Journal of Mathematics and Statistics 1 (3): 234-238, 2005
ISSN 1549-3644

On the Prime Radical of a Hypergroupoid

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Abstract: In this study, we give definitions of a prime ideal, a s-semiprime ideal and a w-semiprime ideal for a hypergroupoid K. For an ideal A of K we show that radical of A (R(A)) can be represented as the intersection of all prime ideals of K containing A and we define a strongly A-nilpotent element. For any ideal A of K, we prove that R(A)=∩(s-semiprime ideals of K containing A)= ∩(w-semiprime ideals of K containing A)={strongly A nilpotent elements}. For an ideal B of K put $B^{(o)}=B$ and $B^{(n+1)}=(B^{(n)})^2$. If a hypergroupoid K satisfies the ascending chain condition for ideals then $(R(A))^{(n)}\subseteq A$ for some n. For an ideal A of K we give a definition of right radical of A ($R_+(A)$). If K is associative then $R(A)=R_+(A)=R\_(A)$.

Key words: Hypergroupoids, s-semiprime ideal, w-semiprime ideal, ascending chain

1. Hypergroupoids and Complete ℓ-Groupoids

Definition 1.1: A groupoid K is a system (K, ·), where K is a set and · is a binary operation on K.

Definition 1.2[1]: A complete ℓ-groupoid is a system (K, ·), where K is a complete lattice and · is a binary operation on K which satisfies the following conditions:

$$a\cdot(\bigvee_{t\in T} b_t) = \bigvee_{t\in T}(a\cdot b_t),\quad (\bigvee_{t\in T} b_t)\cdot a = \bigvee_{t\in T}(b_t\cdot a)$$

for all $a, b_t \in K$

Let K be a set and denote by 2^K the set of all its subsets.

Definition 1.3[2]: A multivariable binary operation on K is a map $\vartheta: KxK\to 2^K$. A hypergroupoid is a system (K, ϑ), where K is a set and ϑ is a multivariable operation on K.

From now on, we write a·b instead of ϑ (a, b)

Let (K, ·) be a hypergroupoid. For A, B $\in 2^K$. $A\neq\emptyset$, $B\neq\emptyset$, put $A\cdot B=\bigcup_{a\in A, b\in B}(a\cdot b)$ and $\emptyset\cdot A=A\cdot\emptyset=\emptyset$ for all $A\in 2^K$. Then $(2^K, \cdot)$ is a complete ℓ-groupoid.

Conversely, If $(2^K, \cdot)$ is a complete ℓ-groupoid then a restriction of the binary operation of 2^K to K is a multivariable operation on K and K is a hypergroupoid, with respect to this operation.

Let w be a ternary relation on K.

For (a, b)∈KxK, put a·b={x∈K| (a, b, x)∈w}, then (K, ·) is a hypergrupoid.

Conversely, let (K, ·) be a hypergroupoid. Denote by w the set (a, b, c)∈KxKxK such that $a\cdot b\neq\emptyset$ and c∈a·b. Then w is a ternary relation on K.

Hypergroupoids contain the following two classes of algebraic systems.

1. A partial binary operation ϑ on K is a map ϑ:A→K, where A is a subset of KxK. A partial groupoid is a system (K, ·), where · is a partial binary operation on K.
   Let (K, ·) be a partial groupoid and A is the definition domain of ·. For (a, b)∉A put $a\cdot b=\emptyset$. Then · is defined for all (a, b)∈KxK and (K, ·) is a hypergroupoid.
2. Let $\{k, \vartheta_V, v\in S\}$ be a universal algebra such that every ϑ_V is a binary operation on K. For (a, b)∈KxK put $a\cdot b=\{\vartheta_v(a, b), v\in S\}$ then (K, ·) is a hypergroupoid.

2. Prime and Semiprime Elements of an Ordered Gruopoid:

Let (G, ·) be an ordered groupoid[1], ch XIV). An ordered groupoid G is called ℓ_o-groupoid if G is a complete lattice. Denote by 1_G the greatest element of G.

