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The Inverse Kinematics of the Plane System 2-3 in a Mechatronic MP2R System, by a Trigonometric Method

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Abstract: Robots have penetrated today in almost all industrial fields, being much more precise than humans in the execution of operations, but also faster, more dynamic, more stable and more resilient, working 24 h of the 24 h possible, in any season, breaks, holidays, vacations and especially without getting sick. The most important fact is that they can also perform special operations that man can't do. So, for example, they can work in toxic, contaminated, mined, dangerous, or airless environments (in the cosmos, under water or underground), they can also position the parts very precisely so they can be used for fine, special processing at high precision operations, excellent mechanical processing, exceptional medical operations and even in all medical operations today, helping the doctor manage the patient's surgery with amazing precision, especially when it comes to open heart, brain, kidneys, liver, etc. Robots should not be fed and yet they can have an extremely long life. They can help us conquer both the underwater spaces and the cosmic spaces. Robots have stunning work precision and a faster execution speed than a man. In addition, their very precise positioning makes robot operations a great advantage, which can no longer be neglected. In repetitive and tiring work they are irreplaceable. The most commonly used anthropomorphic robotic mechatronic systems, which are currently being used, have been studied by eliminating the heavy, matrix 3D spatial system, the study being simplified in a plan by considering the main work plan of the system and the plan, the rotation required to restore the spatial parameters of the anthropomorphic 3D system. In other words, we can greatly ease the work of the anthropomorphic robot engineer by moving from 3D systems to a 2D system. In this study we will study the inverse cinematics of the plan system, 2D, as the most important one. In general, end effector positions are known (required) and the positions of the 2D module elements and the necessary rotations of the actuators for their creation must be determined. In the kinematics we know the kinematic parameters x_M and y_M, which represent the scaled coordinates of the point M (endeffector M) and must be determined by analytical calculations the parameters φ_{20} and φ_{30}.

Keywords: Anthropomorphic Robots, Inverse Kinematics, 3D Calculation, 2D Calculation, A Trigonometric Method

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Introduction

The most commonly used anthropomorphic robotic mechatronic systems, which are currently being used, have been studied by eliminating the heavy, matrix 3D spatial system, the study being simplified in a plan by considering the main work plan of the system and the plan, the rotation required to restore the spatial parameters of the anthropomorphic 3D system.

In other words, we can greatly ease the work of the anthropomorphic robot engineer by moving from 3D systems to a 2D system. In this study we will study the inverse cinematics of the plan system, 2D, as the most important one. In general, end effector positions are known (required) and the positions of the 2D module elements and the necessary rotations of the actuators for their creation must be determined. In the kinematics we know the kinematic parameters x_M and y_M, which represent the scaled coordinates of the point M (endefactor M) and must be determined by analytical calculations the parameters φ_{20} and φ_{30}. First we determine the intermediate parameters d and φ with already known (3) relations.

Robots have penetrated today in almost all industrial fields, being much more precise than humans in the execution of operations, but also faster, more dynamic, more stable and more resilient, working 24 h of the 24 h possible, in any season, breaks, holidays, vacations and especially without getting sick.

The most important fact is that they can also perform special operations that man can't do. So, for example, they can work in toxic, contaminated, mined, dangerous, or airless environments (in the cosmos, under water or underground), they can also position the parts very precisely so they can be used for fine, special processing at high precision operations, excellent mechanical processing, exceptional medical operations and even in all medical operations today, helping the doctor manage the patient's surgery with amazing precision, especially when it comes to open heart, brain, kidneys, liver, etc.

Robots should not be fed and yet they can have an extremely long life. They can help us conquer both the underwater spaces and the cosmic spaces.

Robots have stunning work precision and a faster execution speed than a man. In addition, their very precise positioning makes robot operations a great advantage, which can no longer be neglected. In repetitive and tiring work they are irreplaceable.

Today the moving mechanical systems are utilized in almost all vital sectors of humanity (Reddy *et al*., 2012). The robots are able to process integrated circuits (Aldana *et al*., 2013) sizes micro and nano, on which the man they can be seen only with electron microscopy (Lee, 2013). Dyeing parts in toxic environments, working in chemical and radioactive environments (Padula and Perdereau, 2013; Perumaal and Jawahar, 2013), or at depths and pressures at the deep bottom of huge oceans, or conquest of cosmic space and visiting some new exoplanets, are with robots systems possible (Dong *et al*., 2013) and were turned into from the dream in reality (Garcia *et al*., 2007), because of use of mechanical platforms sequential gearbox (Cao *et al*., 2013; Petrescu, 2009).

The man will be able to carry out its mission supreme (Tang *et al*., 2013; Tong *et al*., 2013), conqueror of new galaxies (Flavio de Melo *et al*., 2012), because of mechanical systems sequential gear-box (robotics systems) (Garcia-Murillo *et al*., 2013).

Robots were developed and diversified (Lin *et al*., 2013), different aspects (He *et al*., 2013), but today, they start to be directed on two major categories: Systems serial (Liu *et al*., 2013; Petrescu and Petrescu, 2011b) and parallel systems (Petrescu and Petrescu, 2012c). Parallel systems are more solid (Tabaković *et al*., 2013; Wang *et al*., 2013) but more difficult to designed and handled and for this reason, the serial systems were those which have developed the most. In medical operations or radioactive environments are preferred mobile systems parallel, because of their high accuracy positioning.

As examples of such combined mechanisms, several kinematic schemes of gears and gears can be observed, presented by Kojevnikov (1969), (Autorenkollektiv, 1968), Şaskin (1963; 1971), Maros (1958), Rehwald and Luck (200; 2001), Antonescu (1993; 2003; Antonescu and Mitrache, 1989).

