

# DYNAMIC MODELING AND GAIN SCHEDULING CONTROL OF A CHILLED WATER COOLING SYSTEM

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## ABSTRACT

A dynamic model of a Chilled Water (ChW) cooling system with a stratified ChW tank was developed. A PI-based Gain Scheduling (GS) control strategy was designed to improve thermal performance of the chiller system. Five different operation strategies were simulated and the energy consumption and cost of operating the system under full load and partial load conditions were compared. The results showed that the operating strategy with optimized chilled water set point saved 7.16% energy (21.5% cost) compared to the case with constant set point under full load conditions. These savings were more significant (8.5% energy and 36% cost) under partial load conditions.

**Keywords:** ChW Cooling System, ChW Tank, Gain-Scheduling Control, Near-Optimal Control

## 1. INTRODUCTION

The ChW cooling systems are widely used for air-conditioning in buildings. In majority of cases, a Thermal Energy Storage (TES) system in conjunction with the ChW cooling system is used for load leveling and to improve energy efficiency. The TES system charges thermal energy within an insulated tank and discharges the stored thermal energy during peak energy demand period in order to reduce the system operation cost when the Real-Time-Price (RTP) utility rate is in effect. The most popular TES system uses chilled water as a medium in a stratified chilled water tank or as an ice-storage tank.

In this study, a ChW cooling system, consisting of a zone, a cooling coil, a chiller and a cooling tower, along with a stratified ChW tank, was considered. The ChW tank was designed to cover a Partial Load Ratio (PLR) of 80% of the design day load. The cooling system performance was investigated using different control strategies. The main focus of this study is to develop a dynamic model of a chilled water cooling system with a ChW tank and design control strategies to improve

energy efficiency of the overall system. To achieve these main goals, the following objectives were defined:

- To model an integrated ChW cooling system with a stratified ChW tank
- To design a Gain Scheduling (GS) controller and implement on the cooling system to improve its control performance. Compare the control performance of the GS controller with a conventional fixed gain PI controller
- To develop a near-optimal algorithm for reducing energy consumption of the chiller plant by increasing the use of cool energy from the stratified ChW tank

## 2. LITERATURE REVIEW

A number of studies on modeling and optimizing the performance of the ChW cooling systems have been conducted. For example (Braun, 2007) presented a near-optimal control method for charging and discharging an ice storage system when real time price utility rate structure is available. The different

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combinations of cooling plants, storage sizes, building locations and RTP rates were investigated and a near-optimal algorithm was proposed.

Swider (2002) studied four types of regression models for chiller simulation and compared with two artificial neural network models. The results of the study showed that the neural network models have a higher level of difficulty but higher accuracy than the regression models.

Li and Sumathy (2002) showed that the storage temperature distribution in a stratified tank significantly affects the overall system performance.

Guorong (1997) proposed two different dynamic models, using bottom-up and top-down approaches, for studying the interactions between the components of the multizone HVAC system and the building shell. Recent studies on control strategies for cooling systems with storage include (Li *et al.*, 2013; Beghi *et al.*, 2013; Ma *et al.*, 2012).

Jian You (2013) presented a steady state model to predict the performance of cooling tower used in air conditioning applications.

Alireza (2006) developed a gain-scheduling scheme controller for improving the performance of a synchronous generator. The controller allows its proportional and integral gains to be varied within a predetermined range. The maximum and minimum values of the proportional gain were predetermined. Results from simulation showed that the gain-scheduling controller keeps the system more stable than the conventional PI controller does.

It can be noted from the above literature review that most studies focussed on component level modeling, steady state analysis or energy storage strategies. In this study, the dynamics and control of an integrated cooling system with a thermal storage tank are explored.

### 3. DESCRIPTION OF THE ChW COOLING SYSTEM

**Figure 1** shows the schematic diagram of a ChW cooling system with a stratified ChW tank.

The operation of the whole ChW system is divided into two stages: One is the charge cycle of the ChW tank; and the other one is the discharge cycle of the tank. The charge cycle of the tank operates from 20:00 to 7:00 h and the discharge cycle is activated during the occupancy period which is from 7:00 to 20:00 h. In other words, the ChW tank is operated under its discharge cycle during the whole occupancy period of the building.

As shown in **Fig. 1**, there are seven control loops in the ChW cooling system. The functions of the control loops are described in the following.

#### 3.1. VAV Box Control Loop

The controller C6 is used to control the supply mass flow rate of the air to the zone in order to maintain the zone air temperature at its set point.

#### 3.2. Cooling Coil Control Loop

The supply air temperature to the zone is controlled by the controller C5.

#### 3.3. ChW Tank Charge and Discharge Control Loop

During the discharge cycle of the tank, the controller C4 adjusts the opening of the throttling valve TV2 which determines the mass flow rate of ChW from both the tank and chiller to maintain the temperature of the mixed supply ChW to the coil.

When the ChW tank is operated under its charge cycle, the controller C3 controls the three-way throttling valve TV1 by supplying the ChW from the chiller; and the rest of the ChW is by-passed to the coil for controlling the supply air temperature to the zone.

It is to be noted that the discharge and charge cycles of the tank are not operated simultaneously, therefore the controller C4 regulating the TV2 would not interact with controller C3.

#### 3.4. Chiller Control Loop

The controller C2 maintains the supply ChW temperature at a chosen set point in the chiller control loop. The water mass flow rate in the evaporator water loop is held constant.

#### 3.5. Cooling Tower Control Loop

The controller C1 maintains the cooled water temperature leaving the cooling tower by adjusting the fan speed in the cooling tower.

#### 3.6. ChW Tank Water Mass Flow Rate Balancing Control Loop

The controller C7 is used to maintain a balance between the water flow rate entering and leaving the chilled water tank by maintaining a constant water level in the ChW tank.

