Journal of Computer Science 7 (5): 749-756, 2011
ISSN 1549-3636


# The Efficiency Measurement of Parallel Production Systems: A Non-radial Data Envelopment Analysis Model

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**Abstract: Problem statement:** Data Envelopment Analysis (DEA) is a non-parametric technique for measuring the relative efficiency of a set of production systems or Decision Making Units (DMU) that have multiple inputs and outputs. Sometimes, DMUs have a parallel structure, in which systems composed of parallel units work individually; the sum of their own inputs/outputs is the input/output of the system. For this type of system, conventional DEA models treat each DMU as a black box and ignore the performance of its units. **Approach:** This study introduces a DEA model in Slacks-Based Measure (SBM) formulation which considers the parallel relationship of the units within the system in measuring the efficiency of the system. Under this framework, the overall efficiency of the system is expressed as a weighted sum of the efficiencies of its units. **Results:** As an SBM model, the proposed model is non-radial and is suitable for measuring the efficiency when inputs and outputs may change non-proportionally. A theoretical result shows that if any unit of a parallel system is inefficient then the system is inefficient. **Conclusion:** This study introduces a non-radial DEA model, takes into account the operation of individual components within the parallel production system, to measure the overall efficiency as well as the efficiencies of sub-processes. This helps the decision makers recognize inefficient units and make later improvements.

**Key words:** Data envelopment analysis, decision making unit, production possibility set, parallel production system, slacks-based measure

## INTRODUCTION

Data Envelopment Analysis (DEA) is a linear programming methodology in Operations Research and Economics that is extensively applied by various research communities (Sohn and Moon, 2004; Seol *et al*., 2007; Rayeni and Saljooghi, 2010 Zreika and Elkanj, 2011). The domain of inquiry of the DEA is a set of production systems or decision making units (DMU), which use multiple inputs to produce multiple outputs. The aim of the DEA is to measure the relative efficiency of each DMU within a data set. The results specify how efficient each DMU has performed as compared to other DMUs in converting inputs to outputs. An issue which is of greater concern to the inefficient DMUs is what factors cause the inefficiency. Several studies have assigned to breaking down the overall efficiency into components so that the sources of inefficiency can be identified. One type of decomposition emphasizes the sub-processes of the production process. In recent years, a great number of DEA studies have focused on two-stage production systems, where all outputs from the first stage are intermediate products that make up the inputs to the second stage. For example see (Sexton and Lewis, 2003; Chen and Zhu, 2004; Liang *et al*., 2006; Kao and Hwang, 2008; Chen *et al*., 2009) among others. Recently, Tone and Tsutsui (2009) and Cook *et al*., (2010) have proposed DEA models for measuring the efficiency of network systems connected in series with linking activities.

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In some situations, DMUs have a parallel structure that is composed of a set of units that work individually; the sum of their own inputs/outputs is the input/output of the DMUs. A typical example of these production systems is a university with faculties. The overall efficiency of the university can be calculated by the total inputs used and total outputs produced by all faculties. Each specific faculty can have an efficiency measured by comparing it with the equivalent faculties of other universities. The study of Färe and Primont (1984), which discusses the efficiency of firms with multiple plants, is probably the first study of such DMUs. Kao (1998) applied the Färe and Primont's methodology for measuring the efficiency of forest districts with multiple working circles in Taiwan. Castelli *et al*. (2004) discussed a hierarchical structure in which if there is only one layer, it becomes a parallel system. The works of Färe *et al*. (1997), Tsai and Molinero (2002) and Yu (2008) extend of the independent parallel system where certain resources are shared by some units.

Conventional DEA models view this type of production system as a black box and ignore the operations of its units. Recently, Kao (2009) has modified a standard DEA model and introduced a radial DEA model that evaluates the overall efficiency of the system as well as the efficiencies of its units. His method decomposes the inefficiency slack of a DMU into the inefficiency slacks of its sub-DMUs. As an application of Kao's parallel model, Rayeni and Saljooghi (2010) examine the performance of the universities in Iran via a parallel production process.