The main problems with plane and spatial gears and gears refer to kinematic analysis and geometric-kinematic synthesis under certain conditions imposed by technological processes, Bruja and Dima (2011), Buda and Mateucă (1989), Luck and Modler (1995), Niemeyer (2000), Tutunaru (1969), Popescu (1977), Braune (2000), Dudita (1989), Lichtenheldt (1995), Lederer (1993), Lin (1999), Modler and Wadewitz (1998; 2001; Modler, 1979), Neumann (1979; 2001), Stoica (1977), Petrescu and Petrescu (2011c; 2011d; Petrescu, 2012d; 2012e); (Petrescu, 2016; Petrescu *et al*., 2017a; 2017b; 2017c; 2017d; 2017e; 2017f; 2017g; 2017h; 2017i; 2017j; 2017k; 2017l; 2017m; 2017n; 2017o; 2017p; 2017q; Aversa *et al*., 2017a; 2017b; 2017c; 2017d; 2017e; 2016a; 2016b; 2016c; 2016d; 2016e; 2016f; 2016g; 2016h; 2016i; 2016j; 2016k; 2016l; 2016m; 2016n; 2016o; Mirsayar *et al*., 2017; Petrescu and Petrescu, 2016a; 2016b; 2016c, 2013a; 2013b; 2013c; 2013d, 2012a; 2012b; 2012c; 2012d; 2011a; 2011b; Petrescu, 2012a; 2012b; 2012c, 2009; Petrescu and Calautit, 2016a; 2016b; Petrescu *et al*., 2016a; 2016b; Maros, 1958; Modler and Wadewitz, 2001; Manolescu, 1968; Margine, 1999).

Materials and Methods; the Trigonometric Method

Figure 1 shows the geometric-kinematic scheme of a base structure 3R.

From this platform you can study by adding any other modern n-R scheme.

The platform (system) of Fig. 1 has three degrees of mobility, made by three actuators (electric motors) or actuators. The first electric motor trains the entire system in a rotation motion around a vertical axis O_0z_0. The motor (actuator) number 1 is mounted on the fixed member (bay, 0) and drives the mobile element 1 in a rotation motion around a vertical axis. On the mobile element 1, then all the other components (components) of the system are built.

There follows a planar (vertical) cinematic chain consisting of two movable elements and two kinematic motor couplings. It is the movable kinematic elements 2 and 3, the assembly 2,3 being moved by the second actuator mounted in the coupling A fixed on the element 1. Thus the second electric motor fixed by the element 1 will drive the element 2 in a relative rotation relative to element 1, but automatically it will move the entire kinematic chain 2-3.

The last actuator (electric motor) fixed by element 2 in B will rotate element 3 (relative to 2).

The rotation φ_{10} made by the first actuator is also relative (between elements 1 and 0) and absolute (between elements 1 and 0).

The rotation φ_{20} of the second actuator is also relative (between elements 2 and 1) and absolute (between elements 2 and 0) due to the positioning of the system.

The rotation $\theta = \varphi_{32}$ of the third actuator is only relative (between elements 3 and 2), the corresponding absolute (between elements 3 and 0) being a function of $\theta = \varphi_{32}$ and φ_{20}.

The kinematic chain 2-3 (made up of moving kinematic elements 2 and 3) is a planar cinematic chain that falls into one plane or one or more parallel planes. It is a special cinematic system that will be studied separately. The kinematic coupler A (O_2) and B (O_3) become the first fixed coupler and the second movable coupler, both of which are C5 cinematic couplers, of rotation.

The kinematic chain 2-3 having the degree of mobility 2 must be driven by two motors.

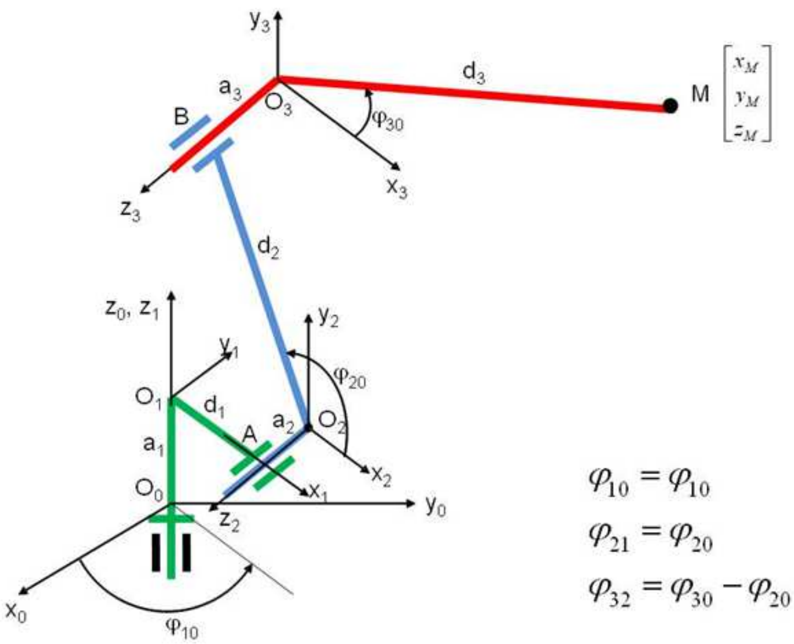
It is preferred that the two actuators are two electric, DC, or alternating motors. The action can also be done with other engines. Hydraulic, pneumatic, sonic, etc.

The schematic diagram of the planar kinematic chain 2-3 (Fig. 2) resembles its kinematic scheme.

