

## Intervened 2-Tier Poisson Distribution for Understanding Hospital Site Infectivity

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**Abstract: Problem statement:** Whether it is a surgical site or medical treatment at a hospital site, the nurses in particular and the entire medical team including surgeons/physicians in general undergo a risk of being infected ironically by the patients whom they intend to disinfect. There are 2-tiers of patients at the site. One type consists of patients who are internally and well pre-disinfected, not to be sourced for infecting the medical team. The second type consists of patients who are influx to the hospital site and are not well pre-disinfected enough. It is the second type which is a source of hospital site infection for the medical team in general and for the nurses in particular. In other words, the nurses who have to deal with the second type of patients get more exposed to the virus from the patients themselves. This is named the nurses' exposure rate. Independently, there is an inactivity rate in general for anyone. To reduce such an infectivity, the hospital management makes an intervention with preventive efforts to reduce the infection rate and the impact of such preventive intervention efforts is captured by a parameter in our model. Using a maximum likelihood estimate of the intervention parameter with the data information, we assess the significance of the intervention efforts.

**Approach:** For this concept to work, there is a need to develop an appropriate model as none exists in the literature to be suitable. The model is an abstraction of the reality in the hospital set up. Such a needed, new probability count model is introduced. It is named an Intervened 2-Tier Poisson (I2TP) distribution in this article. Several statistical properties of the I2TP distribution are derived and illustrated to explain the inactivity rate,  $\theta > 0$  during the treatments of contagious patients in a hospital. Not all nurses are exposed to the virus, while  $0 \leq \pi \leq 1$  is their exposure rate towards infection. The physicians/surgeons, nurses and staffs undergo a risk of being infected during their treatment of infection or surgery on patients in spite of precautions to avoid infection. The hospital management intervenes with several precautions to minimize, if not eliminate the health care personnel's risk of being infected. In this article, a statistical methodology is developed to estimate and test the significance of the management's intervention effect,  $\rho \geq 0$ .

**Results:** The methodology is illustrated using the number of exposed and infected nurses during their healthcare of SARS patients in a Toronto hospital as reported in <http://wwwnc.cdc.gov/>. There were 32 nurses in the Toronto hospital working with the Severe Acute Respiratory Syndrome (SARS) patients. The sixteen activities of the nurses included in our analysis are administration of medication, intubation, bathing, manipulation of bipap mask, radiology procedures among others. In all these activities, the nurses are well trained to use disinfected gloves, nasal masks. As part of the preventive measures to avoid infection from the SARS patients. The exposure rates for the nurses in these activities to SARS patients varied from 0.13 to 0.81. The infectivity ranged from 1.26 to 8 in these activities. The impact of the intervention efforts ranged from 0.25 to 206.3 in all these sixteen activities. The impact of the intervention efforts was insignificant in the activities: endotracheal aspirate, integration of a peripheral, intravenous catheter Intubation, manipulation of bipap mask, Manipulation of bipap mask, manipulation of commodes or bedpans, Nebulizer treatment and Suctioning before intubation. The impact of the intervention was significant in the activities: administration of medication, assessment of patient, bathing or patient transfer, manipulation of oxygen mask, mouth or dental care, performing an electrocardiogram, radiology procedures, suctioning after intubation and venipuncture.

**Conclusion/Recommendations:** It is interesting to notice that the preventive intervention efforts by the hospital management for the nurses to be disinfected from the SARS patients worked in some activities but not in others. This distinction could be made because of the intervened 2-tier Poisson distribution which is introduced in this article. Clues for successful intervention in some but not in other activities perhaps hid in

covariates. Currently, the author is not able to access such data on covariates. The future research work would proceed in this direction using regression concepts.

**Key words:**Count model, likelihood ratio test, p-value, hypothesis testing, exposure rate, physicians/surgeons, management intervenes, statistical properties, infectivity rate

## INTRODUCTION

Infection is a colonization of a virus leading towards a disease. Hosts do normally fight infections via their immune system. The physicians/surgeons, nurses and staffs undergo a risk of being infected during their treatment of infections or surgery patients in spite of precautions to avoid infection. Ironically, the source of infection for physicians/surgeons, nurses and supportive staffs is the patients who are helped by them. This serious phenomenon occurs in surgical or hospice situations. Viable prevention strategies are necessary, though not sufficient, to avoid being infected. Techniques like hand washing, wearing gowns and wearing face masks among others help to prevent infections from being passed on to the healthcare workers from the patients. This article examines the issue and develops a new model. This new model is named Intervened 2-Tier Poisson (I2TP) distribution. The subtitle "2-tier" is appropriate to suit the theme that not all healthcare workers of the contagious patients get exposed in the first place and not all exposed health care workers do end up with an infection.