This study presents an alternative method for estimating the efficiency of a parallel production system and the efficiency of its units. Since DEA models implicitly use Production Possibility Set (PPS) to evaluate the efficiency of DMUs, we first define the PPS of the parallel production systems. Then, based on this PPS, we introduce a non-radial DEA model in Slacks-Based Measure (SBM) formulation for aggregating the units in a parallel production system. Under this framework, the overall efficiency of the system is expressed as a weighted sum of the efficiencies of its units. With decomposition of the overall efficiency, the units which cause the inefficient operation of the system can be identified for future improvements An example from the forest production industry in Taiwan is applied to compare the new approach with Kao's parallel model.

## MATERIALS AND METHODS

**Production possibility set:** Suppose we have n DMUs, where each $DMU_j$ (j=1,…,n) uses m inputs $x_{ij}$(i = 1, …, m) to produce s outputs $y_{rj}$(r = 1,…, s). It is assumed that all inputs and outputs are positive. We denote the $DMU_j$ by $(x_j, y_j)$, where $x_j = (x_{1j}, x_{2j},\ldots, x_{mj})^T$ and $y_j = (y_{1j}, y_{2j},\ldots, y_{mj})^T$ are input and output vectors, respectively. The Production Possibility Set (PPS) T is defined as a set of all inputs and outputs of a production technology in which outputs can be produced from inputs. Under the assumption of Constant Returns to Scale (CRS) the PPS can be represented as follows:

$$T_C = \left\{ (x, y) \;\middle|\; x \geq \sum_{j=1}^{n} \lambda_j x_j,\ y \leq \sum_{j=1}^{n} \lambda_j y_j,\ \lambda_j \geq 0,\ j = 1,\ldots,n \right\}$$

where, $\lambda = (\lambda_1,\ldots,\lambda_n) \in \Re^n$ is the intensity vector.

The PPS under Variable Returns to Scale (VRS) assumption can be defined by adding the convexity constraint $\sum_{j=1}^{n} \lambda_j = 1$ into $T_c$.

**Definition 1:** (Dominance). We say that $DMU_p$ $(x_p, y_p)$ dominates $DMU_q$ $(x_q, y_q)$ if and only if $x_p \leq x_q$ and $y_p \geq y_q$ with strict inequality holding for at least one component in the input or the output vector.

Thus, a DMU of the PPS is not dominated if and only if there is no other DMU (original or virtual) in the PPS which satisfies Definition 1.

**Definition 2:** (Efficiency). $DMU_o=(x_o,y_o)$ is efficient if and only if there is no (x,y) of PPS such that (x,y) dominates $(x_o,y_o)$.

**Radial and non radial DEA model:** DEA provides for two types of measure: radial and non-radial. Radial models assume proportional change of inputs or outputs and usually disregard the existence of slacks in measuring efficiency scores. Radial measures are represented by CCR (Charnes *et al*., 1978) and BCC (Banker *et al*., 1984) Non-radial models, on the other hand, regard the slacks of each input or output and the variations of inputs and outputs as not proportional; in other words in non-radial models the inputs/outputs are allowed to decrease/increase at different rates. Non-radial models include Russell measure (Färe and Lovell, 1978) and Slacks-Based Measure (SBM) (Tone, 2001).

For evaluating the efficiency of $DMU_o$ ($o \in \{1,\ldots,n\}$) the input-oriented CCR model, whose purpose is to minimize input custom while keeping the level of current outputs, is set as follows:

$$\theta^* = \min \theta - \varepsilon(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+)$$
$$\text{s.t.} \quad \theta x_o = \sum_{j=1}^{n} \lambda_j x_j + s^-,$$
$$y_o = \sum_{j=1}^{n} \lambda_j y_j - s^+, \quad (1)$$
$$\lambda, s^-, s^+ \geq 0, \theta \text{ free},$$

where, $\varepsilon$ is non-Archimedean small value and the optimal solution of $\theta$*is efficiency score. Also non-negative vectors $s^- = (s_1^-,...,s_m^-) \in \Re^m$ and $s^+ = (s_1^+,...,s_s^+) \in \Re^s$ indicate input excess and output shortfall slacks, respectively. Likewise the output-oriented CCR model can be defined.

Suppose an optimal solution for model (1) to be $(\theta^*, \lambda^*, s^{-*}, s^{+*})$.