Definition 2.1[1]: Let (G, ·) be an ℓ_o-groupoid. An element p∈G is prime if $p\neq 1_G$ and a·b≤p, for a, b∈G, then a≤p or b≤p.

For a∈G, $a\neq 1_G$, denote by $R_G(a)$ the intersection of all prime elements of G containing a. Put $R_G(a)=1_G$ if there are not any element with this property.

Definition 2.2: An element h∈G is s-semiprime if $h\neq 1_G$ and $a^2\leq h$, for a∈G, implies that a≤h.

For a∈G, $a\neq 1_G$, denote by $r_G^S(a)$ the intersection of all s-semiprime elements of G containing a. Put $r_G^S(a)=1_G$ if there are not any element with this property. For a∈G denote by <a> the groupoid generated by a. An element of the groupoid <a> will be denoted by f(a).

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Definition 2.3: An element $h\in G$ is w-semiprime if $h\neq 1_G$ and $f(a)\leq h$, $a\in G$, $f(a)\in\langle a\rangle$ implies that $a\leq h$.

Therefore every w-semiprime element is s-semiprime. For $a\in G$, $a\neq 1_G$, denote by $r_G^w(a)$ the intersection of all w-semiprime elements of G containing a. Put $r_G^w(a)=1_G$ if there are not any element with this property. It is clear that $r_G^s(a)\leq r_G^w(a)\leq R_G(a)$ for all $a\in G$.

3. The Prime Radical of an Ideal

Definition3.1: Let K be a hypergroupoid. A right (left) ideal of K is a subset H such that $ha\subseteq H$ (respectively $a\cdot h\subseteq H$) for all $a\in K$, $h\in H$. An (two-side) ideal of K is a subset H such that $ha\subseteq H$ and $ah\subseteq H$ for all $a\in K$, $h\in H$. Denote by Id(K) ($Id_+(K)$, $Id\_(K)$) the set of all ideals (respectively, right ideals, left ideals) of K. Put $\varnothing\in Id(K)$, $\varnothing\in Id_+(K)$, $\varnothing\in Id\_(K)$. Then Id(K), $Id_+(K)$, $Id\_(K)$ are complete lattices with respect to the inclusion relation.

Proposition 3.2: Let K be an hypergroupoid. Then:

1. $\bigcap_{t\in T} A_t\in Id(K)$ and $\bigcup_{t\in T} A_t\in Id(K)$ for any $A_t\in Id(K)$;
2. $\bigcap_{t\in T} B_t\in Id_+(K)$ and $\bigcup_{t\in T} B_t\in Id_+(K)$ for any $_t\in Id_+(K)$;
3. $\bigcap_{t\in T} C_t\in Id\_(K)$ and $\bigcup_{t\in T} C_t\in Id\_(K)$ for any $C_t\in Id\_(K)$.

The proof is clear. We next consider the multiplication operation $A\cdot B$ on 2^K.

Definition 3.3: Hypersemigroup is a hypergroupoid K such that $(A\cdot B)\cdot C=A\cdot(B\cdot C)$ for any A, B, $C\in 2^K$.

If K is hypersemigroup then $A\cdot B\in Id(K)$ for any A, $B\in Id(K)$. But there are a hypergroupoid K and A, $B\in Id(K)$ such that $A\cdot B\notin Id(K)$. Therefore for any hypergroupoid K we define a multiplication operation of ideals as follows:

For A, $B\in Id(K)$ denote by $A\cdot B$ the intersection of all ideals of K containing the set $G=\{x|x=a\cdot b,\ a\in A,\ b\in B\}$

Multiplication operations on $Id_+(K)$ and $Id\_(K)$ are introduced similarly.

Proposition 3.4: For any hypergroupoid K, the lattices Id(K), $Id_+(K)$, $Id\_(K)$ are complete ℓ-groupoids with respect to above multiplication operations.