The statistical properties of I2TP distribution are derived to explain health care workers' infectivity rate during a surgery or treatment of patients in hospital. The results are illustrated later in the article using the number of exposed and infected cases nurses during the treatment of SARS patients in a Toronto hospital as reported in Loeb *et al.* (2004). The final thoughts are stated for future research direction in the end.

### Main results: Intervened 2-tier poisson distribution

Let  $0 < \pi \leq 1$ ,  $\theta > 0$  and  $\rho \geq 0$  denote respectively the exposure rate, infectivity rate and intervention effect to minimize (if not eliminate) the infections during the activities rendered to the patients by healthcare workers in general and by nurses in particular. To be specific, let the random variable (rv),  $N$  of healthcare workers have encountered with the contagious patients in a hospital. The rv  $N$  usually follows a Poisson distribution. That is:

$$\Pr(N|\theta) = e^{-\theta} \theta^n / n!; n = 0, 1, 2, 3, \dots; \theta > 0$$

Realizing that not all healthcare workers are exposed, let  $I_k = 1$  if the  $k^{\text{th}}$  healthcare worker is exposed with the probability  $\pi$  and  $I_k = 0$  otherwise. Consider  $X = I_1 + I_2$

$+ \dots + I_n$ . Notice that  $X$  follows a binomial distribution with parameter  $\pi$  conditional on  $N = n$ . That is:

$$\Pr(X|N = n) = \frac{n!}{x!(n-x)!} \pi^x (1-\pi)^{n-x}$$

Unconditionally, the rv  $X$  follows a Poisson distribution because Eq. 1:

$$\begin{aligned} \Pr(X = x) &= \sum_{n=0}^{\infty} \Pr(n) \Pr(x|n) \\ &= \sum_{n=x}^{\infty} \frac{e^{-\theta} \theta^n \pi^x (1-\pi)^{n-x}}{x!(n-x)!} = \frac{e^{-\pi\theta} (\pi\theta)^x}{x!} \end{aligned} \quad (1)$$

The Poisson distribution in (1) with parameter  $\pi\theta$  is a 2-tier type. However, the event  $X = 0$  is not usually observed. The data collection apparatus is activated only when  $X \geq 1$ . In other words, medical intervention takes place to control the infectivity only when a non-zero  $X = 1, 2, 3, \dots$  incidence is noticed. This probability pattern of the non-zero incidence of  $X$  is then a positive Poisson (PP) distribution (2). That is Eq. 2:

$$\begin{aligned} \Pr(X|\pi, \theta) &= (e^{\pi\theta} - 1)^{-1} (\pi\theta)^x / x!; \\ x &= 1, 2, 3, \dots; \theta > 0 \end{aligned} \quad (2)$$

At this stage, the healthcare management intervenes to control and/or eliminate the infectivity by resorting to various preventive actions including training healthcare workers to be disinfected. The effectiveness of this intervention is not observable but an unknown parameter  $\rho \geq 0$ . Let  $Z$  be the number of additional healthcare workers with infection since the time of interventions. The general infectivity rate  $\theta > 0$  is now modified to  $\rho\theta > 0$  because of the intervention efforts. Consequently,  $\Pr(Z = z) = e^{-\rho\theta} (\rho\theta)^z / z!$ . The recorded number of infected healthcare workers is not  $Z$  but rather  $Y = X + Z$ . The rv  $Y$  follows an intervened 2-tier Poisson (I2-TP) distribution Eq. 3. That is:

$$\begin{aligned} \Pr(Y = y|\pi, \theta, \rho) &= \sum_{i=1}^y \Pr(X = i) \Pr(Z = y - i) \\ &= \frac{(e^{\pi\theta} - 1)^{-1} e^{-\pi\theta} \pi^y}{y!} \sum_{i=1}^y \binom{y}{i} \left(\frac{\pi}{\rho}\right)^i \\ &= \frac{(e^{\pi\theta} - 1)^{-1} e^{-\pi\theta} [(\pi + \rho)^y - \rho^y] \theta^y}{y!} \end{aligned} \quad (3)$$

where,  $y = 1, 2, 3, \dots$ . Is the expression (3) a bona-fide probability distribution? The answer is affirmative because  $\Pr(Y = y | \pi, \theta, \rho) > 0$  and  $\sum_{y=1}^{\infty} \Pr(Y = y | \pi, \theta, \rho) = 1$ .

After algebraic simplifications, its mean and variance are obtained. The mean of I2-TP distribution is in Eq. 4:

$$\begin{aligned} \mu_y &= E(Y | \pi, \theta, \rho) \\ &= \sum_{y=1}^{\infty} y \frac{(e^{\pi\theta} - 1)^{-1} e^{-\rho\theta} [(\pi + \rho)^y - \rho^y] \theta^y}{y!} \\ &= \left[ \rho + \frac{\pi e^{\pi\theta}}{(e^{\pi\theta} - 1)} \right] \end{aligned} \quad (4)$$

and the variance is a quadratic function of the mean as in Eq. 5. That is:

$$\sigma_y^2 = \rho + (\mu_y - \rho)(1 + \rho + \pi - \mu_y) \quad (5)$$

The survival function of the I2-TP distribution is:

$$\begin{aligned} S(r | \pi, \theta, \rho) &= \Pr(Y > r) \\ &= \sum_{y=r}^{\infty} \frac{(e^{\pi\theta} - 1)^{-1} e^{-\rho\theta} [(\pi + \rho)^y - \rho^y] \theta^y}{y!} \\ &= \left\{ \frac{e^{\pi\theta} \Pr[\chi_{2r}^2 \leq 2(\pi + \rho)\theta] - \Pr[\chi_{2r}^2 \leq 2\rho\theta]}{(e^{\pi\theta} - 1)} \right\} \end{aligned}$$

The odds of having no more than r infected healthcare workers is then:

$$\begin{aligned} \text{odds}_r &= \frac{1 - S(r | \pi, \theta, \rho)}{S(r | \pi, \theta, \rho)} \\ &= \left\{ \frac{e^{\pi\theta} \Pr[\chi_{2r}^2 \leq 2(\pi + \rho)\theta] - \Pr[\chi_{2r}^2 \leq 2\rho\theta]}{(e^{\pi\theta} - 1)} \right\}^{-1} - 1 \end{aligned}$$

The odds are popular in healthcare studies. The epidemiologists are fond of the odds. In particular, the odds of having one infected healthcare worker in a hospital where contagious patients are treated by healthcare workers is given in Eq. 6:

$$\text{odds}_1 = \left\{ \frac{(e^{\pi\theta} - 1)e^{\rho\theta}}{\pi\theta} - 1 \right\}^{-1} \quad (6)$$

An estimation procedure is necessary for the model parameters based on a collected sample  $y_1, y_2, \dots, y_n$  of size n from I2-TP distribution in (3). The Maximum Likelihood Estimates (MLE) are preferable because the MLE possess invariance property. That is the MLE of a

function is the function of MLE. First, the MLE of  $\pi$  is the solution of the score functions  $\partial_{\pi} \ln L(x_1, x_2, \dots, x_n | n) = 0$  where  $\partial_a$  is the derivative with respect to a. It is in Eq. 7:

$$\hat{\pi}_{mle} = \frac{x}{n} \quad (7)$$

Secondly, the conditional score functions  $\partial_{\theta} \ln L(\theta, \rho | \pi) = 0$  and  $\partial_{\rho} \ln L(\theta, \rho | \pi) = 0$  need to be solved. To obtain the conditional score functions, the log likelihood function is in Eq. 8:

$$\begin{aligned} \ln L(\rho, \theta | \hat{\pi}_{mle}) &= -n \ln(e^{\hat{\pi}_{mle}\theta} - 1) - n\rho\theta \\ &+ \sum_{i=1}^n \ln[(\hat{\pi}_{mle} + \rho)^{y_i} - \rho^{y_i}] \\ &+ n\bar{y} \ln \theta - \sum_{i=1}^n \ln y_i \\ &\approx -n[\ln \hat{\pi}_{mle} + \ln \theta + \rho\theta] \\ &+ n\bar{y}[\ln(\hat{\pi}_{mle} + \rho) + \ln \theta] - \sum_{i=1}^n \ln y_i \end{aligned} \quad (8)$$

ignoring the small amount  $(\frac{\rho}{\hat{\pi}_{mle} + \rho})^{y_i} < 1$ . The log likelihood function in (8) is differentiated with respect to the parameters  $\theta$  and  $\rho$  to obtain the score functions. The score functions are in Eq. 9 through Eq. 10:

$$\partial_{\rho} \ln L(\rho, \theta | \hat{\pi}_{mle}) = 0 \approx -n\theta + \frac{n\bar{y}}{\hat{\pi}_{mle} + \rho} \quad (9)$$

and:

$$\begin{aligned} \partial_{\theta} \ln L(\rho, \theta | \hat{\pi}_{mle}) &= 0 \\ &\approx -n(1 + \rho) - \frac{n}{\theta} + \frac{n\bar{y}}{\theta} \end{aligned} \quad (10)$$

The simultaneous and conditional MLE of the parameters are therefore in Eq. 11 and 12:

$$\hat{\rho}_{mle} \approx \hat{\pi}_{mle} (\bar{y} - 1) \quad (11)$$

and:

$$\hat{\theta}_{mle} \approx \frac{(\bar{y} - 1)}{\hat{\rho}_{mle}} \quad (12)$$

For administrative reasons, the healthcare management might want to assess the significance of



$$\text{Power} = \Pr\left[\chi_{(1+\hat{\delta}_p)^2}^2 \frac{df}{(1+2\hat{\delta}_p)}\right] > \frac{(1 + \frac{\hat{\delta}_{p=0}}{1 + \hat{\delta}_{p=0}})(-\ln \Lambda_{\rho=p^*}) \chi_{(1+\hat{\delta}_{p=0})^2}^2 \frac{df, \alpha}{(1+2\hat{\delta}_{p=0})}}{(1 + \frac{\hat{\delta}_{p^*}}{1 + \hat{\delta}_{p^*}})(-\ln \Lambda_{\rho=0})} \quad (16)$$

**Illustration using sars infections:** Severe Acute Respiratory Syndrome (SARS) patients were treated by nurses who worked in two Toronto critical care units. Some nurses were infected. Chen *et al.* (2008) and Poutanen *et al.* (2003) for details about SARS. McKibben *et al.* (2005), Understanding Infectious Diseases, 2010 and Preventing Infections Adequately, 2010. for details about the recommended preventive actions to be disinfected. The Table 1 provides infected data during services to contagious patients on treatment care activities and the results for the methodology in this article. We considered only sixteen healthcare activities. The excluded healthcare activities had either missing entry or just one infected nurse which is not enough for modeling. Our methodology is suitable for the activities which had two or more infected nurses. The Fig. 1 confirms an upward relationship between the number of exposed nurses (X) and the number of infected nurses (Y). More exposed nurses resulted in more infected nurses.

The MLE of exposure rate, infectivity rate and intervention effect are indicated by notations  $\hat{\pi}_{mle}$ ,  $\hat{\rho}_{mle}$  and  $\hat{\theta}_{mle}$  respectively in Table 1.

The notations df0 and df1 denote the degrees of freedom under the null hypothesis  $H_0: \rho = 0$  and alternative hypothesis  $H_a: \rho \neq 0$  respectively. The p-value is the chance for the null hypothesis to be true meaning that its smaller value refers rejection of the null hypothesis. An interpretation is that the intervention was effective only in the healthcare activities: administration of medication, assessment of patients, bathing or patient transfer, manipulation of oxygen mask, mouth or dental care, performing an electro cardiogram, radiology procedures, suctioning after intubation and vein puncture. The expression (15) is used to perform the hypothesis testing. In most of the above healthcare activities, the statistical power to accept the specific alternative  $H_a: \rho = \rho^* = 1$  is excellent as they are shown in Table 1.