**Definition 3:** (CCR-efficiency). $DMU_o$ is CCR-efficient if and only if $\theta^* = 1$ and $s^{-*} = s^{+*} = 0$.

The model presented in (1) is called the CCR envelopment model. The dual of model (1) (without $\varepsilon$, i.e., $\varepsilon = 0$), or the CCR multiplier model, is given by:

$$\theta^* = \max \quad u y_o$$
$$\text{s.t.} \quad v x_o = 1,$$
$$u y_j - v x_j \leq 0, \ j = 1,...,n, \quad (2)$$
$$u, v \geq 0,$$

where, $v = (v_1,..., v_m) \in \Re^m$ and $u = (u_1,..., u_s) \in \Re^s$ are dual variable vectors corresponding to the constraints of model (1).

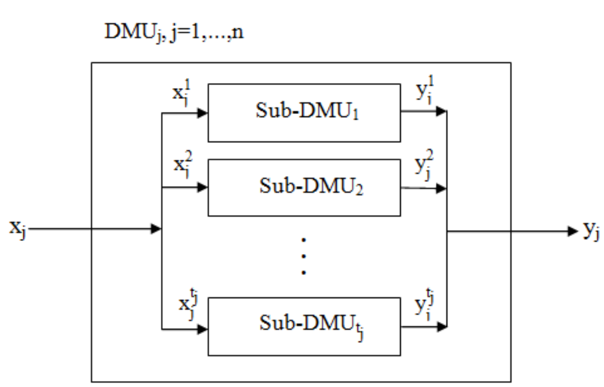


Fig. 1: The parallel production system

The SBM model, as a non-oriented and non-radial DEA model, for evaluating the efficiency of DMUo is defined as follows:

$$\rho^* = \min \rho = \frac{1 - \frac{1}{m}\sum_{i=1}^{m} \frac{s_i^-}{x_{io}}}{1 + \frac{1}{s}\sum_{r=1}^{s} \frac{s_r^+}{y_{ro}}}$$
$$\text{s.t.} \quad x_o = \sum_{j=1}^{n} \lambda_j x_j + s^-, \quad (3)$$
$$y_o = \sum_{j=1}^{n} \lambda_j y_j - s^+,$$
$$\lambda, s^-, s^+ \geq 0$$

The optimal solution of $\rho$* is the SBM efficiency score. It can be obviously identified that $0 < \rho^* \leq 1$ and supports the properties of unit invariance and monotone.

Suppose an optimal solution for model (3) to be $(\rho^*, \lambda^*, s^{-*}, s^{+*})$.

**Definition 4:** (SBM-efficiency). $DMU_o$ is SBM-efficient if and only if $\rho^* = 1$.

This definition is equivalent to $s^{-*} = s^{+*} = 0$. It means that there are no input excesses and output shortfalls in any optimal solution.

The input (output)-oriented SBM model can be defined by ignoring the denominator (numerator) of the objective function.

Here we emphasize that, as demonstrated by Tone (2001), the efficiency score measured by the SBM model is not greater than the efficiency score measured by the CCR model. Moreover, a DMU is SBM-efficient if and only if it is CCR-efficient.

Parallel production system: Consider a parallel production process as shown in Fig. 1. Suppose we have n DMUs, of which each $DMU_j$ (j=1,…,n) is composed of $t_j$ units (sub-DMUs) connected in parallel. Each sub-DMU uses the same inputs to produce the same outputs, individually. $\text{Sub-DMU}_p$ (p=1,…,$t_j$) has m inputs $x_{ij}^p$ (i=1,…,m) and s outputs $y_{rj}^p$ (r=1,...,s). The sum of all $x_{ij}^p$ over p, namely, $\sum_{p=1}^{t_j} x_{ij}^p$ and the sum of all $y_{rj}^p$ over p, namely, $\sum_{p=1}^{t_j} y_{rj}^p$, are the ith input and the rth output of the system $DMU_j$, respectively. In other words, we have:

$$\sum_{p=1}^{t_j} x_j^p = x_j, \qquad \sum_{p=1}^{t_j} y_j^p = y_j \tag{4}$$

**Kao's parallel model:** Kao (2009) has developed a DEA model based on the input-oriented CCR multiplier model such that by minimizing the inefficiency slack instead of maximizing the efficiency, we are able to decompose the inefficiency slack of a DMU into the inefficiency slacks of its sub-DMUs. Kao's model for measuring the inefficiency of $DMU_o$ is given by:

$$\begin{aligned} &\min \sum_{p=1}^{t_o} s_o^p \\ &\text{s.t.}\quad vx_o = 1, \\ &\quad uy_o - vx_o + s_o = 0, \\ &\quad uy_o^p - vx_o^p + s_o^p = 0,\ p = 1,...,t_o, \\ &\quad uy_j^p - vx_j^p \le 0,\ j = 1,...,n, j \ne o, p = 1,...,t_j, \\ &\quad uy_j - vx_j \le 0,\ j = 1,...,n,\ j \ne o, \\ &\quad u,v \ge 0 \end{aligned} \tag{5}$$

where, $s_o$ and $s_o^p$ $(p = 1,...,t_o)$ are the inefficiency slacks of $DMU_o$ and its sub-DMUs, respectively.

On optimality, the efficiency scores for $DMU_o$ and sub-$DMU_p$ (p = 1,…,$t_o$) can be calculated as follows:

$$\begin{aligned} &E_o = 1 - s_o^* = 1 - \sum_{p=1}^{t_o} s_o^{p*} \\ &E_o^p = 1 - \frac{s_o^{p*}}{v^* x_o^p},\ p = 1,...,t_o, \end{aligned} \tag{6}$$

where, (*) shows the optimal value from model (5).

**The parallel SBM model:** The PPS $T_C^{parallel}$ under the CRS assumption for $n\times t_j$ sub-DMUs of n parallel production systems, is defined by:

$$T_C^{parallel} = \left\{ (x,y) \;\middle|\; x \ge \sum_{j=1}^{n}\sum_{p=1}^{t_j} \lambda_j^p x_j^p,\ y \le \sum_{j=1}^{n}\sum_{p=1}^{t_j} \lambda_j^p y_j^p,\ \lambda_j^p \ge 0 \right\}$$

Note that if all $\lambda_j^p, p = 1,...,t_j$ associated with the sub-DMUs within the $DMU_j$ are the same, then $T_C^{parallel}$ converts to the conventional PPC, namely $T_C$.

Suppose, $DMU_o$ $(o\in\{1,\ldots,n\})$ to be the DMU under evaluation. In an effort to measure the overall efficiency of $DMU_o$, first by using the input-oriented SBM model, we calculate the efficiency score of each sub-$DMU_p$ (p=1,…,$t_o$) as follows:

$$\begin{aligned} &E_o^p = \min\ 1 - \frac{1}{m}\sum_{i=1}^{m}\frac{s_i^-}{x_{io}^p} \\ &\text{s.t.}\ \sum_{j=1}^{n}\sum_{p=1}^{t_j} \lambda_j^p x_j^p + s^- = x_o^p, \\ &\quad \sum_{j=1}^{n}\sum_{p=1}^{t_j} \lambda_j^p y_j^p - s^+ = y_o^p, \\ &\quad \lambda_j^p, s^-, s^+ \ge 0,\ p = 1,...,t_j,\ j = 1,...,n \end{aligned} \tag{7}$$

After evaluating the efficiency scores of all Sub-DMUs, we define the overall efficiency score of $DMU_o$ as a weighted combination of the efficiency scores of its sub-DMUs. This is shown in the following equation:

$$E_o = w_o^1 E_o^1 + w_o^2 E_o^2 + \ldots + w_o^{t_o} E_o^{t_o} = \sum_{p=1}^{t_o} w_o^p E_o^p \tag{8}$$

where, the weight $w_o^p$ of each sub-$DMU_p$ is

$$w_o^p = \frac{1}{m}\sum_{i=1}^{m}\frac{x_{io}^p}{x_{io}},\ p = 1,...,t_o \tag{9}$$

Hence $w_o^p$ is the arithmetic mean of the portion of resources devoted to each sub-DMU by $DMU_o$. From (4), it can be verified that $\sum_{p=1}^{t_o} w_o^p = 1$.

**Definition 5:** $DMU_o$ is said to be efficient in sub-$DMU_p$, if $E_o^p = 1$.

**Definition 6:** $DMU_o$ is said to be efficient if its overall efficiency score is equal to one, i.e., $E_o = 1$.

Decomposing the overall efficiency of a system into the weighted combination of its unit's efficiencies helps us to identify the units that cause inefficiency. By using the model (7), we are able to recognize the inefficient sub-DMUs and make later improvements. Also, using Eq. 8 we can evaluate the overall efficiency of the $DMU_o$ in a way that takes into account the operations of all its sub-DMUs.