Proof: We give a proof for Id(K) and the proofs for $Id_+(K)$ and $Id\_(K)$ are similar. Suppose A, $B_t\in Id(K)$, $t\in T$. It is clear that $A\cdot(\bigcup_{t\in T} B_t)\supseteq \bigcup_{t\in T}(A\cdot B_t)$

Conversely the ideal $A\cdot(\cup B_t)$ is the smallest ideal containing all elements $a\cdot b$, where $a\in A$, $b\in \bigcup_{t\in T} B_t$.

Let a, $b\in A\cdot(\bigcup_{t\in T} B_t)$.

Since $b\in B_t$ for some $t\in T$ then $a\cdot b\in A\cdot B_t$. Therefore $A\cdot(\bigcup_{t\in T} B_t)\subseteq \bigcup_{t\in T}(A\cdot B_t)$

Now, we apply the definitions and designations of the prime and semiprime elements of ordered groupoids to 2^K, Id(K), $Id_+(K)$, $Id\_(K)$. Put

$R_G(A)=R(A)$, $r_G^s(A)=r^s(A)$, $r_G^w(A)=r^w(A)$ for $G=Id(K)$, $A\in Id(K)$

$R_G(A)=R_+(A)$, $r_G^s(A)=r_+^s(A)$, $r_G^w(A)=r_+^w(A)$ for $G=Id_+(K)$, $A\in Id_+(K)$

$R_G(A)=R\_(A)$, $r_G^s(A)=r_-^s(A)$, $r_G^w(A)=r_-^w(A)$ for $G=Id\_(K)$, $A\in Id\_(K)$

$R_G(A)=R_o(A)$, $r_G^s(A)=r_o^s(A)$, $r_G^w(A)=r_o^w(A)$ for $G=2^K$, $A\in 2^K$.

For $A\in Id(K)$ the ideal R(A) will be called radical of A. An ideal A is called radical if A=R(A)

Definition 3.5: An ideal H is maximal if $H\neq K$ and $H\subseteq B\subseteq K$, $B\in Id(K)$ implies that H=B or B=K.

For $a\in K$ denote by [a] the intersection of all ideals of K containing a.

Proposition 3.6: Let K be a hypergruopoid. Then any maximal ideal of K is prime if and only if $K=K^2$.

Proof: Let $K=K^2$ and M be a maximal ideal of K. Assume that $A\cdot B\subseteq M$, A, $B\in Id(K)$. If $A\not\subset B$ and $B\not\subset M$ then $A\cup M=K$, $B\cup M=K$. Therefore $K\cdot K=(A\cup M)(B\cup M)=A\cdot B\cup AM\cup MB\cup MM\subseteq M\subseteq K$ by Proposition 3.4. Hence M=K. This is a contradiction. Thus M is prime.

Conversely, Let $K^2\neq K$ and $a\in K\backslash K^2$. We prove that $M=K\backslash\{a\}$ is a maximal ideal of K and it is not prime. Let $b\in M\backslash\{a\}$. Then $hb\in M$ and $bh\in M$ for all $h\in K$. Indeed, if there is $h\in K$ such that $hb\notin M$ then $a\in hb$.

Hence $a\in K^2$. It is a contradiction. Thus $hb\in M$ and $bh\in M$ for any $h\in K$. It is clear that M is a maximal ideal. Prove that M is not prime. By $a\notin M$ we have $[a]\not\subset M$. But $[a]\subseteq K^2\subseteq M$. Therefore M is not prime.

Remark: This proposition is known for semigroups[5].

Every sequence $\{x_o, x_1,..., x_n,...\}$, where $x_o=a$, $x_{n+1}\in[x_n]^2$, will be called an s-sequence of the element a.

Definition 3.7: Let $A\in Id(K)$. An element $a\in K$ is strongly A-nilpotent if every s-sequence of a meets A.