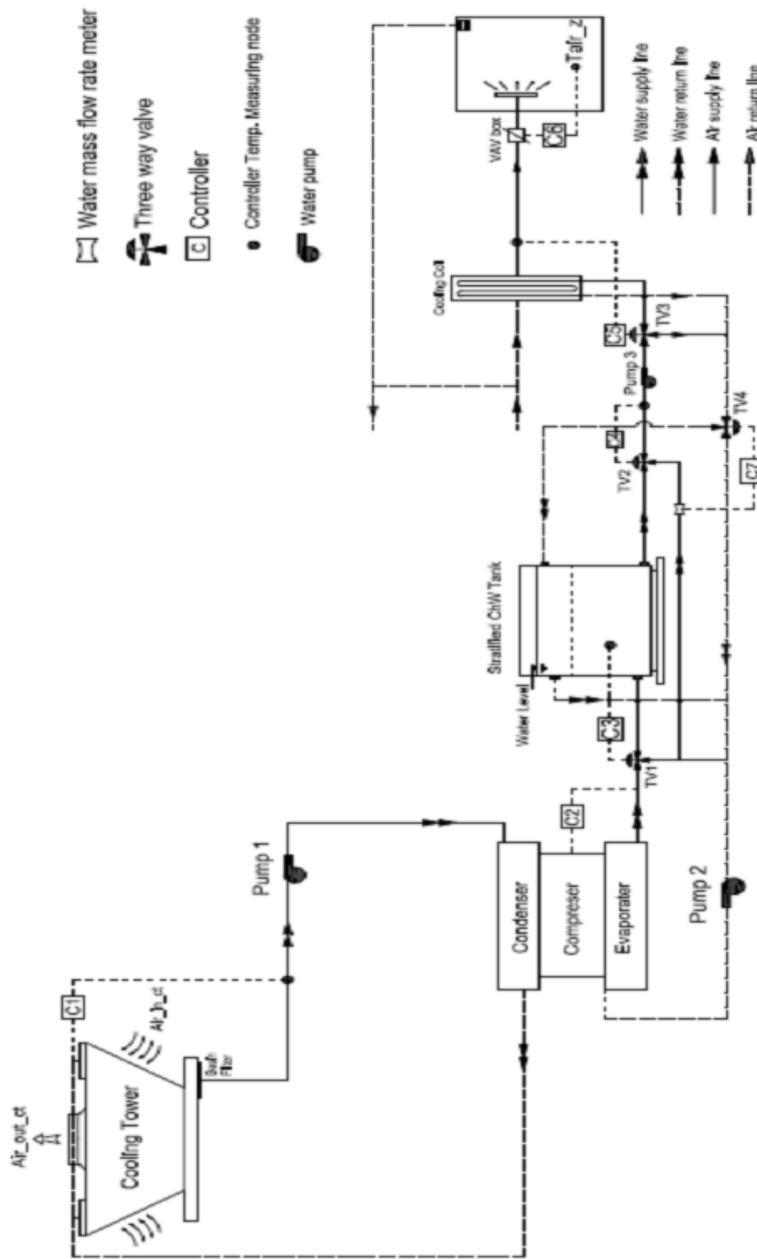


Fig. 1. Schematic diagram of a ChW cooling system with a stratified ChW tank

#### 4. THE COMPONENT MODELS

The dynamic models of the chiller plant consist of the following component models:

- Zone model
- Cooling coil model

- Chiller model
- Cooling tower model
- Stratified ChW tank model

##### 4.1. Zone Model

A single story single zone building is considered in this study. The building measures  $80 \times 50 \times 3 \text{ m}^3$ . The

window area is 30 percent of each wall area. The Air Change rate (ACH) of 0.25 was assumed. The thermal resistance of the wall and roof are 3.36 RSI and 5.34 RSI, respectively. The temperature of each layer of the wall was assumed to be uniformly distributed. Therefore one-dimensional heat transfer was considered to model the heat transfer process in the wall and roof elements. The total sensible cooling load of this building, which included the heat gain due to heat conduction through the building envelope, heat gain due to air infiltration, lights, occupants and equipment, was determined assuming design day conditions. Dynamic equations for the zone air, wall and roof temperatures were written based on the energy heat balance principle Equation 1 to 3:

$$\frac{dT_{wall,d}}{dt} = \left( \frac{A_{nw,d}}{R_{wa}} (T_{sat,d} - T_{wall,d}) - \frac{A_{mw,d}}{R_{wz}} (T_{wall,d} - T_{az}) \right) / C_{brsw} \quad (1)$$

$$\frac{dT_{roof}}{dt} = \left( \frac{A_{roof}}{R_{ra}} (T_{sat,r} - T_{roof}) - \frac{A_{roof}}{R_{rz}} (T_{roof} - T_{az}) \right) / C_{rzd} \quad (2)$$

$$\frac{dT_{az}}{dt} = \left( Q_{hs,cond} + Q_{hs,solar} + Q_{hs,opt} + Q_{hs,inf} + Q_{hs,light} + Q_{hs,eqp} - Q_{cz} \right) / C_{az} \quad (3)$$

Where:

$$Q_{cz} = m_{az} cp_a (T_{az} - T_{asc})$$

#### 4.2. Cooling Coil Model

A four-row cross flow cooling coil was designed based on the estimated total cooling load plus a safety factor. The cooling coil was divided into four control volumes. The air-side and water-side heat transfer coefficients of the tube wall were calculated as a function of the Reynolds and Nusselt numbers of the fluids (McQuiston *et al.*, 2005). The dynamic equations for the air, ChW and tube wall temperatures in each control volume were written by applying fundamental principles of mass and energy balance in the cooling coil Equation 4 to 6:

$$\frac{dT_{ac,cv_i}}{dt} = (m_{asc} cp_a (T_{ac,cv_{i-1}} - T_{ac,cv_i}) + h_{oc} A_{ot,cv_i} (T_{ib,cv_i} - (T_{ac,cv_{i-1}} + T_{ac,cv_i}) / 2)) / C_{ac,cv_i} \quad (4)$$

$$\frac{dT_{ib,cv_i}}{dt} = (h_{oc} A_{ot,cv_i} ((T_{ac,cv_{i-1}} + T_{ac,cv_i}) / 2 - T_{ib,cv_i}) + h_{ic} A_{it,cv_i} ((T_{cw,cv_i} + T_{cw,cv_{i+1}}) / 2 - T_{ib,cv_i})) / C_{ib,cv_i} \quad (5)$$

$$\frac{dT_{cw,cv_i}}{dt} = (m_{cwc} cp_{cw} (T_{cw,cv_{i+1}} - T_{cw,cv_i}) + h_{ic} A_{it,cv_i} (T_{ib,cv_i} - (T_{cw,cv_{i+1}} + T_{cw,cv_i}) / 2)) / C_{cw,cv_i} \quad (6)$$