The guide element 2 is connected to the fixed element 1 by the motor coupler O_2 and the drive element 3 is connected to the mobile element 2 by the motor coupler O_3.

This results in a two-degree open cinematic chain made by the two actuators, ie the two electric motors mounted in the kinematic couplers A and B or O_2 or O_3.

Figure 3 shows the kinematic diagram of the open 2-3 chain (Petrescu, 2014).



$$\varphi_{10} = \varphi_{10}$$
$$\varphi_{21} = \varphi_{20}$$
$$\varphi_{32} = \varphi_{30} - \varphi_{20}$$

Fig. 1: The geometric-kinematic scheme of a base structure 3R

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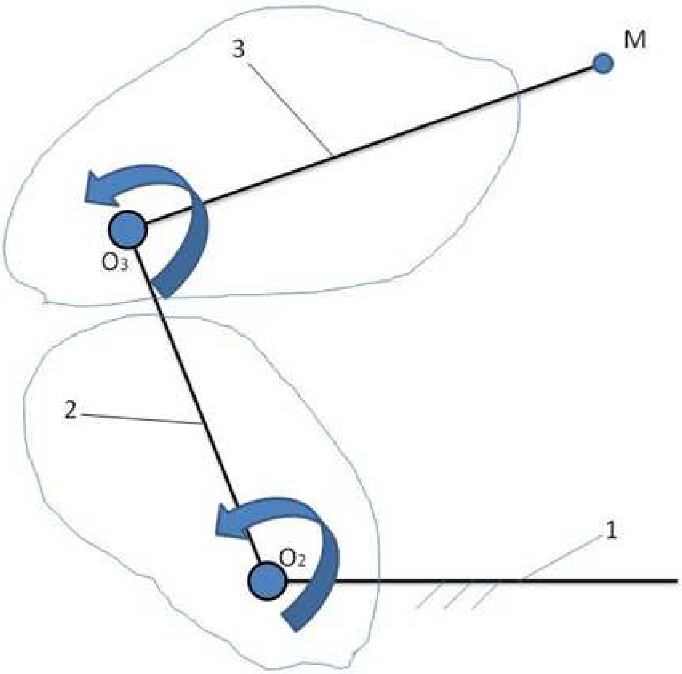

Fig. 2: The schematic diagram of the planar kinematic chain 2-3 bound to the element 1 considered to be fixed

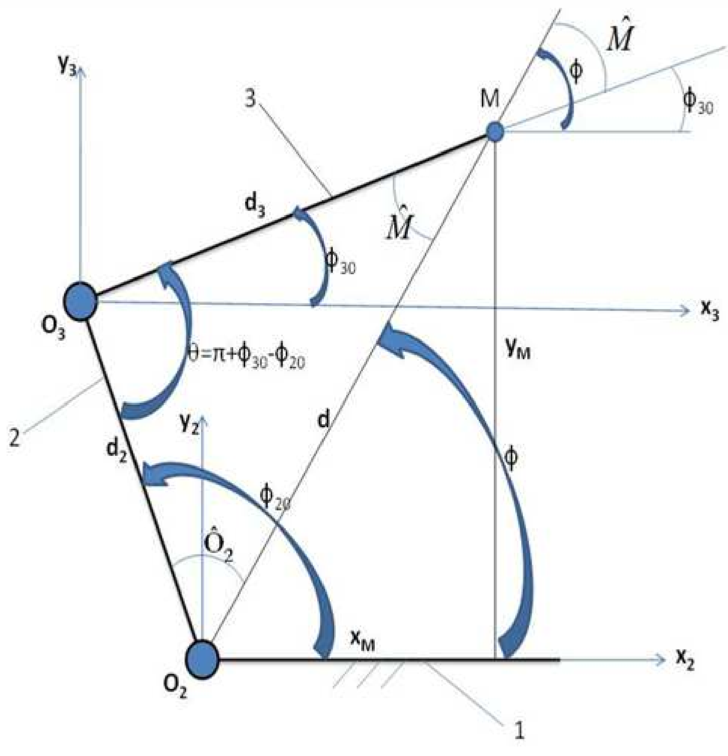

Fig. 3: The kinematic scheme of the planar kinematic chain 2-3 bound to element 1 considered fixed

In the kinematics we know the kinematic parameters x_M and y_M, which represent the scaled coordinates of the point M (endefactor M) and must be determined by analytical calculations the parameters φ_{20} and φ_{30}. First we determine the intermediate parameters *d* and φ with already known (1) relations:

$$\begin{cases} d^2 = x_M^2 + y_M^2; d = \sqrt{x_M^2 + y_M^2} \\ \cos\varphi = \dfrac{x_M}{d} = \dfrac{x_M}{\sqrt{x_M^2 + y_M^2}}; \\ \sin\varphi = \dfrac{y_M}{d} = \dfrac{y_M}{\sqrt{x_M^2 + y_M^2}} \\ \varphi = semn(\sin\varphi)\cdot\arccos(\cos\varphi) \end{cases} \tag{1}$$

In the triangle of some O_2O_3M we know the lengths of its three sides, d^2, d^3 (constants) and d (variable), so that all the other elements of the triangle, namely its angles and the trigonometric functions of (we are particularly interested in sin and cos).

Different methods can be used to determine the angles φ_{20} and φ_{30}, of which two of them (as being most representative) will be presented: The trigonometric method and the geometric method.