Remark: This definition is similar to the definition of the n-sequence[6].

Denote by n(A) the set of all strongly A-nilpotent elements of K.

Theorem 3.8: Let K be a hypergroupoid. Then for any ideal A of K, we have $n(A)=r^s(A)=r^w(A)=R(A)$.

Proof: From the definitions $r^s(A)$, $r^w(A)$, $R(A)$ we obtain $r^s(A)\subseteq r^w(A)\subseteq R(A)$ for any $A\in Id(K)$.

We prove that $n(A)\subseteq r^s(A)$. If there is not an s-semiprime ideal of K containing A then $r^s(A)=K$ and $n(A)\subseteq r^s(A)$.

Assume that there exists an s-semiprime ideal of K containing A. Let $a\in n(A)$ and S be s-semiprime ideal of K containing A. We first prove that $a\in S$. If $a\notin S$, then $[x_o]\not\subset S$, where $x_o=a$. There exists $x_1\in [x_o]^2$ such that $x_1\notin S$ since $[x_o]^2\not\subset S$.

By continuing in this manner we obtain an s-sequence $\{x_o, x_1, ..., x_n, ...\}$ of the element a such that $x_n\notin S$ for all n. But this is a contradiction since every s-sequence of the element a meets A. Thus $a\in S$ and $a\in r^s(A)$ since S is any semiprime ideal containing A. Hence $n(A)\subseteq r^s(A)\subseteq r^w(A)\subseteq R(A)$.

Now we prove that $R(A)=n(A)$. If $n(A)=K$ then $n(A)=r^s(A)=r^w(A)=R(A)=K$. Let $n(A)\neq K$. Hence there exists $b\in K$ such that $b\notin n(A)$. Then there exists an s-sequence $X=\{x_o, x_1, ..., x_n, ...\}$ of the element b such that $X\cap A=\varnothing$. Denote by Σ the set of ideals M in K such that $X\cap M=\varnothing$, $M\supseteq A$. Σ is not empty since $A\in\Sigma$.

We can apply Zorn's lemma to the set Σ so there exists a maximal element P of Σ. We show that P is prime.

First, P is proper since $b\notin P$. Let $B, C\in Id(K)$, $B\not\subset P$, $C\not\subset P$. Then $P\cup B\neq P$ and $P\cup C\neq P$. By the maximality of P in Σ. We have $P\cup B\notin\Sigma$ and $P\cup C\notin\Sigma$. Hence there exist $x_m\in X$, $x_q\in X$ such that $x_m\in P\cup B$, $x_q\in P\cup C$. Then $[x_m]\subseteq P\cup B$, $[x_q]\subseteq P\cup C$. Hence $x_{m+1}\in [x_m]^2\subseteq P\cup B$, $x_{q+1}\in [x_q]^2\subseteq P\cup C$. By continuing in this manner we find $x_{m+1}\in P\cup B$, $x_{q+1}\in P\cup C$ for all t. Put n=max(m, q). Then $x_n\in P\cup B$, $x_n\in P\cup C$. Hence, $x_{n+1}\in [x_n]^2 \subseteq (P\cup B)\cdot(P\cup C)\subseteq P\cup B\cdot C$ by the Proposition 3.4. But $x_{n+1}\notin P$. Hence $B\cdot C\not\subset P$. Therefore P is prime. Thus there exists a prime ideal P such that $b\notin P$. Thus $n(A)=r^s(A)=r^w(A)=R(A)$. From the Theorem 3.8, we obtain that every s-semiprime ideal of K is radical.

The ideal $R(\varnothing)$ will be called the prime radical of the hypergroupoid K and will be denoted by Pr. rad(K).

Corollary 3.9: For any ideal A of K the following conditions are equivalent:

1. $R(A)=A$;
2. If $B^{(n)}\subseteq A$, $B\in Id(K)$, for some n then $B\subseteq A$.
3. If $B^2\subseteq A$, $B\in Id(K)$, then $B\subseteq A$.