#### 4.3. Chiller Model

The COP of the evaporator is modeled by using a multivariate polynomial regression equation which is a function of the ChW temperature leaving the evaporator, cooled water temperature entering the condenser, mass flow rate of ChW in the evaporator water loop and mass flow rate of cooled water in the condenser water loop. There are ten regressive coefficients which were obtained by fitting a polynomial equation to the data from manufacturer's catalogue. The regression equation is shown below Equation 7:

$$COP_{evap} = C_0 + C_1 T_{cws,cond} + C_2 T_{cws,chi} + C_3 m_{cow,cond} + C_4 m_{cws,chi} + C_5 T_{cws,cond}^2 + C_6 T_{cws,chi}^2 + C_7 m_{cow,cond}^2 + C_8 m_{cws,chi}^2 + C_9 T_{cws,cond} T_{cws,chi} + C_{10} m_{cow,cond} m_{cws,chi} \quad (7)$$

Dynamic equations for the ChW temperature leaving the evaporator and cooled water temperature leaving the condenser were written based on the energy balance principle Equation 8 and 9:

$$\frac{dT_{cowr,cond}}{dt} = (m_{cow,cond} cp_{cow} (T_{cws,cond} - T_{cowr,cond}) + Q_{comp} (COP_{evap} + 1)) / C_{cow} \quad (8)$$

$$\frac{dT_{cws,chi}}{dt} = (m_{cws,chi} cp_{cw} (T_{cwr,chi} - T_{cws,chi}) - Q_{comp} COP_{evap}) / C_{cw} \quad (9)$$

#### 4.4. Cooling Tower Model

A steady state model (Beghi *et al.*, 2013) was used to predict the performance of the cooling tower in this study. According to energy conservation principle, the heat rejected by air should be equal to the energy removed by the cooling water. Therefore, the following equation was used to calculate the mass flow rate of air entering to the cooling tower Equation 10:

$$m_{a_{ctin}} (h_{a_{ctout}} - h_{a_{ctin}}) = m_{cow_{condin}} cp_{cow} (T_{cow_{condout}} - T_{cow_{condin}}) \quad (10)$$

The steady state Equation 11 for the energy transfer in the condenser is given by:

$$m_{cow_{ctin}} T_{cow_{ctin}} cp_{cow} - Q_{rej} = m_{cow_{ctout}} T_{cow_{ctout}} cp_{cow} \quad (11)$$

Where:

$$\varepsilon = \frac{1 - \exp[-NTU(1-C)]}{1 - C \exp[-NTU(1-C)]}$$

#### 4.5. Stratified ChW Tank Model

The stratified tank with an inside diameter of 8 meters and a height of 4.2 m was sized assuming that the ChW tank would cover up to a maximum of 80% of the design load from 9:00 to 14:00 h of building operation. The chilled water in the tank was divided into two control volumes. The chilled water temperature in each control volume is affected by the outside ambient air temperature, the fluid temperature in the evaporator loop and the cooling coil loop. The ChW in each control volume was assumed to be well mixed. Therefore the ChW temperature stratification in the storage tank was analyzed by applying one dimensional heat transfer approach to each control volume. The dynamic equations for the ChW temperature in each control volume of the tank during the charge cycle are shown below Equation 12 and 13:

$$\frac{dT_{cw_{tank_{cv1}}}}{dt} = (m_{cw_{s_{chi}}} cp_{cw} (T_{cw_{s_{chi}}} - T_{cw_{tank_{cv1}}}) + \frac{A_{w_{tank_{cv1}}}}{R_{tank}} (T_{o_{tank}} - T_{cw_{tank_{cv1}}})) / C_{cw_{tank_{cv1}}} \quad (12)$$

$$\frac{dT_{cw_{tank_{cv2}}}}{dt} = (m_{cw_{s_{chi}}} cp_{cw} (T_{cw_{s_{chi}}} - T_{cw_{tank_{cv2}}}) + \frac{A_{w_{tank_{cv2}}}}{R_{tank}} (T_{o_{tank}} - T_{cw_{tank_{cv2}}})) / C_{cw_{tank_{cv2}}} \quad (13)$$

And for the discharge cycle, the dynamic equations for the ChW temperatures are Equation 14:

$$\frac{dT_{cw_{tank_{cv1}}}}{dt} = (m_{cw_{s_{tank}}} cp_{cw} (T_{cw_{rc}} - T_{cw_{tank_{cv1}}}) + \frac{A_{w_{tank_{cv1}}}}{R_{tank}} (T_{o_{tank}} - T_{cw_{tank_{cv1}}})) / C_{cw_{tank_{cv1}}} \quad (14)$$

$$\frac{dT_{cw_{tank_{cv2}}}}{dt} = (m_{cw_{s_{tank}}} cp_{cw} (T_{cw_{tank_{cv1}}} - T_{cw_{tank_{cv2}}}) + \frac{A_{w_{tank_{cv2}}}}{R_{tank}} (T_{o_{tank}} - T_{cw_{tank_{cv2}}})) / C_{cw_{tank_{cv2}}} \quad (15)$$

### 5. GAIN SCHEDULING (GS) CONTROL

To maintain thermal comfort in the zone, a good controller for regulating the operation of Heating, Ventilating and Air Conditioning (HVAC) system has to be designed. The conventional PI controller is one of the most popular controllers in HVAC control. It can provide temperature regulation fairly well in an efficient way. However the balance between the proportional gain and integral gain of a well-tuned PI controller would be difficult to maintain when the ChW cooling system encounters a sudden change in cooling loads. Therefore a Gain Scheduling (GS) controller is proposed and its performance will be compared with the conventional PI controller. The basic concept of the GS control is to adjust the proportional gain and integral gain as a function of error.