Note that, similarly, we can introduce an output-oriented SBM model for parallel production systems such that the weights associated to sup-DMU of $DMU_o$ can be defined as:

$$w_o^p = \frac{1}{s}\sum_{r=1}^{s}\frac{y_{ro}^p}{y_{ro}},\ \ p = 1,\ldots,t_o \tag{10}$$

## RESULTS

The following theorem explains the relationship between the efficiency of a parallel production system and its production units.

**Theorem 1:** If any of $\text{sub-DMU}_p = (x_k^p,\ y_k^p)\ (p = 1,\ldots,t_k)$ of $DMU_k$ is CRS-inefficient, then $DMU_k = (x_k, y_k)$ is CRS-inefficient.

**Proof:** Suppose any of $\text{sub-DMU}_p = (x_k^p,\ y_k^p)$ (p=1,…,$t_k$) to be CRS-inefficient. We will show that there is $(\bar{x},\bar{y}) \in T_C^{\text{parallel}}$ such that $(x_k, y_k)$ is dominated by $(\bar{x},\bar{y})$.

Without loss of generality, we assume that $\text{sub-DMU}_1 = (x_k^1,\ y_k^1)$ is CRS-inefficient. Then, the following system has a solution $\{\lambda_j^{p^*}, p = 1\ldots,t_j,\forall j; s^{-*}; s^{+*}\}$ with $(s^{-*},s^{+*}) > (0,0)$:

$$\begin{aligned} x_k^1 &= \sum_{j=1}^{n}\sum_{p=1}^{t_j}\lambda_j^{p*}x_j^p + s^{-*}, \\ y_k^1 &= \sum_{j=1}^{n}\sum_{p=1}^{t_j}\lambda_j^{p*}y_j^p - s^{+*} \end{aligned} \tag{11}$$

We set:

$$\bar{x}_k^1 = x_k^1 - s^{-*} = \sum_{j=1}^{n}\sum_{p=1}^{t_j}\lambda_j^{p*}x_j^p, \bar{y}_k^1 = y_k^1 + s^{+*} = \sum_{j=1}^{n}\sum_{p=1}^{t_j}\lambda_j^{p*}y_j^p \tag{12}$$

From (4) and (12) we have

$$x_k = \bar{x}_k^1 + \sum_{p=2}^{t_k}x_k^p + s^{-*}, y_k = \bar{y}_k^1 + \sum_{p=2}^{t_k}y_k^p - s^{+*} \tag{13}$$

Now we define:

$$\bar{x} = \bar{x}_k^1 + \sum_{p=2}^{t_k}x_k^p, \bar{y} = \bar{y}_k^1 + \sum_{p=2}^{t_k}y_k^p\ . \tag{14}$$

Since $(\bar{x}_k^1,\bar{y}_k^1) \in P_C^{\text{parallel}}$, thus $(\bar{x},\bar{y}) \in T_C^{\text{parallel}}$. Hence we have:

$$x_k = \bar{x} + s^{-*}, y_k = \bar{y} - s^{+*} \tag{15}$$

Since $(s^{-*},s^{+*}) \neq (0,0)$, $(x_k,\ y_k)$ is dominated by $(\bar{x},\bar{y})$. Hence, according to Definition 2, $DMU_k$ is CRS-inefficient.

As a contraposition to Theorem 1, we have.

**Corollary 1:** If $DMU_k(x_k,\ y_k)$ is CRS-efficient, then each $\text{sub-DMU}_p = (x_k^p,\ y_k^p)$ (p=1,…,$t_k$) of $DMU_k$ is CRS-efficient.

**Empirical example:** Now, we apply the proposed model to the national forests of Taiwan as studied by Kao (2009). In Taiwan, the forest lands are divided into eight regions, each of which is divided into four or five sub-regions called working circles (WCs). These WCs are the basic component in the management of the forest. The forest production process is a characteristic parallel production process, in that each region has several subordinated WCs operating individually. There are four inputs:

- Land ($x_1$): area in thousand hectares
- Labor ($x_2$): number of employees in persons
- Expenditures ($x_3$): money spent each year in ten thousand new Taiwan dollars
- Initial stocks ($x_4$): volume of forest stock before the period of evaluation in 10000 $m^3$
- The outputs are
- Timber production ($y_1$): timber produced each year in cubic meters
- Soil conservation ($y_2$): volume of forest stock in 10000 $m^3$, as higher stock level leads to less soil erosion; and
- Recreation ($y_3$): visitors serviced by forests every year in thousands of visits

The data are shown in Table 1. For each input (output) the quantity of a region is the sum of its sub-regions.