Determination of Positions

Scalar position equations are written (2):

$$\begin{cases} d_2\cdot\cos\varphi_{20} + d_3\cdot\cos\varphi_{30} = x_M \\ d_2\cdot\sin\varphi_{20} + d_3\cdot\sin\varphi_{30} = y_M \\ \cos^2\varphi_{20} + \sin^2\varphi_{20} = 1 \\ \cos^2\varphi_{30} + \sin^2\varphi_{30} = 1 \end{cases} \tag{2}$$

The problem of this two scalar, trigonometric, two unknown equations (φ_{20} and φ_{30}) is that they have been transcribed (they are trigonometric, transcendental equations, where the unknown does not appear directly φ_{20} but in the form of $\cos\varphi_{20}$ and $\sin\varphi_{20}$, so in reality in the two trigonometric equations we no longer have two unknown but four: $\cos\varphi_{20}$, $\sin\varphi_{20}$, $\cos\varphi_{30}$ and $\sin\varphi_{30}$).

To solve the system we need two more equations, so in the system (2) we have added two more trigonometric equations, namely the golden basic trigonometric equations as it is called for the angle φ_{20} and separated for the angle φ_{30}.

In order to solve the first two equations of the system (2) we write in the form (3):

$$\begin{cases} d_2\cdot\cos\varphi_{20} - x_M = -d_3\cdot\cos\varphi_{30} \\ d_2\cdot\sin\varphi_{20} - y_M = -d_3\cdot\sin\varphi_{30} \end{cases} \tag{3}$$

Each equation of the system (3) rises to square, then sums up both equations (raised to square) and obtains the equation of form (4):

$$\begin{aligned} & d_2^2\cdot(\cos^2\varphi_{20} + \sin^2\varphi_{20}) + x_M^2 + y_M^2 \\ & -2\cdot d_2\cdot x_M\cdot\cos\varphi_{20} - 2\cdot d_2\cdot y_M\cdot\sin\varphi_{20} \\ & = d_3^2\cdot(\cos^2\varphi_{30} + \sin^2\varphi_{30}) \end{aligned} \tag{4}$$

Now is the time to use the two trigonometric gold "equations of gold" written at the end of the system (2), with which Equation 4 gets simplified form (5):

$$\begin{aligned} & d_2^2 + x_M^2 + y_M^2 - 2\cdot d_2\cdot x_M\cdot\cos\varphi_{20} \\ & -2\cdot d_2\cdot y_M\cdot\sin\varphi_{20} = d_3^2 \end{aligned} \tag{5}$$

The terms of Equation 5 are arranged in the more convenient form (6):

$$\begin{aligned} & d_2^2 - d_3^2 + x_M^2 + y_M^2 \\ & = 2\cdot d_2\cdot\left(x_M\cdot\cos\varphi_{20} + y_M\cdot\sin\varphi_{20}\right) \end{aligned} \tag{6}$$

Divide Equation 6 with $2.d_2$ and result in a new form (7):

$$x_M\cdot\cos\varphi_{20} + y_M\cdot\sin\varphi_{20} = \frac{d_2^2 - d_3^2 + x_M^2 + y_M^2}{2\cdot d_2} \tag{7}$$

From Fig. 3 or System (1) the relationship (8) is noted:

$$x_M^2 + y_M^2 = d^2 \tag{8}$$

Insert (8) into (7) and amplify the right fraction with d so that the expression (7) takes the convenient form (9):

$$x_M\cdot\cos\varphi_{20} + y_M\cdot\sin\varphi_{20} = \frac{d_2^2 + d^2 - d_3^2}{2\cdot d_2\cdot d}\cdot d \tag{9}$$

Now is the moment of entering the cosine expression of the O_2 angle, depending on the sides of the triangle of some O_2O_3M (10):

$$\cos\hat{O}_2 = \frac{d_2^2 + d^2 - d_3^2}{2\cdot d_2\cdot d} \tag{10}$$

With relation (10) Equation 9 takes the simplified form (11):

$$x_M\cdot\cos\varphi_{20} - d\cdot\cos\hat{O}_2 = -y_M\cdot\sin\varphi_{20} \tag{11}$$

We want to eliminate $\sin\varphi_{20}$, which is why we have isolated the term in sin and square Equation 11, because by using the trigonometric gold equation of the angle φ_{20}, we transform sin into cos, the equation becoming one of the second degree in $\cos\varphi_{20}$. After picking up the square (11) it takes shape (12):

$$\begin{aligned} & x_M^2\cdot\cos^2\varphi_{20} + d^2\cdot\cos^2\hat{O}_2 \\ & -2\cdot d\cdot x_M\cdot\cos\hat{O}_2\cdot\cos\varphi_{20} \\ & = y_M^2\cdot\sin^2\varphi_{20} \end{aligned} \tag{12}$$

The gold formula is used so that the expression (12) takes the form (13) which is conveniently arranged by grouping the terms into the shape (14):

$$\begin{aligned}&x_M^2\cdot\cos^2\varphi_{20}+d^2\cdot\cos^2\hat{O}_2\\&-2\cdot d\cdot x_M\cdot\cos\hat{O}_2\cdot\cos\varphi_{20}\\&=y_M^2-y_M^2\cdot\cos^2\varphi_{20}\end{aligned}\tag{13}$$

$$\begin{aligned}&\left(x_M^2+y_M^2\right)\cdot\cos^2\varphi_{20}-2\cdot d\cdot x_M\cdot\cos\hat{O}_2\cdot\cos\varphi_{20}\\&-\left(y_M^2-d^2\cdot\cos^2\hat{O}_2\right)=0\end{aligned}\tag{14}$$