Following the method proposed by (Alireza, 2006), the proportional gain for the gain scheduling control was expressed mathematically as follows Equation 16:

$$k_p(t) = k_{p(max)} - (k_{p(max)} - k_{p(min)})e^{-(k_1 e(t))} \quad (16)$$

$k_{p(max)}$  and  $k_{p(min)}$  are the maximum and minimum values of proportional gain, respectively. The  $k_1$  is a constant that is predetermined when designing the

controller. Therefore when the  $e(t)$  is increasing, the exponential term approaches 0 which leads the  $k_p(t)$  to reach  $k_{pmax}$  and vice versa to  $k_{pmin}$ . The integral gain of the GS controller was calculated as follows Equation 17:

$$k_i(t) = k_{i(max)}(1 - \tanh \tanh(k_2 |e(t)|)) \quad (17)$$

The  $k_{i(max)}$  is the maximum integral gain of the controller.

### 5.1. Comparison of Control Performance

The conventional PI control and GS control strategies will be simulated for the system undergoing a step change in cooling loads.

#### Case 1

The ChW cooling system is simulated to undergo a change from low demand for cooling to higher demand for cooling. In the lower cooling load condition, the outdoor air temperature was set at 30°C which is 2°C lower than the higher demand case. The heat gain due to the solar radiation and the occupants were reduced to 60% of their peak values.

#### Case 2

In this case, the ChW cooling system experiences a change in load from higher cooling load to lower cooling load which is opposite to the case 1.

#### 5.1.1. Results and Discussion for Case 1 and 2

The simulated performance results of the conventional PI controller (figure at the left side) and GS control (figures at the right side) are shown in Fig. 2-5 for both cases 1 and 2.

#### Case 1: Load Change from Lower to Higher Level

- Conventional PI control
- Gain scheduling control
- Conventional PI control
- Gain scheduling control

#### Case 2: Load Change from Higher to Lower Level

- Conventional PI control
- Gain scheduling control

- Conventional PI control
- Gain scheduling control

By comparing the responses from the PI control and GS control (Fig. 2 and 3), it can be noted that the GS control responses are much smoother and are lot more stable resulting in a good set point tracking performance. This observation is applied to the zone control loop and chiller control loops. In other words, the controller gains are continuously adjusted to give smooth and stable responses.

Likewise, when the cooling load changes from high to low load, the GS control provides smooth and stable temperature control responses (Fig. 4 and 5) as compared to the corresponding responses from the PI control.

#### 5.1.2. Results and Discussion for Cases 3 and 4

A typical day operation of the system under design and partial load conditions was simulated. The set point of the zone air temperature was set at 23°C for the occupied period and 28°C under unoccupied period; and the set point for the supply air to the zone was set at 13°C and 18°C for the occupied and unoccupied periods, respectively. The results of the responses of the zone air temperatures are shown in Fig. 6 and 7. These figures are grouped into two cases identified as Case 3 (system operation under design load) and Case 4 (system operation under partial load conditions).

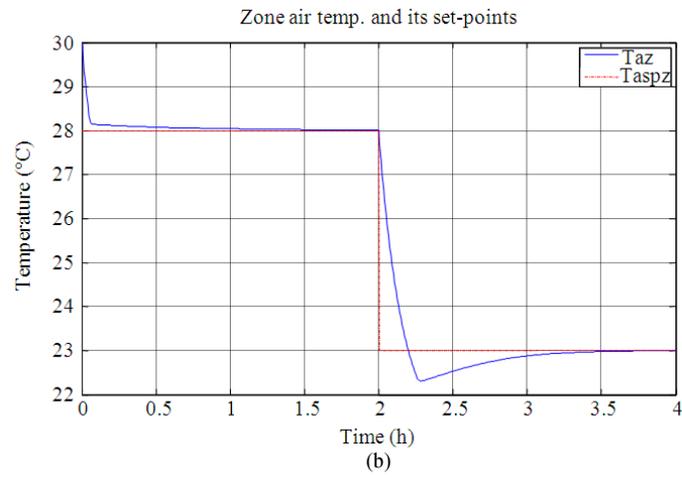
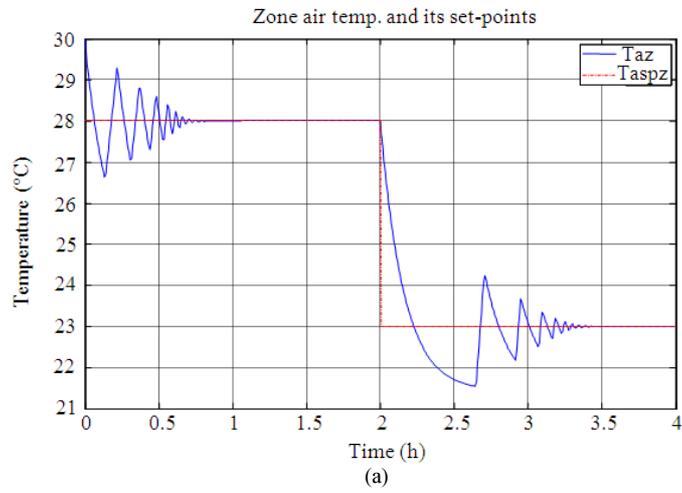
#### Case 3: Typical Day Operation (Design day Conditions)