Proof: (1)⇒(2)⇒(3) is clear. (3)⇒(2): Let $B^{(n)}\subseteq A$, $B\in Id(K)$, for some n. Then $B^{(n)}=(B^{(n-1)})^2\subseteq A$ implies that $B^{(n-1)}\subseteq A$. By induction on n we obtain $B\subseteq A$. (2)⇒(1): The condition (2) implies that $r^s(A)=A$. By the Theorem 3.8 we see $R(A)=r^s(A)=A$.

Corollary 3.10: For a hypergropoid K the following conditions are equivalent:

1. Every ideal of K is radical;
2. $A\cdot B=A\cap B$ for all $A, B\in Id(K)$;
3. $[a]^2=[a]$ for all $a\in K$.

Proof: We use the following lemma:

Lemma 3.11: $R(A\cdot B)=R(A\cap B)=R(A)\cap R(B)$ for any $A, B\in Id(K)$.

The proof of this lemma follows from the Proposition 1.6[7].

(1)⇒(2): If every ideal of K is radical then using the lemma we obtain $A\cdot B=R(A\cdot B)=R(A)\cap R(B)=A\cap B$. (2)⇒(3): Let $A\cdot B=A\cap B$ for all $A, B\in Id(K)$. Then $A^2=A$ for all $A\in Id(K)$. (3)⇒(1): We prove that every ideal of K is s-semiprime. Let A be an ideal of K. Then $A=\bigcup_{a\in A}[a]$. Using the Proposition 3.4 we have $A^2=(\bigcup_{a\in A}[a])^2=(\bigcup_{a\in A}[a]^2)\cup(\bigcup_{a\in A}[a][b])=\bigcup_{a\in A}[a]=A$ since $[a]\cdot[b]\subseteq[a]\cap[b]$ for any $a, b\in A$. Thus $A^2=A$ for all $A\in Id(K)$. Assume that $B^2\subseteq A$, $B\in Id(K)$. Then $B=B^2\subseteq A$. Therefore A is s-semiprime. From the Theorem 3.8 we obtain that A is radical.

Remark: This corollary is an analog of the similar theorem for associative rings[8].

Definition 3.12: Let $A\in Id(K)$. An ideal B of K is A_s-nilpotent if $B^{(n)}\subseteq A$ for some n.

Proposition 3.13: Let K be hypergroupoid and $A, B\in Id(K)$. If C is B_s-nilpotent and B is A_s-nilpotent then C is A_s-nilpotent.

Proof: Since C is B_s-nilpotent then $C^{(n)}\subseteq B$ for some n. Hence $C^{(n+m)}=(C^{(n)})^{(m)}\subseteq B^{(m)}\subseteq A$ for some m.

Theorem 3.14: Let K be a hypergroupoid satisfying the ascending chain condition for ideals. Then for any ideals A of K, R(A) is A_s-nilpotent.

Proof: Let $A\in Id(K)$. Denote by Σ the set of all A_s-nilpotent ideals H of K such that $H\supseteq A$. Σ is not empty since $A\in\Sigma$. There exists a maximal element P in Σ. We prove that P is s-semiprime. Let $B^2\subseteq P$. Then $(B\cup P)^2=B^2\cup BP\cup PB\cup P^2\subseteq P$. By Proposition 3.13 the ideal $B\cup P$ is A_s-nilpotent. By the maximality of P we have $B\cup P=P$. Hence $B\subseteq P$. This means that P is s-semiprime. Since $P\supseteq A$ then $R(A)\subseteq P$ by Theorem 3.8. But $P^{(n)}\subseteq A\subseteq R(A)$ for some n. Since R(A) is s-semiprime then $P\subseteq R(A)$. Thus $P=R(A)$

Remark: This theorem is similar to the proposition for associative rings[9].