The determinant of the second-degree Equation 14 just obtained, is calculated with the relation (15):

$$\begin{aligned}&\Delta=d^2\cdot x_M^2\cdot\cos^2\hat{O}_2+d^2\cdot\left(y_M^2-d^2\cdot\cos^2\hat{O}_2\right)\\&=d^2\cdot\left(x_M^2\cdot\cos^2\hat{O}_2+y_M^2-d^2\cdot\cos^2\hat{O}_2\right)\\&=d^2\cdot\left(y_M^2-y_M^2\cdot\cos^2\hat{O}_2\right)\\&=d^2\cdot y_M^2\cdot\left(1-\cos^2\hat{O}_2\right)=d^2\cdot y_M^2\cdot\sin^2\hat{O}_2\end{aligned}\tag{15}$$

The second order radical of the determinant is expressed as (16):

$$R=\sqrt{\Delta}=\sqrt{d^2\cdot y_M^2\cdot\sin^2\hat{O}_2}=d\cdot y_M\cdot\sin\hat{O}_2\tag{16}$$

The solutions of the second-degree equation in cosine (14) are written as (17):

$$\begin{aligned}&\cos\varphi_{20_{1,2}}=\frac{d\cdot x_M\cdot\cos\hat{O}_2\mp d\cdot y_M\cdot\sin\hat{O}_2}{d^2}\\&=\frac{x_M\cdot\cos\hat{O}_2\mp y_M\cdot\sin\hat{O}_2}{d}\\&=\frac{x_M}{d}\cdot\cos\hat{O}_2\mp\frac{y_M}{d}\cdot\sin\hat{O}_2\end{aligned}\tag{17}$$

Next, in solutions (17) we replace the corresponding trigonometric angle of functions φ, the expressions (17) obtaining the form (18):

$$\begin{aligned}&\cos\varphi_{20_{1,2}}=\frac{x_M}{d}\cdot\cos\hat{O}_2\mp\frac{y_M}{d}\cdot\sin\hat{O}_2\\&=\cos\varphi\cdot\cos\hat{O}_2\mp\sin\varphi\cdot\sin\hat{O}_2=\cos\left(\varphi\pm\hat{O}_2\right)\\&\cos\varphi_{20}=\cos\left(\varphi\pm\hat{O}_2\right)\end{aligned}\tag{18}$$

We now return to the Equation 11 that we order in form (19), in order to solve it in the sin. Equation 19 rises to square and using the trigonometric gold equation of the angle φ_{20}, form (20) is obtained:

$$x_M\cdot\cos\varphi_{20}=d\cdot\cos\hat{O}_2-y_M\cdot\sin\varphi_{20}\tag{19}$$

$$\begin{cases}x_M^2\cdot\cos^2\varphi_{20}=d^2\cdot\cos^2\hat{O}_2+y_M^2\cdot\sin^2\varphi_{20}\\-2\cdot y_M\cdot d\cdot\cos\hat{O}_2\cdot\sin\varphi_{20}\\x_M^2-x_M^2\cdot\sin^2\varphi_{20}=d^2\cdot\cos^2\hat{O}_2+y_M^2\cdot\sin^2\varphi_{20}\\-2\cdot y_M\cdot d\cdot\cos\hat{O}_2\cdot\sin\varphi_{20}\\\left(x_M^2+y_M^2\right)\cdot\sin^2\varphi_{20}-2\cdot y_M\cdot d\cdot\cos\hat{O}_2\cdot\sin\varphi_{20}\\-\left(x_M^2-d^2\cdot\cos^2\hat{O}_2\right)=0\\d^2\cdot\sin^2\varphi_{20}-2\cdot y_M\cdot d\cdot\cos\hat{O}_2\cdot\sin\varphi_{20}\\-\left(x_M^2-d^2\cdot\cos^2\hat{O}_2\right)=0\end{cases}\tag{20}$$

The discriminative of the second-degree Equation 20 in cosine takes the form (21):

$$\begin{aligned}&\Delta=y_M^2\cdot d^2\cdot\cos^2\hat{O}_2+d^2\cdot\left(x_M^2-d^2\cdot\cos^2\hat{O}_2\right)\\&=d^2\cdot\left(x_M^2+y_M^2\cdot\cos^2\hat{O}_2-x_M^2\cdot\cos^2\hat{O}_2-y_M^2\cdot\cos^2\hat{O}_2\right)\\&=d^2\cdot\left(x_M^2-x_M^2\cdot\cos^2\hat{O}_2\right)=d^2\cdot x_M^2\cdot\sin^2\hat{O}_2\end{aligned}\tag{21}$$

The solutions of Equation 20 are written as (22):

$$\begin{aligned}&\sin\varphi_{20}=\frac{y_M\cdot d\cdot\cos\hat{O}_2\pm x_M\cdot d\cdot\sin\hat{O}_2}{d^2}\\&=\frac{y_M\cdot\cos\hat{O}_2\pm x_M\cdot\sin\hat{O}_2}{d}=\frac{y_M}{d}\cdot\cos\hat{O}_2\pm\frac{x_M}{d}\cdot\sin\hat{O}_2\\&=\sin\varphi\cdot\cos\hat{O}_2\pm\cos\varphi\cdot\sin\hat{O}_2=\sin\left(\varphi\pm\hat{O}_2\right)\\&\sin\varphi_{20}=\sin\left(\varphi\pm\hat{O}_2\right)\end{aligned}\tag{22}$$