- Conventional PI control
- Gain scheduling control

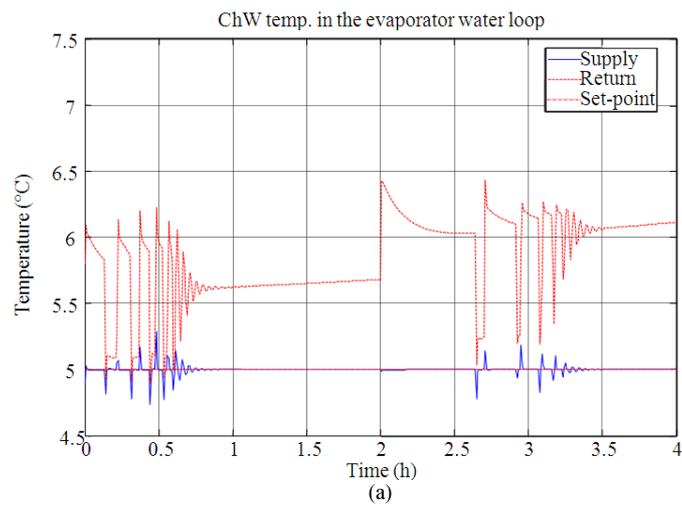
#### Case 4: Typical Day Operation (Partial Load Conditions-PLR 50%)

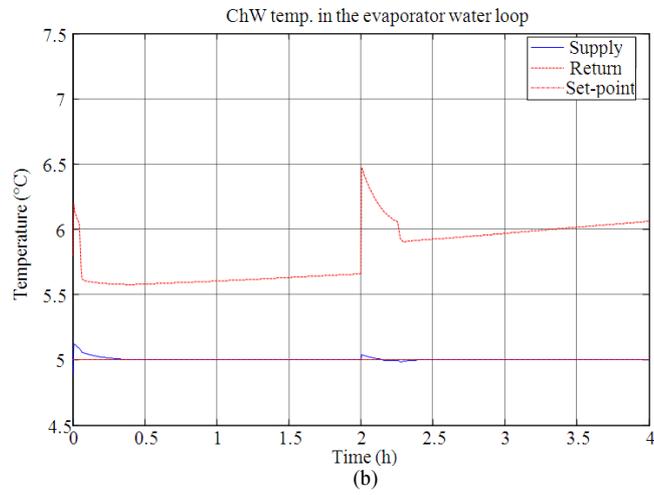
- Conventional PI control
- Gain scheduling control

It is worth noting that the evolution of the zone air temperature for the system with PI control shows more oscillations under partial load conditions compared with the design load operating conditions. On the other hand, the responses of the zone air temperatures of the GS controlled system are stable due to its continuous tuning ability compared with the PI control.

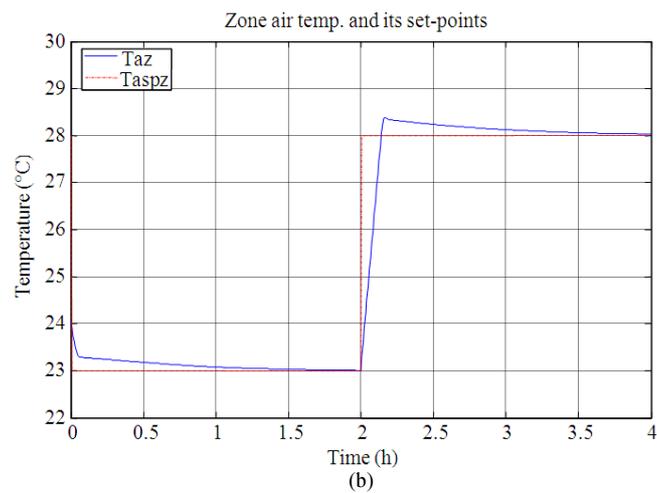
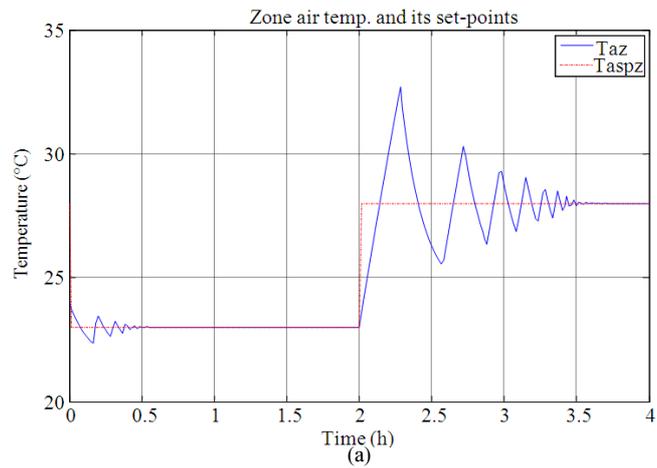