The results of this measurement of efficiency are reported in Table 2, where the second column shows the weights calculated from (9) for each sub-DMU, the third column is the efficiency score calculated using model (7) and Eq. 8. The efficiency scores of the eight DMUs calculated by the input-oriented of model (3) without taking into account the operations of sub-DMUs are shown in the last column under the heading conventional SBM model.

Table 1: Taiwan forest data

| | Inputs | | | | Outputs | | |
|---|---|---|---|---|---|---|---|
| Working circles | Land | Labor | Expenditures | Initial stocks | Timber | Soil cons. | Recreation |
| Lotung Region | 175.73 | 248.33 | 1581.60 | 1604.38 | 746.04 | 1604.01 | 207.59 |
| 1. Taipei | 18.23 | 45.33 | 608.32 | 125.46 | 19.59 | 125.46 | 0.00 |
| 2. Tai-ping-shan | 55.49 | 98.00 | 336.33 | 584.85 | 17.70 | 584.85 | 207.59 |
| 3. Chao-chi | 31.44 | 51.00 | 263.99 | 147.76 | 0.00 | 147.39 | 0.00 |
| 4. Nan-au | 28.94 | 27.33 | 166.78 | 263.02 | 38.00 | 263.02 | 0.00 |
| 5. Ho-ping | 41.63 | 26.67 | 206.18 | 483.29 | 670.75 | 483.29 | 0.00 |
| Hsinchu Region | 162.81 | 316.67 | 850.05 | 2609.79 | 16823.42 | 2603.99 | 308.97 |
| 6. Guay-shan | 41.48 | 86.33 | 158.49 | 386.03 | 26.37 | 386.03 | 114.16 |
| 7. Ta-chi | 29.72 | 58.00 | 260.02 | 638.87 | 42.53 | 638.87 | 181.01 |
| 8. Chu-tung | 59.28 | 77.67 | 220.97 | 1218.07 | 1350.65 | 1214.48 | 13.80 |
| 9. Ta-hu | 32.33 | 94.67 | 210.57 | 366.82 | 15403.87 | 364.61 | 0.00 |
| Tungshi Region | 138.42 | 310.34 | 864.42 | 2348.03 | 4778.32 | 2819.48 | 264.92 |
| 10. Shan-chi | 10.40 | 50.67 | 218.55 | 103.86 | 2842.34 | 165.63 | 0.00 |
| 11. An-ma-shan | 33.64 | 111.33 | 153.07 | 731.43 | 0.00 | 728.19 | 38.98 |
| 12. Li-yang | 38.01 | 97.67 | 272.32 | 421.41 | 1935.98 | 558.17 | 111.26 |
| 13. Li-shan | 56.37 | 50.67 | 220.48 | 1091.33 | 0.00 | 1367.49 | 114.68 |
| Nantou Region | 211.82 | 287.32 | 1835.20 | 2352.10 | 11429.54 | 2343.86 | 0.00 |