We obtained relations (23), from which the basic relationship is deduced (24):

$$\begin{cases}\cos\varphi_{20}=\cos\left(\varphi\pm\hat{O}_2\right)\\\sin\varphi_{20}=\sin\left(\varphi\pm\hat{O}_2\right)\end{cases}\tag{23}$$

$$\varphi_{20}=\varphi\pm\hat{O}_2\tag{24}$$

Repeat the procedure for determining the angle φ_{30}, starting again from the system (2), in which the first two transcendental equations are rewritten in the form (25) in order to eliminate the angle φ_{20} this time:

$$\begin{cases}d_2\cdot\cos\varphi_{20}+d_3\cdot\cos\varphi_{30}=x_M\\d_2\cdot\sin\varphi_{20}+d_3\cdot\sin\varphi_{30}=y_M\\\cos^2\varphi_{20}+\sin^2\varphi_{20}=1\\\cos^2\varphi_{30}+\sin^2\varphi_{30}=1\end{cases}\tag{2}$$

$$\begin{cases} d_2 \cdot \cos\varphi_{20} = x_M - d_3 \cdot \cos\varphi_{30} \\ d_2 \cdot \sin\varphi_{20} = y_M - d_3 \cdot \sin\varphi_{30} \end{cases} \tag{25}$$

Raises the two equations of the system (25) to the square and assembles, resulting in the equation of form (26), which is arranged in more convenient forms (27) and (28):

$$\begin{aligned} d_2^2 = x_M^2 + y_M^2 + d_3^2 - 2 \cdot d_3 \cdot x_M \cdot \cos\varphi_{30} \\ -2 \cdot d_3 \cdot y_M \cdot \sin\varphi_{30} \end{aligned} \tag{26}$$

$$x_M \cdot \cos\varphi_{30} + y_M \cdot \sin\varphi_{30} = d \cdot \frac{d^2 + d_3^2 - d_2^2}{2 \cdot d \cdot d_3} \tag{27}$$

$$x_M \cdot \cos\varphi_{30} + y_M \cdot \sin\varphi_{30} = d \cdot \cos\hat{M} \tag{28}$$

We want to determine it first on the basket so that we will initially isolate the term in the sin, Equation 28 being in the form (29), which by square-up generates expression (30), the expression which is arranged in the form (31):

$$x_M \cdot \cos\varphi_{30} - d \cdot \cos\hat{M} = -y_M \cdot \sin\varphi_{30} \tag{29}$$

$$\begin{aligned} & x_M^2 \cdot \cos^2\varphi_{30} + d^2 \cdot \cos^2\hat{M} \\ & -2 \cdot d \cdot x_M \cdot \cos\hat{M} \cdot \cos\varphi_{30} \\ & = y_M^2 - y_M^2 \cdot \cos^2\varphi_{30} \end{aligned} \tag{30}$$

$$\begin{aligned} & d^2 \cdot \cos^2\varphi_{30} - 2 \cdot d \cdot x_M \cdot \cos\hat{M} \cdot \cos\varphi_{30} \\ & -(y_M^2 - d^2 \cdot \cos^2\hat{M}) = 0 \end{aligned} \tag{31}$$

Equation 31 is a second-degree equation in the cosine with the solutions given by the expression (32):

$$\begin{aligned} & \cos\varphi_{30} \\ & = \frac{d \cdot x_M \cdot \cos\hat{M} \pm \sqrt{d^2 \cdot x_M^2 \cdot \cos^2\hat{M} + d^2 \cdot \left(y_M^2 - d^2 \cdot \cos^2\hat{M}\right)}}{d^2} \\ & = \frac{d \cdot x_M \cdot \cos\hat{M} \pm \sqrt{d^2 \cdot y_M^2 \cdot \left(1 - \cos^2\hat{M}\right)}}{d^2} \\ & = \frac{d \cdot x_M \cdot \cos\hat{M} \pm d \cdot y_M \cdot \sin\hat{M}}{d^2} \\ & = \frac{x_M}{d} \cdot \cos\hat{M} \pm \frac{y_M}{d} \cdot \sin\hat{M} = \cos\varphi \cdot \cos\hat{M} \pm \sin\varphi \cdot \sin\hat{M} \\ & = \cos\left(\varphi \mp \hat{M}\right) \\ & \cos\varphi_{30} = \cos\left(\varphi \mp \hat{M}\right) \end{aligned} \tag{32}$$

We continue to write Equation 28 in the form (33), where the term is now isolated in the cosine for its elimination, in order to determine the sin:

$$x_M \cdot \cos\varphi_{30} = d \cdot \cos\hat{M} - y_M \cdot \sin\varphi_{30} \tag{33}$$

Equation 33 rises to square and obtains the equation of form (34), which is arranged in a convenient form (35):

$$\begin{aligned} & x_M^2 \cdot \left(1 - \sin^2\varphi_{30}\right) = d^2 \cdot \cos^2\hat{M} + \\ & + y_M^2 \cdot \sin^2\varphi_{30} - 2 \cdot y_M \cdot d \cdot \cos\hat{M} \cdot \sin\varphi_{30} \end{aligned} \tag{34}$$

$$\begin{aligned} & d^2 \cdot \sin^2\varphi_{30} - 2 \cdot y_M \cdot d \cdot \cos\hat{M} \cdot \sin\varphi_{30} \\ & -\left(x_M^2 - d^2 \cdot \cos^2\hat{M}\right) = 0 \end{aligned} \tag{35}$$