**Fig. 2.** Responses of the zone air temperature with PI and GS control (Case 1)





**Fig. 3.** Responses of ChW temperatures in the evaporator water loop Case-1)



**Fig. 4.** Responses of the zone air temperature with PI and GS control (Case 2)

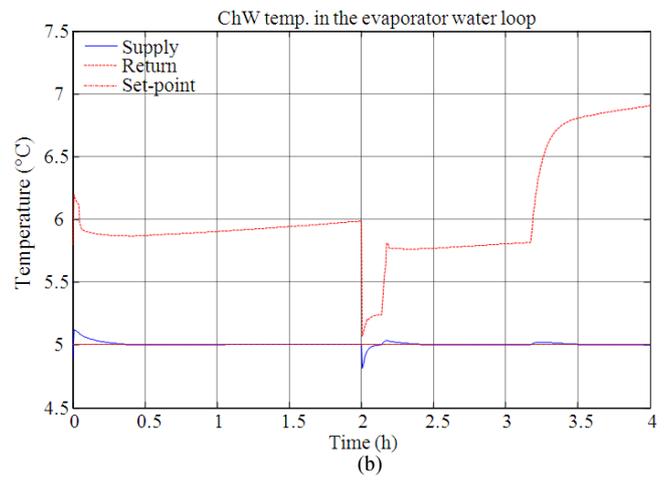
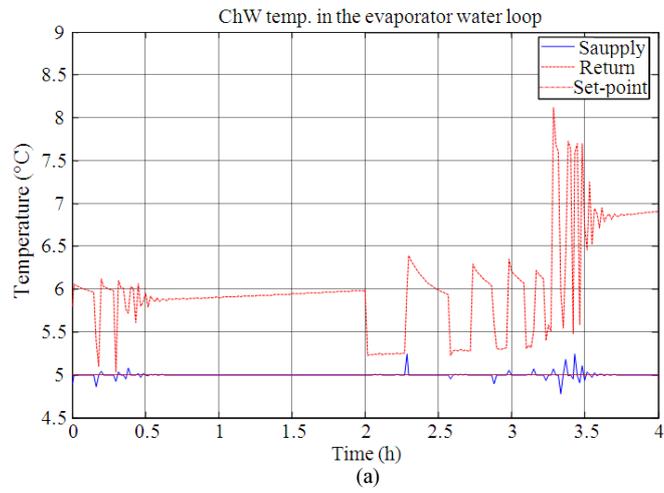
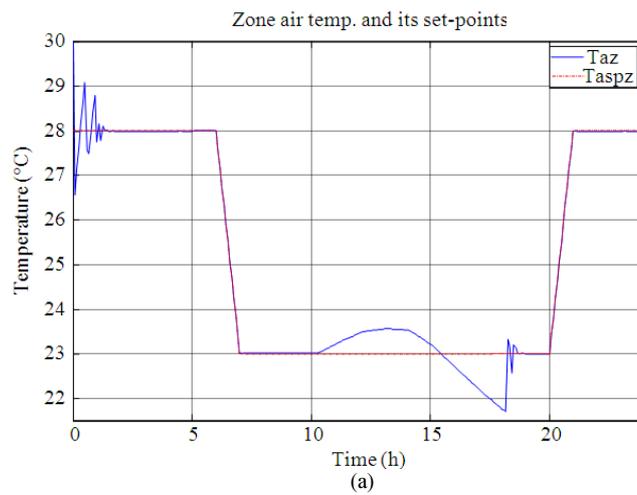


Fig. 5. Responses of ChW temperatures in evaporator water loop (Case 2)



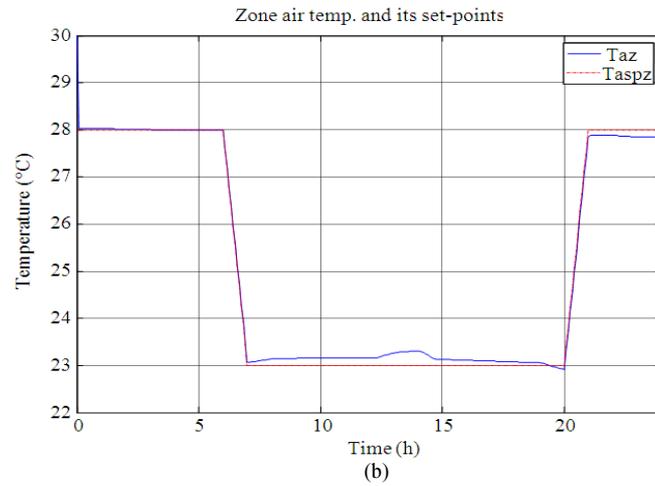


Fig. 6. Typical daily responses of the zone air temperature with PI and GS control (Case 3)

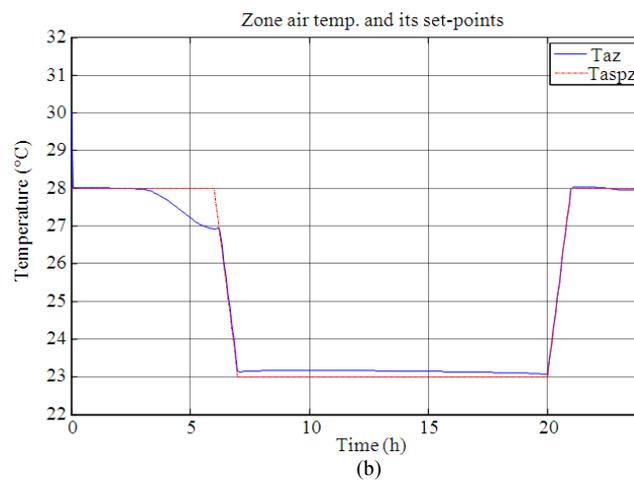
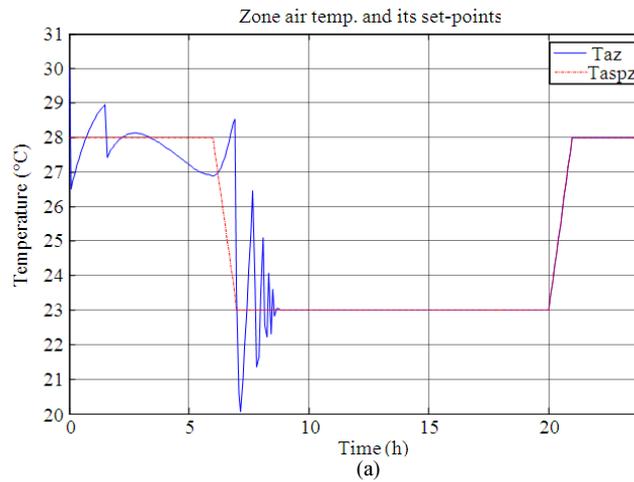


Fig. 7. Typical daily responses of the zone air temperature with PI and GS control (Case 4)

