\text{Var}\left(\hat{\mu}_{YERRSh}\right) = \frac{1}{m}\left(\frac{(K_h-\beta)^2}{2}\left(\sigma^2_{X(1)} + \sigma^2_{X(m)}\right) + \sigma^2_Y\left(1-\rho^2\right)\right)$$

For odd sample size the proof is the same. From the two lemmas (1) and (2) it can be noted that the suggested estimators are approximately unbiased for the first degree of Taylor series expansion.

Now, we will show mathematically that $\hat{\mu}_{YERSSh}$ is more efficient than $\hat{\mu}_{YSRSh}$ for first degree of approximation. The efficiency of $\hat{\mu}_{YERSSh}$ with respect to $\hat{\mu}_{YSRSh}$ for estimating the population mean of Y is defined as:

$$\text{eff}\left(\hat{\mu}_{YSRSh}, \hat{\mu}_{YERSSh}\right) = \frac{\text{Var}\left(\hat{\mu}_{YSRSh}\right)}{\text{Var}\left(\hat{\mu}_{YERSSh}\right)}$$

$$\cong \begin{cases}\dfrac{(K_h-\beta)^2\sigma^2_X + \left(1-\rho^2\right)\sigma^2_Y}{\frac{(K_h-\beta)^2}{2}\left(\sigma^2_{X(1)} + \sigma^2_{X(m)}\right) + \left(1-\rho^2\right)\sigma^2_Y}, \text{m is even} \\ \dfrac{(K_h-\beta)^2\sigma^2_X + \left(1-\rho^2\right)\sigma^2_Y}{\frac{(K_h-\beta)^2}{m}\left(\begin{array}{l}\frac{m-1}{2}\left(\sigma^2_{X(1)} + \sigma^2_{X(m)}\right)\\ +\sigma^2_{X\left(\frac{m+1}{2}\right)}\end{array}\right) + \sigma^2_Y\left(1-\rho^2\right)}, \text{m is odd}\end{cases} \tag{11}$$

Since both $(K_h-\beta)^2$ and $\frac{\sigma^2_Y}{m}\left(1-\rho^2\right)$ are fixed in the numerator and denominator in the case of an even and odd sample size and since for symmetric distributions if m even we have $\frac{1}{2}\left(\sigma^2_{X(1)} + \sigma^2_{X(m)}\right) < \sigma^2_X$ and if m is odd $\frac{1}{m}\left(\frac{m-1}{2}\left(\sigma^2_{X(1)} + \sigma^2_{X(m)}\right) + \sigma^2_{X\left(\frac{m+1}{2}\right)}\right) < \sigma^2_X$ see Samawi *et al.*[9] and Jemain *et al.*[5], then $\text{eff}\left(\hat{\mu}_{YSRSh}, \hat{\mu}_{YERSSh}\right) > 1$. This implies that $\hat{\mu}_{YERSSh}$ is more efficient than $\hat{\mu}_{YSRSh}$ for estimating the population mean.

RESULTS AND DISCUSSION

Simulation study: In this section, a simulation study is conducted to study the properties of ERSS estimators in estimating the population mean of Y. The samples were generated from bivariate normal distribution $BN(2,4,1,1,\rho)$ where $\rho = \pm 0.99$, ± 0.90, ± 0.80, ± 0.70 ± 0.50. In this simulation the approximation will be for higher degrees of Taylor expansion, so that small bias will appear throughout the simulation. Therefore, the efficiency of $\hat{\mu}_{YERSSh}$ with respect to $\hat{\mu}_{YSRSh}$ for estimating the population mean of *Y* is defined as:

$$\text{eff}\left(\hat{\mu}_{YSRSh}, \hat{\mu}_{YERSSh}\right) = \frac{\text{MSE}\left(\hat{\mu}_{YSRSh}\right)}{\text{MSE}\left(\hat{\mu}_{YERSSh}\right)}$$

$$\cong \frac{\text{Var}\left(\hat{\mu}_{YSRSh}\right) + \left(\text{Bias}\left(\hat{\mu}_{YSRSh}\right)\right)^2}{\text{Var}\left(\hat{\mu}_{YERRSh}\right) + \left(\text{Bias}\left(\hat{\mu}_{YERRSh}\right)\right)^2} \tag{12}$$