Expression (35) is an equation of the grade II in the sine which admits the solutions given by the relation (36):

$$\begin{aligned} & \sin\varphi_{30} \\ & = \frac{d \cdot y_M \cdot \cos\hat{M} \mp \sqrt{d^2 \cdot y_M^2 \cdot \cos^2\hat{M} + d^2 \cdot \left(x_M^2 - d^2 \cdot \cos^2\hat{M}\right)}}{d^2} \\ & = \frac{d \cdot y_M \cdot \cos\hat{M} \mp \sqrt{d^2 \cdot x_M^2 \cdot \left(1 - \cos^2\hat{M}\right)}}{d^2} \\ & = \frac{d \cdot y_M \cdot \cos\hat{M} \mp d \cdot x_M \cdot \sin\hat{M}}{d^2} \\ & = \frac{y_M}{d} \cdot \cos\hat{M} \mp \frac{x_M}{d} \cdot \sin\hat{M} = \sin\varphi \cdot \cos\hat{M} \mp \cos\varphi \cdot \sin\hat{M} \\ & = \sin(\varphi \mp \hat{M}) \\ & \sin\varphi_{30} = \sin(\varphi \mp \hat{M}) \end{aligned} \tag{36}$$

The relations (37) from which the expression (38) is deduced are also retained:

$$\begin{cases} \cos\varphi_{30} = \cos\left(\varphi \mp \hat{M}\right) \\ \sin\varphi_{30} = \sin\left(\varphi \mp \hat{M}\right) \end{cases} \tag{37}$$

$$\varphi_{30} = \varphi \mp \hat{M} \tag{38}$$

Results: Determination of Speeds and Accelerations

Velocities

From the system (2), only the relations (39) required in the study of velocities at the reverse kinematics are retained. It starts from the relationship that links the cosine of the angle to the sides of the triangle, a relationship that is time dependent and thus the simpler written value (relations 40) is obtained:

$$\begin{cases} \dot{\varphi} = \dfrac{\dot{y}_M \cdot \cos\varphi - \dot{x}_M \cdot \sin\varphi}{d} \\ \dot{d} = \dfrac{x_M \cdot \dot{x}_M + y_M \cdot \dot{y}_M}{d} \end{cases} \tag{39}$$

$$\begin{cases} 2\cdot d_2\cdot d\cdot\cos O_2 = d_2^2 - d_3^2 + d^2 \\ 2\cdot d_2\cdot \dot{d}\cdot\cos O_2 - 2\cdot d_2\cdot d\cdot\sin O_2\cdot\dot{O}_2 = 2\cdot d\cdot\dot{d} \\ \Rightarrow \dot{O}_2 = \dfrac{d_2\cdot\dot{d}\cdot\cos O_2 - d\cdot\dot{d}}{d_2\cdot d\cdot\sin O_2} \end{cases} \quad (40)$$

The relationship (24) is derived and the angular velocity $\omega_{20}\equiv\dot{\varphi}_{20}$ (relationship 41) is obtained:

$$\varphi_{20} = \varphi \pm \hat{O}_2 \quad (24)$$

$$\omega_{20}\equiv\dot{\varphi}_{20} = \dot{\varphi}\pm\dot{O}_2 \quad (41)$$

To determine ω20 (relation 41) we need $\dot{\varphi}$ to be computed from (39) and $\dot{O}_2$ determined by (40). In turn $\dot{O}_2$ it requires $\dot{d}$ for its calculation, which is also calculated from the system (39):

Input speeds $\dot{x}_M$ *and* $\dot{y}_M$ are known, are imposed as inputs, or are conveniently chosen, or can be calculated on the basis of imposed criteria.

Similarly, the angular velocity $\omega_{30}\equiv\dot{\varphi}_{30}$ is determined (42):

$$\begin{cases} 2\cdot d_3\cdot d\cdot\cos M = d_3^2 - d_2^2 + d^2 \\ 2\cdot d_3\cdot \dot{d}\cdot\cos M - 2\cdot d_3\cdot d\cdot\sin M\cdot\dot{M} = 2\cdot d\cdot\dot{d} \\ \Rightarrow \dot{M} = \dfrac{d_3\cdot\dot{d}\cdot\cos M - d\cdot\dot{d}}{d_3\cdot d\cdot\sin M} \end{cases} \quad (42)$$

The relationship (38) is derived to obtain the angular velocity $\omega_{30}\equiv\dot{\varphi}_{30}$, (expression 43). $\dot{\varphi}$ is calculated with the already known expression in the system (39) and $\dot{M}$ it is determined from the system (42) and with the help of the system (39) which determines also and $\dot{d}$:

$$\varphi_{30} = \varphi \mp \hat{M} \quad (38)$$

$$\omega_{30}\equiv\dot{\varphi}_{30} = \dot{\varphi}\mp\dot{M} \quad (43)$$

Accelerations

Derive (39) and we obtain the necessary relations (44) in the study of the accelerations in the inverse kinematics. The relation in the system (40) is derived a second time with the time and the system (45) is obtained:

$$\begin{cases} \ddot{\varphi} = \dfrac{\ddot{y}_M\cdot\cos\varphi - \ddot{x}_M\cdot\sin\varphi - \dot{y}_M\cdot\sin\varphi\cdot\dot{\varphi} - \dot{x}_M\cdot\cos\varphi\cdot\dot{\varphi} - \dot{d}\cdot\dot{\varphi}}{d} \\ \ddot{d} = \dfrac{\dot{x}_M^2 + x_M\cdot\ddot{x}_M + \dot{y}_M^2 + y_M\cdot\ddot{y}_M - \dot{d}^2}{d} \end{cases} \quad (44)$$

$$\begin{cases} 2\cdot d_2\cdot d\cdot\cos O_2 = d_2^2 - d_3^2 + d^2 \\ 2\cdot d_2\cdot \dot{d}\cdot\cos O_2 - 2\cdot d_2\cdot d\cdot\sin O_2\cdot\dot{O}_2 = 2\cdot d\cdot\dot{d} \\ \Rightarrow d_2\cdot d\cdot\sin O_2\cdot\dot{O}_2 = d_2\cdot\dot{d}\cdot\cos O_2 - d\cdot\dot{d} \\ \ddot{O}_2 = \\ \dfrac{\ddot{d}d_2\cos O_2 - \ddot{d}d - 2\dot{d}d_2\sin O_2\cdot\dot{O}_2 - dd_2\cos O_2\cdot\dot{O}_2^2 - \dot{d}^2}{d_2\cdot d\cdot\sin O_2} \end{cases} \quad (45)$$