The intervention was not effective in healthcare activities: Endotracheal aspirate, insertion of a peripheral, intubation, manipulation of BiPAP mask, manipulation of commodes or bedpans, nebulizer treatment and suctioning before intubation. In these activities, the statistical power to accept the specific alternative  $H_a: \rho = \rho^* = 1$  is poor as noticeable so in Table 1.

The odds for a nurse to get infected are given in Table 2 for all sixteen activities. Note the odds are high in endotracheal aspirate, insertion of a peripheral, intubation, manipulation of BiPAP mask, manipulation of commodes or bedpans and nebulizer treatment. The other activities have lesser odds as shown in Table 2.

Table 1: # infected nurses while they provided patients personal care activities

Patient care activity	y	x	n	$\hat{\pi}_{mle}$	$\hat{\rho}_{mle}$	$\hat{\theta}_{mle}$	dfo	p value	de11	power
Administration of medication	5	23	32	0.72	2.88	1.39	1.60E+01	3E-05	21.0	0.7292
Assessment of patient	6	23	32	0.72	3.59	1.39	1.60E+01	3E-05	23.0	0.7775
Bathing or patient transfer	7	26	32	0.81	4.88	1.26	1.60E+01	6E-11	25.0	0.8368
Endotracheal aspirate	3	12	32	0.38	0.75	2.67	1.60E+01	0.420	11.0	0.2647
Intertion of a peripheral intravenous cathet	3	5	32	0.16	0.31	6.40	1.60E+01	0.420	70.0	0.0088
Intubation	3	4	32	0.13	0.25	8.00	1.60E+01	0.420	96.0	0.0020
Manipulation of bipap mask	3	6	32	0.19	0.38	5.33	1.60E+01	0.420	53.0	0.0274
Mainpulation of commodes or bedpans	3	5	32	0.16	0.31	6.40	1.60E+01	0.420	70.0	0.0088
Mainpulation of oxygen mask	7	14	32	0.44	2.63	2.29	1.60E+01	6E-11	20.0	0.6290
Mouth or dental care	5	21	32	0.66	206.30	1.52	1.60E+01	3E-05	20.0	0.6568
Nebulizer treatment	3	5	32	0.16	0.31	6.40	1.60E+01	0.420	70.0	0.0088
Performing an electrocardiogram	4	12	32	0.38	1.13	2.67	1.60E+01	0.007	3.6	0.3576
Radiology procedures	4	15	32	0.47	1.41	2.13	1.60E+01	0.007	9.2	0.4555
Suctioning after intubation	4	19	32	0.59	1.78	1.68	1.60E+01	0.007	14.0	0.4883
Suctioning before intubation	3	4	32	0.13	0.25	8.00	1.60E+01	0.420	96.0	0.0020
Venipuncture	6	17	32	0.53	2.66	1.88	1.60E+01	5E-08	20.0	0.6408
Y= infected nurses X= exposed nurses	n = nurses in hospital									