| 14. Tai-chung | 10.57 | 64.33 | 319.51 | 39.12 | 3330.16 | 39.12 | 0.00 |
| 15. Tan-ta | 52.69 | 49.00 | 340.54 | 688.60 | 1242.50 | 688.60 | 0.00 |
| 16. Pu-li | 77.22 | 68.33 | 652.53 | 966.44 | 4134.43 | 966.44 | 0.00 |
| 17. Shui-li | 54.29 | 59.33 | 348.33 | 602.24 | 2574.87 | 602.24 | 0.00 |
| 18. Chu-shan | 17.05 | 46.33 | 174.29 | 55.70 | 147.58 | 47.46 | 0.00 |
| Chiayi Region | 139.65 | 203.00 | 215.77 | 1316.48 | 1086.00 | 1330.10 | 845.05 |
| 19. A-li-shan | 42.81 | 69.33 | 62.51 | 527.44 | 0.00 | 527.40 | 845.05 |
| 20. Fan-chi-hu | 19.28 | 35.33 | 54.71 | 96.00 | 1086.00 | 95.97 | 0.00 |
| 21. Ta-pu | 32.86 | 44.67 | 60.41 | 196.30 | 0.00 | 195.85 | 0.00 |
| 22. Tai-nan | 44.70 | 53.67 | 38.14 | 496.74 | 0.00 | 510.88 | 0.00 |
| Pingtung Region | 196.06 | 250.33 | 1230.56 | 1588.02 | 7236.45 | 1588.02 | 939.69 |
| 23. Chih-shan | 35.64 | 61.33 | 37.92 | 150.90 | 1405.76 | 150.90 | 0.00 |
| 24. Chao-chou | 70.19 | 62.00 | 188.12 | 624.80 | 1802.85 | 624.80 | 0.00 |
| 25. Liu-guay | 70.96 | 55.67 | 461.42 | 722.46 | 4027.84 | 722.46 | 8.08 |
| 26. Heng-chun | 19.27 | 71.33 | 543.10 | 89.86 | 0.00 | 89.86 | 931.61 |
| Taitung Region | 226.54 | 141.67 | 755.20 | 2679.98 | 8086.47 | 2679.98 | 161.38 |
| 27. Kuan-shan | 113.42 | 54.67 | 272.35 | 1607.90 | 7669.57 | 1607.90 | 57.87 |
| 28. Chi-ben | 44.54 | 41.00 | 184.65 | 552.13 | 416.90 | 552.13 | 103.51 |
| 29. Ta-wu | 44.03 | 20.33 | 100.70 | 394.03 | 0.00 | 394.03 | 0.00 |
| 30. Chan- kong | 24.55 | 25.67 | 197.50 | 125.92 | 0.00 | 125.92 | 0.00 |
| Hualien Region | 320.43 | 284.00 | 1092.92 | 4001.21 | 2263.01 | 4410.58 | 53.19 |
| 31. Shin-chan | 85.95 | 64.00 | 314.71 | 1074.86 | 17.77 | 1085.88 | 0.00 |
| 32. Nan-hua | 51.60 | 76.00 | 228.40 | 886.07 | 110.28 | 882.20 | 16.50 |
| 33. Wan-yong | 59.53 | 74.00 | 282.01 | 829.11 | 339.91 | 819.16 | 0.00 |
| 34. Yu-li | 123.35 | 70.00 | 267.80 | 1611.17 | 1795.05 | 1623.34 | 36.6 |