Corollary 3.15: Let K be hypergroupoid satisfying the ascending chain condition for ideals. Then the following conditions are equivalent:

1. $K^{(n)}=\varnothing$ for some n.
2. K doesn't have a prime ideal;
3. K doesn't have a s-semiprime ideal.

A proof follows from Theorem 3.14 and the definition of Pr. rad(K). Denote by $Id_r(K)$ the set of all radical ideals of K. $Id_r(K)$ is a complete lattice with respect to the inclusion relation. Denote by $\vee$ and $\wedge$ the lattice operations in $Id_r(K)$.

Theorem 3.16: Let K be a hypergroupoid. Then the lattice $Id_r(K)$ satisfies the infinite $\wedge$-distributive condition:

$A\wedge(\vee_{t\in T} B_t)=\vee_{t\in T}(A\wedge B_t)$ for any A, $B_t\in Id_r(K)$

Proof: The proof follows from Theorem 1.3[7].

Theorem 3.17: Let K be a hypergroupoid satisfying the ascending chain candition for ideals. Then any radical ideal of K is an intersection of finite prime ideals and a such representation is unique.

Proof: First we prove the following lemma.

Lemma: $H\in Id_r(K)$ is prime ideal if and only if H is an $\wedge$-indecomposable element of the lattice $Id_r(K)$.

Proof: Let A be a prime ideal of K and $A=A_1\wedge A_2$, A_1, $A_2\in Id_r(K)$. Then[7].

$A_1A_2\subseteq A_1\cap A_2\subseteq R(A_1\cap A_2)=A_1\wedge A_2=A$. Hence $A_1\subseteq A$ or $A_2\subseteq A$. Then $A=A_1$ or $A=A_2$ Let A be an $\wedge$-indecomposable element in $Id_r(K)$ and $BC\subseteq A$, B, $C\in Id(K)$. Then $R(B\cdot C)\subseteq A$. By the lemma 1.6[7] we have $R(B)\wedge R(C)=R(B\cdot C)\subseteq A$. By the distributivity $Id_r(K)$ we obtain $A=A\vee(R(B)\wedge R(C))=(A\vee R(B))\wedge(A\vee R(C))$. Then $A=A\vee R(B)$ or $A=A\vee R(C)$ since A is $\wedge$-indecomposable. This means that $B\subseteq R(B)\subseteq A$ or $C\subseteq R(C)\subseteq A$.

Thus A is prime. The lemma is proved. By the lemma and the Corollary[1] we obtain that every radical ideal of K is an intersection of finite prime ideals and a such represantation is unique.

4. The Right Prime Radical of an Ideal

Definition 4.1: A right ideal H of K is maximal if $H\neq K$ and $H\subseteq B\subseteq K$, $B\in Id_+(K)$, implies that H=B or B=K.

Proposition 4.2: Let K be a hypergroupoid such that $A\subseteq K\cdot A$ for all $A\in Id_+(K)$. Then any maximal right ideal of K is prime element of $Id_+(K)$.

Proof: Let M be a maximal right ideal of K and $A\cdot B\subseteq M$, A, $B\in Id_+(K)$. If $A\not\subseteq M$ then $M\cup A=K$. By Proposition 3.4 we have $B\subseteq K\cdot B=(M\cup A)\cdot B=MB\cup AB\subseteq M$.

Definition 4.3: An element $1\in K$ is called identity of K if $1\cdot a=a\cdot 1=a$ for all $a\in K$.

Remark: The conditions of Proposition 4.2 are satisfied for groupoids with 1. Thus there exists a prime right ideal in such groupoids.

For an element $a\in K$ denote by $[a]_+$ the intersection of all right ideals containing a. Every sequence $\{x_o, ..., x_n, ...\}$, where $x_o=a$, $x_{m+1}\in[x_m]_+^2$, is called an s_+-sequence of the element a.