Then the expression (41) is derived and the relation (46) generating the absolute angular acceleration $\varepsilon_2\equiv\varepsilon_{20}$, which is calculated with $\ddot{\varphi}$ the output from the system (44) and with $\ddot{O}_2$ extracted from the system (45) and for $\ddot{O}_2$ to be obtained it is also necessary $\ddot{d}$ dermined also from (44):

$$\omega_{20}\equiv\dot{\varphi}_{20} = \dot{\varphi}\pm\dot{O}_2 \quad (41)$$

$$\varepsilon_2\equiv\varepsilon_{20} = \dot{\omega}_{20}\equiv\ddot{\varphi}_{20} = \ddot{\varphi}\pm\ddot{O}_2 \quad (46)$$

The second time is now derived (42) and the system (47) is obtained:

$$\begin{cases} 2\cdot d_3\cdot d\cdot\cos M = d_3^2 - d_2^2 + d^2 \\ 2\cdot d_3\cdot \dot{d}\cdot\cos M - 2\cdot d_3\cdot d\cdot\sin M\cdot\dot{M} = 2\cdot d\cdot\dot{d} \\ \Rightarrow d_3\cdot d\cdot\sin M\cdot\dot{M} = d_3\cdot\dot{d}\cdot\cos M - d\cdot\dot{d} \\ \ddot{M} = \\ \dfrac{\ddot{d}d_3\cos M - \ddot{d}d - 2\dot{d}d_3\sin M\cdot\dot{M} - dd_3\cos M\cdot\dot{M}^2 - \dot{d}^2}{d_3\cdot d\cdot\sin M} \end{cases} \quad (47)$$

The relationship (43) is again obtained with time and the expression (48) of the absolute angular acceleration $\varepsilon_3\equiv\varepsilon_{30}$ which is determined with $\ddot{\varphi}$ and $\ddot{M}$.

$\ddot{\varphi}$ it is determined from the system (44) and $\ddot{M}$ withdrawn from the system (47) and needs and the $\ddot{d}$ which is extracted from the system (44):

$$\omega_{30}\equiv\ddot{\varphi}_{30} = \ddot{\varphi}\mp\dot{M} \quad (43)$$

$$\varepsilon_3\equiv\varepsilon_{30} = \dot{\omega}_{30}\equiv\ddot{\varphi}_{30} = \ddot{\varphi}\mp\ddot{M} \quad (48)$$

Discussion

Robots have penetrated today in almost all industrial fields, being much more precise than humans in the execution of operations, but also faster, more dynamic, more stable and more resilient, working 24 h of the 24 h possible, in any season, breaks, holidays, vacations and especially without getting sick. The most important fact is that they can also perform special operations that man can't do. So, for example, they can work in toxic, contaminated, mined, dangerous, or airless environments (in the cosmos, under water or underground), they can also position the parts very precisely so they can be used for fine, special processing at high precision operations, excellent mechanical processing, exceptional medical operations and even in all medical operations today, helping the doctor manage the patient's surgery with amazing precision, especially when it comes to open heart, brain, kidneys, liver, etc. Robots should not be fed and yet they can have an extremely long life. They can help us conquer both the underwater spaces and the cosmic spaces. Robots have stunning work precision and a faster execution speed than a man. In addition, their very precise positioning makes robot operations a great advantage, which can no longer be neglected. In repetitive and tiring work they are irreplaceable. The most commonly used anthropomorphic robotic mechatronic systems, which are currently being used, have been studied by eliminating the heavy, matrix 3D spatial system, the study being simplified in a plan by considering the main work plan of the system and the plan, the rotation required to restore the spatial parameters of the anthropomorphic 3D system. In other words, we can greatly ease the work of the anthropomorphic robot engineer by moving from 3D systems to a 2D system. In this study we will study the inverse kinematics of the plan system, 2D, as the most important one. In general, end effector positions are known (required) and the positions of the 2D module elements and the necessary rotations of the actuators for their creation must be determined. In the kinematics we know the kinematic parameters x_M and y_M, which represent the scaled coordinates of the point M (endefactor M) and must be determined by analytical calculations the parameters φ_{20} and φ_{30}.

Conclusion

The most commonly used anthropomorphic robotic mechatronic systems, which are currently being used, have been studied by eliminating the heavy, matrix 3D spatial system, the study being simplified in a plan by considering the main work plan of the system and the plan, the rotation required to restore the spatial parameters of the anthropomorphic 3D system. In other words, we can greatly ease the work of the anthropomorphic robot engineer by moving from 3D systems to a 2D system. In this study we will study the inverse kinematics of the plan system, 2D, as the most important one. In general, end effector positions are known (required) and the positions of the 2D module elements and the necessary rotations of the actuators for their creation must be determined. In the kinematics we know the kinematic parameters x_M and y_M, which represent the scaled coordinates of the point M (endefactor M) and must be determined by analytical calculations the parameters φ_{20} and φ_{30}.

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Author's Contributions

This section should state the contributions made by each author in the preparation, development and publication of this manuscript.

Ethics

Authors should address any ethical issues that may arise after the publication of this manuscript.

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