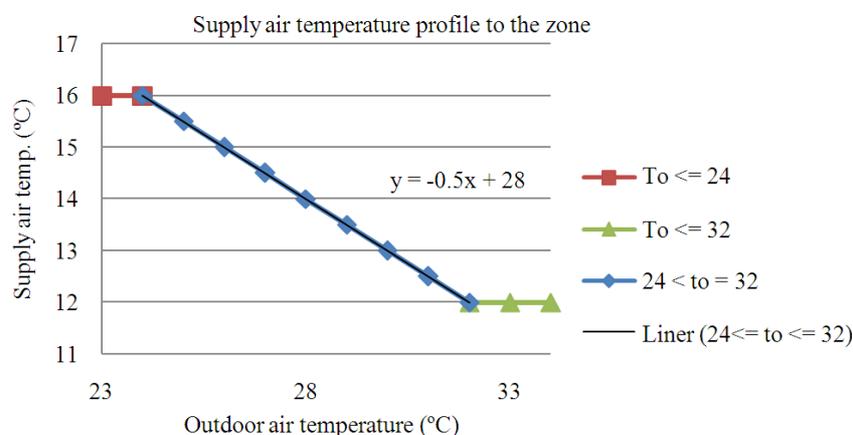


Fig. 8. Supply air temperature profile

## 6. SIMULATION OF OPERATION STRATEGIES

In this section, energy consumption and cost of operating the system under different control strategies are compared. The price of energy was chosen arbitrarily at \$ 0.1/kWh during night time (18:00-7:00 h) and at rate of \$ 0.3/kWh during the day time (7:00-8:00 h). The simulations were run until the system reached steady periodic conditions and the next day's results were used to make the comparisons. The GS controllers were used to regulate the control loops. Five different operation strategies were simulated. These are.

### 6.1. Cooling System without ChW Tank (OS-1)

In this case, the cooling system is simulated without the ChW tank. The set points for the controllers in each control loop are selected based on the design day conditions. The day time simulation period runs from 7:00-20:00 and the night time period spans the remaining hours of the day.

### 6.2. Cooling System with ChW Tank (OS-2)

In this operating strategy the ChW tank is used to store cool energy and use it during the peak cooling period. The ChW tank is charged by the chiller to maintain a set point temperature of 5.5°C during the off-peak period. The discharge cycle is initiated during the peak period.

### 6.3. Optimised Cooling System with ChW Tank (OS-3)

In this case, the operation of the cooling system is locally optimized by applying the following logic:

- A supply air temperature profile as a function of outdoor air temperature was assumed as shown in the Fig. 8
- With the assumed value of supply air temperature, the maximum supply ChW temperature to the cooling coil was determined using a steady state optimization method
- The supply ChW temperature to the coil was calculated by using the following Equation 18:

$$T_{cws_c} = (m_{cws_{chi}} T_{cws_{chi}} + m_{cws_{tank}} T_{cws_{tank}}) / (m_{cws_{chi}} + m_{cws_{tank}}) \quad (18)$$

The set point of the ChW temperature leaving the evaporator was calculated as follows Equation 19:

$$T_{sp_{cws_{chi}}} = ((m_{cws_{chi}} + m_{cws_{tank}}) T_{sp_{cws_c}} - m_{cws_{tank}} T_{cws_{tank}}) / m_{cws_{chi}} \quad (19)$$

In the energy consumption simulations, the mass flow rate of the ChW from the tank to the coil was set at 80% of the maximum ChW flow rate to the coil in order not to deplete the tank at a faster rate.

### 6.4. Cooling System with ChW Tank at 50% Load (OS-4)

In this operating strategy, the cooling system with chilled water tank is operated at partial load of 50% of full load conditions.

### 6.5. Optimized Cooling System with ChW Tank at 50% Load (OS-5)

The cooling system is operated under the same optimized control logic as OS-2 but under partial load conditions (50% of full load conditions).

### 7. RESULTS AND DISCUSSION

The results for the energy consumption and cost for the system operated under various operating strategies are shown in Fig. 9 and 10. The system operation with OS-2 resulted in a total energy consumption of 973.72 kWh which is 58 kWh higher than the results from OS-1. However the cost (\$206.89) is 12.6% less than the OS-1 strategy. The optimized operating strategy resulted in a total energy consumption of 904.046 kWh and the cost was \$162.5. The percentage savings in energy

consumption and system operation cost compared to the OS-2 are 7.16 and 21.46%, respectively.

The OS-4 operation strategy resulted in energy consumption of 458.326 kWh at a cost of \$ 89.20. The total energy consumption of OS-5 was 419.469 kWh which is 8.48% less compared with the OS-4. However the cost \$ 57.07 is about 36.02% of the cost in OS-4. The savings are higher under partial load conditions. Also the cost saving is mainly due to the load demand shifting from day time to low-cost period in the night time. The Fig. 9 and 10 shows the total energy consumption for each operating strategy and the associated cost.