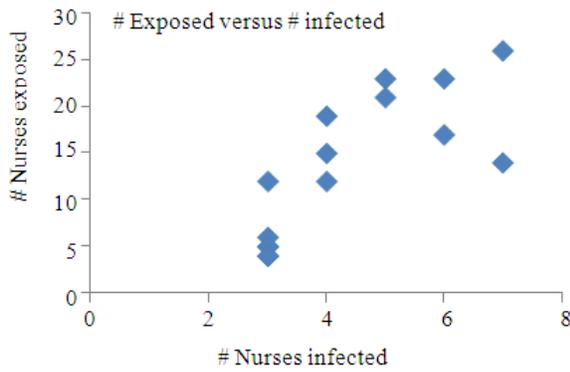


Fig. 1: # nurses exposed versus infected

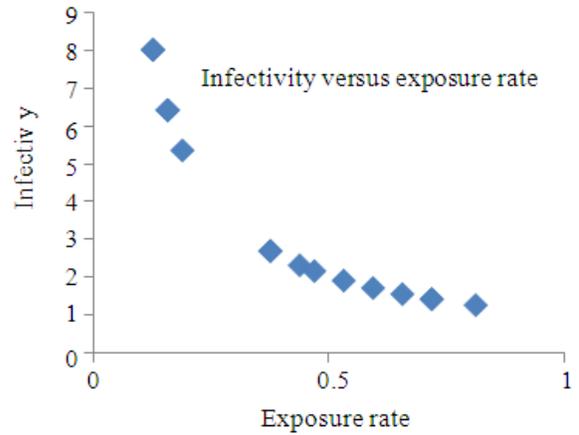


Fig. 4: Infectivity versus exposure rate

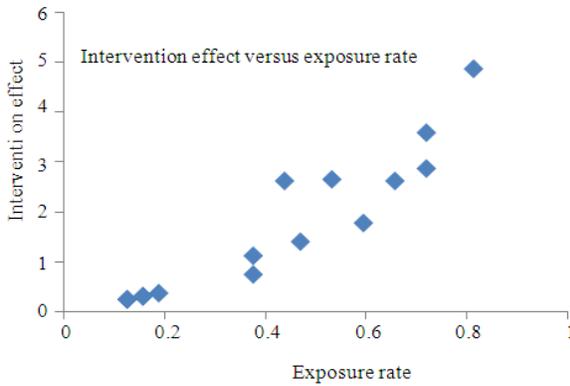


Fig.2: Intervention effect versus exposure rate

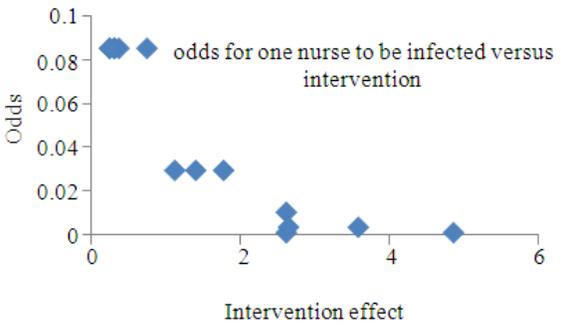


Fig. 5: Odds for one nurse to get infected versus intervention effect

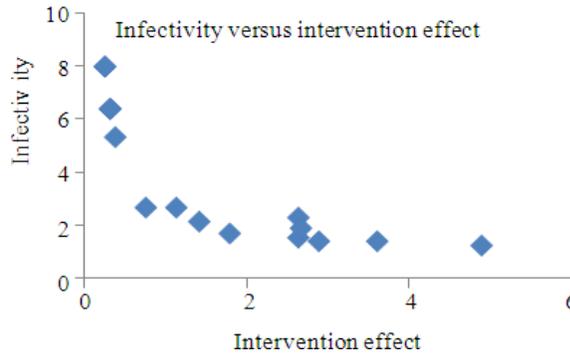


Fig. 3: Infectivity versus intervention effect

The Figure 2 illustrates that when the exposure rate increase, the intervention effect has also increased as one would expect. The Fig. 3 illustrates that when the intervention effect is high, the infectivity rate is lower as one would expect. The Fig. 4 illustrates that when the exposure rate is high, the infectivity is low and it is not quite intuitive. It is so because the intervention effect has an impact on both of them.

Table 2: Odds for one nurse to be infected in treating SARS patients at Toronto

Patient care activity	odds1
Administration of medication	0.011
Assessment of patient	0.004
Bathing or patient transfer	0.001
Endotracheal aspirate	0.085
Insertion of a peripheral intravenous catheter	0.085
Intubation	0.085
Mainpulation of Bipap mask	0.085
Mainpulation of commodes or bedpans	0.085
Manipulation of oxygen mask	0.001
Mouth or dental care	0.011
Nebulizer treatment	0.085
Performing an electrocardiogram	0.030
Radiology procedures	0.030
Suctioning after intubation	0.030
Suctioning before intubation	0.085
Venipuncture	0.004

The Fig. 5 illustrates that the odds for one nurse to be infected is high only when the intervention effect is low and vice versa. The importance of considering the intervention effect in analyzing exposure versus infection data could not be overstated.

## CONCLUSION

The intervened 2-tier Poisson distribution and the methodology of this article are quite useful to analyze similar data in engineering, marketing, economics and sociology and business studies. Also, generalized regression methodology will be useful to assess what extraneous factors which induce the intervention more effective. This research work will be pursued in future STUDY.

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