## DISCUSSION

As pointed out by many authors including Kao and Hwang (2008), Kao (2009), Chen *et al*. (2009), Tone and Tsutsui (2009) and Cook *et al*. (2010), the conventional DEA models apply a single process to evaluate the transforming efficiency of multiple inputs and outputs such that they fail to measure the efforts of different processes and sub-processes within the production systems. Thus, we cannot evaluate the impact of sub-process inefficiencies on the overall efficiency of the system as a whole. In these cases, it is possible that the conventional DEA models evaluate a system as efficient even if none of its component processes is efficient

From Table 2, it can be seen that six DMUs are efficient under the conventional SBM model while according to Theorem 1, under the parallel SBM model, since none of DMUs performs efficiently in all its own sub-DMUs, none of them performs efficiently as a whole. Thus, by using the results of this efficiency measurement we are able to identify the inefficient sub-DMUs and make future improvement. The rankings of the overall efficiency scores of the eight regions taking our approach and taking Kao's approach are shown in Table 3. Comparison of the two sets of scores shows then to have almost identical ranking. The Spearman Rank Correlation coefficient for the rankings in Table 3 is 0.976, showing that the correlation between our results and Kao's results is very high.

Table 2: Efficiency scores

| Working circles | Weight ($w^p_o$) | Parallel SBM model | Conventional SBM model |
|---|---|---|---|
| Lotung Region | | 0.4410 | 0.589 |
| 1. Taipei | 0.187 | 0.3050 | |
| 2. Tai-ping-shan | 0.322 | 0.4840 | |
| 3. Chao-chi | 0.161 | 0.2960 | |
| 4. Nan-au | 0.136 | 0.4490 | |
| 5. Ho-ping | 0.194 | 0.6180 | |
| Hsinchu Region | | 0.7544 | 1.000 |
| 6. Guay-shan | 0.216 | 0.4670 | |
| 7. Ta-chi | 0.229 | 0.6900 | |
| 8. Chu-tung | 0.334 | 0.8220 | |
| 9. Ta-hu | 0.221 | 1.0000 | |
| Tungshi Region | | 0.9110 | 1.000 |
| 10. Shan-chi | 0.134 | 1.0000 | |
| 11. An-ma-shan | 0.272 | 0.6740 | |
| 12. Li-yang | 0.271 | 1.0000 | |
| 13. Li-shan | 0.323 | 1.0000 | |
| Nantou Region | | 0.6180 | 0.732 |
| 14. Tai-chung | 0.116 | 1.0000 | |
| 15. Tan-ta | 0.224 | 0.5970 | |
| 16. Pu-li | 0.343 | 0.6350 | |
| 17. Shui-li | 0.227 | 0.5710 | |
| 18. Chu-shan | 0.090 | 0.2310 | |
| Chiayi Region | | 0.7890 | 1.000 |
| 19. A-li-shan | 0.335 | 1.0000 | |
| 20. Fan-chi-hu | 0.160 | 0.4660 | |
| 21. Ta-pu | 0.221 | 0.4320 | |
| 22. Tai-nan | 0.284 | 1.0000 | |
| Pingtung Region | | 0.6900 | 1.000 |
| 23. Chih-shan | 0.138 | 0.5710 | |
| 24. Chao-chou | 0.288 | 0.5880 | |
| 25. Liu-guay | 0.354 | 0.6260 | |
| 26. Heng-chun | 0.220 | 1.0000 | |
| Taitung Region | | 0.7542 | 1.000 |
| 27. Kuan-shan | 0.462 | 1.0000 | |
| 28. Chi-ben | 0.234 | 0.6270 | |
| 29. Ta-wu | 0.154 | 0.6290 | |
| 30. Chan- kong | 0.150 | 0.3230 | |
| Hualien Region | | 0.6730 | 1.000 |
| 31. Shin-chan | 0.256 | 0.6280 | |
| 32. Nan-hua | 0.210 | 0.6100 | |
| 33. Wan-yong | 0.223 | 0.5700 | |
| 34. Yu-li | 0.311 | 0.8280 | |

Table 3: Ranking of efficiency scores

| | | Kao's results | |
|---|---|---|---|
| Regions | Our ranking | Ranking | Overall efficiency |
| Lotung Region | 8 | 8 | 0.752 |
| Hsinchu Region | 3 | 4 | 0.823 |
| Tungshi Region | 1 | 1 | 0.937 |
| Nantou Region | 7 | 7 | 0.773 |
| Chiayi Region | 2 | 2 | 0.701 |
| Pingtung Region | 5 | 5 | 0.799 |
| Taitung Region | 4 | 3 | 0.860 |
| Hualien Region | 6 | 6 | 0.794 |

Thus the new approach is suitable for measuring the overall efficiency of the whole system with the added benefit of allowing inefficient sub-DMUs to be identified and potentially rectified.

## CONCLUSION

In an earlier study, a radial DEA model was introduced by Kao (2009) for measuring the efficiency of a system composed of parallel units operating independently and where the sum of inputs/outputs for all units is equal to the input/output of the system. In this study, we have introduced a non-radial model based on a Slacks-Based Measure (SBM) framework that evaluates the overall efficiency of the system by considering the operations of its units. Under this framework, the overall efficiency of the system is expressed as a weighted sum of the efficiencies of its units. With decomposition of the overall efficiency, the units which cause the inefficient operation of the system can be identified for future improvements

The proposed model is based on the assumption of Constant Returns to Scale (CRS). By adding the convexity constraint into the PPS which is built by $n \times t_j$ sub-DMUs, the discussion can be expanded to use the Variable Returns to Scale (VRS) assumption.

It is noteworthy that real systems are generally more complex than the parallel system discussed in this study. Tone and Tsutsui (2009) developed a network DEA model based on a weighted SBM approach that can be applied in series systems. Since the series and parallel structure are two basic structures of a network system, we can transform a network system into a combination of series and parallel structures to evaluate the overall efficiency and the efficiencies of sub-processes.

## ACKNOWLEDGMENT

The researchers thank Professor Azizollah Memariani for his comments and suggestions. This work is supported by Graduate Research Assistance of University of Putra Malaysia (Grand no. 5523936).

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