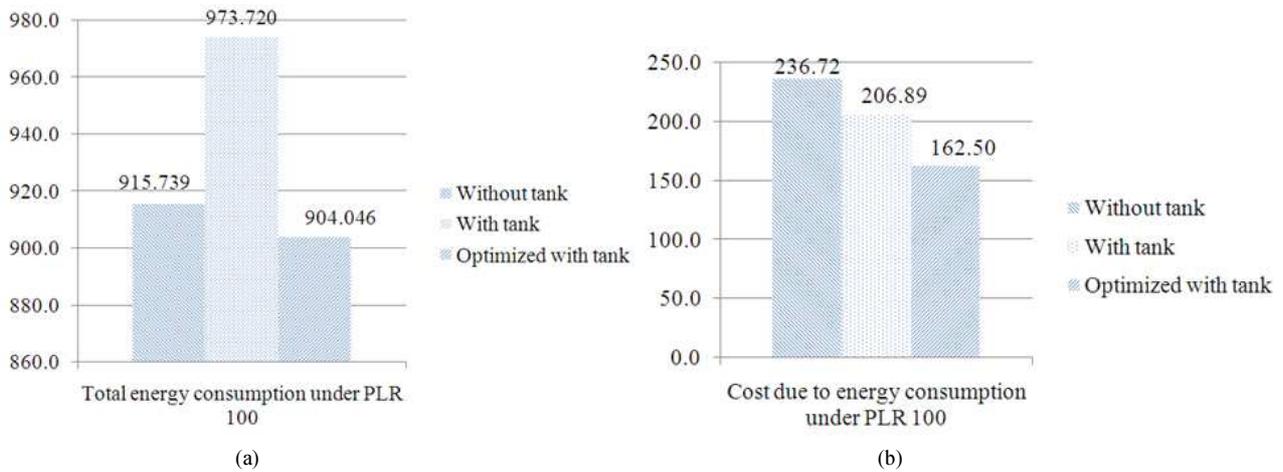


Fig. 9. Total energy consumption and cost for OS-1, OS-2 and OS-3 strategies

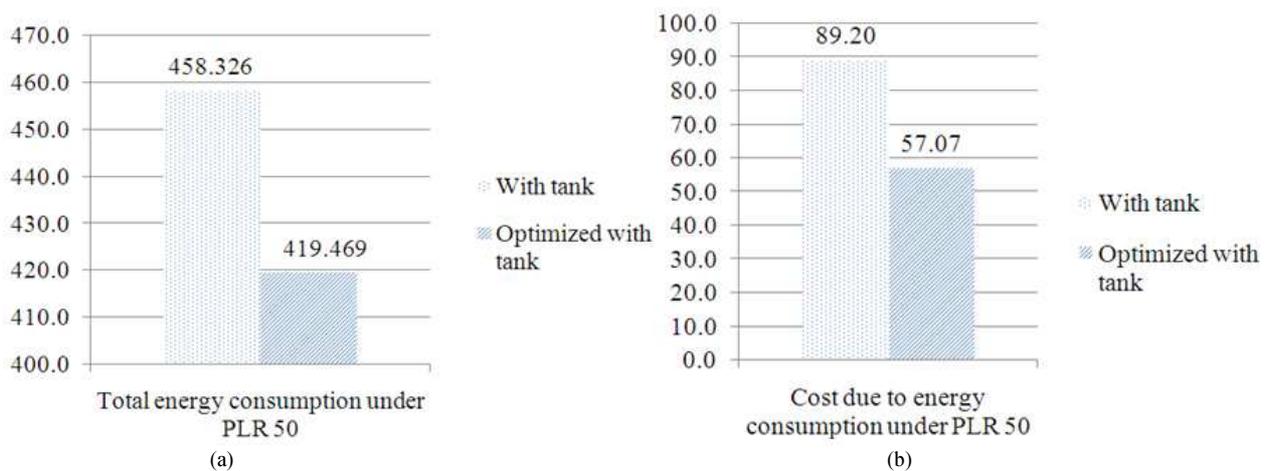


Fig. 10. Total energy consumption and cost for OS-4 and OS-5 strategies

## 8. CONCLUSION

A dynamic model of a chilled water cooling system with a chilled water storage tank for air conditioning of a 4000 m<sup>2</sup> commercial-office building was developed. The chilled water cooling system consists of a chiller, a cooling tower, an air handling unit and a storage tank. Gain scheduling and optimal control strategies were designed. Simulation runs were made to evaluate the performance of the system. The summary and conclusions of this study are given below:

- A dynamic model of a chilled water cooling system with a chilled water storage tank was developed to study dynamic responses of the system subject to variable cooling loads
- A gain scheduling controller was designed. It was shown that by scheduling the gains the control performance of the system can be significantly improved compared with constant gain controller
- The temperature responses with GS controller had less overshoot and the system performance was stable and smooth throughout the operating range
- A near-optimal algorithm for operating the stratified ChW tank with the chiller plant was developed. The results showed that the system operation cost decreased by shifting large proportion of energy consumption into off-peak period when the utility rate structure is in effect. The energy consumption was also reduced due to optimal operation
- The energy consumption was determined under various operating conditions. The results showed that energy savings ranging from 21 to 36% can be achieved by using the near optimal control strategy

## 9. ACKNOWLEDGMENT

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## 10. REFERENCES

Alireza, S., 2006. A PI Controller based on gain-scheduling for synchronous generator. *Turk. J. Elec. Eng.*, 14: 241-251.

- Beghi, A., L. Cecchinato, M. Rampazzo and F. Simmini, 2013. Modeling and control of HVAC systems with Ice cold thermal energy storage. *Proceedings of the 52nd IEEE Conference on Decision and Control*, December 10-13, Firenze, Italy, IEEE Xplore Press, DOI: 10.1109/CDC.2013.6760643
- Braun, J.E., 2007. A near-optimal control strategy for cool storage systems with dynamic electric rates. *HVACR Res.*, 13: 557-580. DOI: 10.1080/10789669.2007.10390972
- Guorong, Z., 1997. Dynamic modeling and global optimal operation of multizone variable air volume hvac system, Thesis (Ph.D.)- Concordia University.
- Jian You, L., 2013. Steady state model of cooling tower used in air conditioning. *Proceedings of the 2nd International Conference on Computer Science and Electronics Engineering*, (SEE'13), Atlantis Press, Paris, France, pp: 760-762. DOI: 10.4028/www.scientific.net/AMR.760-762.1187
- Li, X., Y. Li, J.E. Seem and P. Li, 2013. Dynamic modeling and self-optimizing operation of chilled water systems using extremum seeking control. *Energy Build.*, 58: 172-182. DOI: 10.1016/j.enbuild.2012.12.010
- Li, Z. F. and K. Sumathy, 2002. Performance study of a partitioned thermally stratified storage tank in a solar powered absorption air conditioning system. *Applied Thermal Eng.*, 22: 1207-1216. DOI: 10.1016/S1359-4311(02)00048-0
- Ma, Y. A. Kelman, A. Daly and F. Borrelli, 2012. Predictive control for energy efficient buildings with thermal storage: Modeling, simulation and experiments, *IEEE Control Syst. Magazine*. 32: 44-64. DOI: 10.1109/MCS.2011.2172532
- McQuiston, F.C., D. Jerald, J. Parker and D. Spitler, 2005. *Heating, Ventilating and Air Conditioning: Analysis and Design*. Chapter 14 Extended Surface Heat Exchangers, 6th Edn., John Wiley, ISBN-10: 0471661325, 483-513.
- Swider, D.J., 2002. A comparison of empirically based steady-state models for vapor-compression liquid chillers. *Applied Thermal Eng.*, 23: 539-556. DOI: 10.1016/S1359-4311(02)00242-9