Table 1: The efficiency and bias values of $\hat{\mu}_{YERSS1}$ with respect to $\hat{\mu}_{YSRS1}$ for $m = 2,3,4,5,6$ with different values of ρ using q_1

ρ		m = 2	m = 3	m = 4	m = 5	m = 6
0.99	eff	1.7046	1.7461	1.7628	1.9254	1.8603
	Bias ERSS	0.0243	0.0119	0.0080	0.0049	0.0041
	Bias SRS	0.0387	0.0242	0.0173	0.0140	0.0108
0.90	eff	1.3199	1.3150	1.3037	1.3158	1.3084
	Bias ERSS	0.0320	0.0167	0.0112	0.0095	0.0051
	Bias SRS	0.0542	0.0350	0.0258	0.0204	0.0162
0.80	eff	1.3160	1.3104	1.2822	1.3114	1.3021
	Bias ERSS	0.0454	0.0200	0.0127	0.0109	0.0076
	Bias SRS	0.0743	0.0436	0.0302	0.0232	0.0213
0.70	eff	1.3263	1.3319	1.3204	1.3484	1.3372
	Bias ERSS	0.0563	0.0270	0.0202	0.0123	0.1142
	Bias SRS	0.0868	0.0536	0.0378	0.0343	0.0265
0.50	eff	1.5212	1.3892	1.3809	1.4318	1.3782
	Bias ERSS	0.0814	0.0446	0.0268	0.0149	0.0144
	Bias SRS	0.1251	0.0795	0.0576	0.0444	0.0400
-0.99	eff	1.8421	2.2376	2.4043	2.7614	2.9622
	Bias ERSS	0.2459	0.1235	0.0807	0.0531	0.0443
	Bias SRS	0.3766	0.2459	0.1760	0.1382	0.1127
-0.90	eff	1.7112	2.2001	2.2833	2.6774	2.5738
	Bias ERSS	0.2410	0.1191	0.0794	0.0543	0.0494
	Bias SRS	0.3721	0.2366	0.1732	0.1404	0.1119
-0.80	eff	1.6609	2.0880	2.2235	2.4885	2.4089
	Bias ERSS	0.2374	0.1081	0.0712	0.0513	0.0437
	Bias SRS	0.3601	0.2199	0.1614	0.1301	0.1056
-0.70	eff	1.6640	2.0088	2.0822	2.3533	2.3258
	Bias ERSS	0.2090	0.1062	0.0655	0.0501	0.0352
	Bias SRS	0.3326	0.2174	0.1514	0.1209	0.1006
-0.50	eff	1.6429	1.9234	1.9161	2.1547	2.0841
	Bias ERSS	0.1879	0.0929	0.0672	0.0392	0.0363
	Bias SRS	0.3008	0.1929	0.1362	0.1123	0.0923

The efficiency and bias values of $\hat{\mu}_{YSRSh}$ and $\hat{\mu}_{YERSSh}$ are obtained based on 60,000 replications. The results involving $q_1 = 1.32551$ and $q_3 = 2.67449$ for $m = 2,3,4,5,6$ are presented in Tables 1 and 2, respectively.

Based on Table 1 and 2, the following remarks can be concluded:

- The use of the first or third quartiles of the auxiliary variable improved the efficiency of the suggested estimators. As an example, for $m = 5$, $\rho = 0.80$ and $q_1 = 1.32551$ the efficiency of $\hat{\mu}_{YERSS1}$ is 1.3114
- It is noted that, for the same quartile, the bias of $\hat{\mu}_{YERSSh}$ is less than the bias of $\hat{\mu}_{YSRSh}$ for different values of the sample size or correlation coefficient
- The negative values of ρ produces higher values of the efficiency than the positive values. For example, for $q_1 = 1.32551$, $m = 4$ with $\rho = 0.70$ the efficiency is 1.3204 while with $\rho = -0.70$ is the efficiency 2.0822