Definition 4.4: Let $A\in Id_+(K)$. An element $a\in K$ is strongly A_+-nilpotent if every its s_+-sequence meets A.

Denote by $n_+(A)$ the set of all strongly A_+-nilpotent elements of K.

Proposition 4.5: Let K be a hypergroupoid. For any right ideal A of K are satisfied the following inequalities:

$R(A)\subseteq n_+(A)\subseteq r_+^S(A)\subseteq r_+^W(A)\subseteq R_+(A)$.

Proof: A proof of $n_+(A)\subseteq r_+^S(A)$ is similar to the proof of $n(A)\subseteq r^s(A)$ as in the Theorem 3.8. The inequality $R(A)\subseteq n_+(A)$ immediately follows from the equality $R(A)=n(A)$ and definitions of n(A) and $n_+(A)$.

Theorem 4.6: Let K be a hypergroupoid satisfying the following conditions:

$(K\cdot A)\cdot B=K\cdot(A\cdot B)$, $(A\cdot K)\cdot B=A\cdot(K\cdot B)$ for all A, $B\in Id_+(K)$. Then

$R(A)=n_+(A)=r_+^S(A)=r_+^W(A)=R_+(A)$ for any $A\in Id(K)$.

Proof: By Proposition 4.5 it is enough to prove that $R_+(A)\subseteq R(A)$.

Denote by P(K) the set of all prime ideals of K and by $P_+(K)$ the set of all prime right ideals of K. We prove that $P(K)\subseteq P_+(K)$. Let $Q\in P(K)$ and $B\cdot C\subseteq Q$, B, $C\in Id_+(K)$. Then, $(B\cup K\cdot B)(C\cup K\cdot C)=(B\cdot C)\cup(B\cdot(KC))\cup((K\cdot B)\cdot C)\cup(K\cdot B)\cdot(K\cdot C)\subseteq Q$.

Note that $B\cup KB$ and $C\cup KC$ are ideals of K. Indeed $K\cdot(B\cup KB)=K\cdot B\cup(K\cdot(K\cdot B))\subseteq B\cup KB$.

From $(B\cup KB)(C\cup KC)\subseteq Q$ we obtain $B\subseteq B\cup KB\subseteq Q$ or $C\subseteq C\cup KC\subseteq Q$ since Q is prime. This means $Q\in P_+(K)$.

Thus $P(K)\subseteq P_+(K)$. Therefore we have $R_+(A)\subseteq R(A)$.

Remark: The conditions of this theorem are satisfied for hypersemigroup. Therefore the same theorem is given for nonasociative hypergroupoid K and $A\in Id(K)$ such that $R(A)=R_+(A)$ and $R(A)\neq R\_(A)$.
Let $A\in Id_+(K)$. For $b\in K$ put $b^{(o)}=b$, $b^{(n+1)}=(b^{(n)})^2$.

Definition 4.7: An element $b\in K$ is A_s-nilpotent if $b^{(n)}\subset A$ for some n. An element $b\in K$ is A_w-nilpotent if $f(b)\subseteq A$ for some $f(b)\subseteq\langle b\rangle$.

Denote by $n_o^s(A)$ ($n_o^w(A)$) the set of all A_s-nilpotent (respectively, A_w-nilpotent) elements of K.

Proposition 4.8: For any ideal A of K are hold the following inequalities:

$R(A)\subseteq n_+(A)\subseteq n_o^s(A)\subseteq r_o^s(A)\subseteq R_o(A)$

$R(A)\subseteq n_+(A)\subseteq n_o^w(A)\subseteq r_o^w(A)\in R_o(A)$

The proof is smilar to the proof of Proposition 4.5.

Theorem 4.9: Let K be a hypersemigroup satisfying the condition $K\cdot a=a\cdot K$ for all $a\in K$. Then $R(A)=n_o(A)=r_o(A)=R_o(A)$ for all $A\in Id(K)$.

The proof is smilar to the proof of Theorem 4.6.

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