Table 2: The efficiency and bias values of $\hat{\mu}_{YERSS3}$ with respect to $\hat{\mu}_{YSRS3}$ for $m = 2,3,4,5,6$ with different values of ρ using q_3

ρ		m = 2	m = 3	m = 4	m = 5	m = 6
0.99	eff	1.2766	1.3523	1.4036	1.4559	1.4337
	Bias ERSS	-0.0098	-0.0054	-0.0035	-0.0024	-0.0014
	Bias SRS	-0.0154	-0.0098	-0.0070	-0.0065	-0.0047
0.90	eff	1.0352	1.0301	1.0130	1.0144	1.0257
	Bias ERSS	-0.0033	-0.0023	-0.0014	-0.0004	-0.0008
	Bias SRS	-0.0062	-0.0024	-0.0015	-0.0019	-0.0021
0.80	eff	1.0371	1.0326	1.0367	1.0278	1.0267
	Bias ERSS	0.0057	0.0023	0.0011	0.0009	0.0013
	Bias SRS	0.0060	0.0059	0.0017	0.0036	0.0025
0.70	eff	1.0434	1.0478	1.0516	1.0488	1.0466
	Bias ERSS	0.0119	0.0072	0.0025	0.0041	0.0031
	Bias SRS	0.0156	0.0108	0.0075	0.0058	0.0041
0.50	eff	1.0906	1.1068	1.1155	1.1203	1.1113
	Bias ERSS	0.0252	0.0120	0.0080	0.0087	0.0045
	Bias SRS	0.0425	0.2687	0.0164	0.0172	0.0135
-0.99	eff	1.5852	2.0205	2.1850	2.6091	2.5755
	Bias ERSS	0.1435	0.0789	0.0562	0.0324	0.0265
	Bias SRS	0.2094	0.1393	0.1045	0.0835	0.0678
-0.90	eff	1.5631	1.9597	2.1021	2.4196	2.3398
	Bias ERSS	0.1405	0.0648	0.0443	0.0322	0.0244
	Bias SRS	0.2138	0.1284	0.1021	0.0736	0.0666
-0.80	eff	1.4675	1.8502	1.9685	2.2440	2.1979
	Bias ERSS	0.1274	0.0606	0.0472	0.0293	0.0217
	Bias SRS	0.1882	0.1252	0.0935	0.0680	0.0592
-0.70	eff	1.4532	1.7386	1.8658	2.0651	2.0466
	Bias ERSS	0.1251	0.0621	0.0354	0.0263	0.0244
	Bias SRS	0.1699	0.1112	0.0819	0.0663	0.0571
-0.50	eff	1.3934	1.6031	1.6571	1.8208	1.7905
	Bias ERSS	0.1046	0.0546	0.0382	0.0253	0.0206
	Bias SRS	0.1638	0.0980	0.0728	0.0663	0.0507

- The efficiency is increasing as the magnitude of the correlation coefficient increases for negative values of ρ or example with $q_3 = 2.67449$, $m = 3$ and $\rho = -0.50, -0.70$, -0.80,-0.90,-0.99 the efficiency values are 1.6031, 1.7386, 1.8502, 1.9597 and 2.0205, respectively. For positive values of p the efficiency depends on ρ. For example, for $q_3 = 2.67449$, $m = 5$ the efficiency values for $\rho = 0.99$, 0.90, 0.80, 0.70 and 0.50 are 1.4559, 1.0144, 1.0278, 1.0448 and 1.1203, respectively

For the same value of ρ and quartile, the efficiency is increasing in the sample size for the most cases considered in this study

CONCLUSIONS

In this study, new ratio-type estimators of the population mean are suggested using ERSS when the first or third quartile of the auxiliary variable is known. It is found that, to the first order approximation the SRS and ERSS estimators are approximately unbiased. But the ERSS estimators have small bias than its counterparts using SRS. Also, it is found that the efficiency based on the first quartile is greater than that based on the third quartile.

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