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Confidence Intervals for Signal to Noise Ratio of a Poisson Distribution

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ABSTRACT

The Poisson distribution is one of the most useful probability distributions to fit rare event data. Confidence interval for the SNR is an important issue among the researchers in image processing. This study considers several confidence intervals for the SNR of a Poisson distribution. Different confidence intervals available in literature are reviewed and compared based on the coverage probability and average width of the intervals. Since a theoretical comparison is not possible, a simulation study has been conducted to compare the performance of the interval estimators. Based on the simulation study we observed that most of our proposed interval estimators are performing well in the sense of attaining nominal size and they have been recommended for the researchers. Most of the proposed intervals except methods Wald, Waldz and bootstrap are performing well in the sense of attaining nominal size. The exact method performed the best followed by VSS, Wald B and Bayes in the sense of attaining nominal size and shorter width when the SNR is large.

Keywords: Average Width, Confidence Interval, Coverage Probability, Poisson Distribution, Signal to Noise Ratio, Simulation Study

1. INTRODUCTION

In some situations, the mean describes what is being measured, while the standard deviation represents noise and other interference. In these cases, the standard deviation is not important in itself, but only in comparison to the mean. This gives rise to the term: Signal-to-Noise Ratio (SNR), which is equal to the mean divided by the standard deviation. It is commonly used in image processing (for examples, Tania, 2008; Jitendra, 2009; John, 2007), where the SNR of an image is usually calculated as the ratio of the mean pixel value to the standard deviation of the pixel values over a given neighborhood. SNR measures how much signal has been corrupted by noise (McGibney and Smith, 1993). The population SNR is defined as a ratio of the population mean (μ) to the population standard deviation (σ), i.e., $\mathrm{SNR} = \frac{\mu}{\sigma}$.

In real life examples like image processing, Signal-to-Noise Ratio (SNR) describes the quality of a measurement. In Charge-Coupled Device (CCD) imaging, SNR refers to the relative magnitude of the signal compared to the uncertainty in that signal on a per-pixel basis. Specifically, it is the ratio of the measured signal to the overall measured noise at that pixel. High SNR is particularly important in applications requiring precise light measurement. The detected photons in a CCD camera or a photomultiplier tube follow a Poisson distribution, which is responsible for the Photon Noise and determines the Signal to Noise Ratio of the acquired image (Lee, 2009; Willkinson and Schut, 1998). Since the SNR of Poisson distribution has special interest in imaging, we will be discussing different methods of confidence interval for SNR of Poisson distribution here.

To test the significance of the SNR, a hypothesis test can be conducted and a confidence interval can be generated to reject or accept the null hypothesis. Confidence intervals associated with point estimates

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provide more specific knowledge about the population characteristics than the p-values in the test of hypothesis (Visintainer and Tejani, 1998). The precision of a confidence interval can be determined through the width and coverage probability of the interval. Given constant coverage, as the width of the (1-a) 100% confidence interval decreases, the accuracy of the estimate increase (Kelley, 2007). The coverage level is the probability that the estimated interval will capture the true CV value (Banik and Kibria, 2010).

There are various methods available for estimating the confidence interval for a population CV, such as, parametric, nonparametric, modified and bootstrapping (Banik and Kibria, 2010). The bootstrap approach is a non-parametric and computer-intensive tool used for estimating and making inferences about the parameters that was introduced by Efron (1979). It will be especially useful because, unlike other methods, this technique does not require any assumptions to be made about the underlying population of interest (Banik and Kibria, 2010). Therefore, bootstrapping technique can be applied for estimating or hypothesis testing to all situations. This method is implemented by simulating an original data set then randomly selecting data several times with replacement to estimate the parameter of a distribution. For more information on the confidence interval for the CV, we refer Koopmans *et al*. (1964); Miller (1991); Sharma and Krishna (1994); McKay (1932); Vangel (1996) and very recently Banik and Kibria (2011) among others.

The literature on the confidence intervals for the SNR of a poisson distribution is very limited. The objective of this study is to propose some confidence interval estimators for SNR and find some good estimators for the practitioners. Six confidence intervals that already exist in literature for CV and they are considered for SNR by using the inverse relationship between CV and SNR. Additionally, we have made median and bootstrap modifications to several existing CV intervals in an attempt to improve the interval behavior. Since a theoretical comparison is not possible, a simulation study has been conducted to compare the performance of the interval estimators. Finally, based on the simulation results, the intervals with high coverage probability and smaller width were recommended for practitioners.

2. MATERIALS AND METHODS

Let X_1, X_2, $X_3 \ldots X_n$ be an independently and identically distributed (iid) random sample of size n from a poisson distribution with mean λ. Then the population SNR for Poisson distribution is $\sqrt{\lambda}$ and $\sqrt{\bar{x}}$ would be the estimated value of the population $\mathrm{SNR} = \sqrt{\lambda}$. We want to find the (1-a) 100% confidence intervals for the population SNR. We will review some interval estimators and propose some new interval estimators for SNR. Following 13 methods will be considered.

2.1. Wald Method

Historically, this is one of the first confidence intervals proposed for any parameter (Barker, 2002). The Wald confidence interval results from inverting the Wald test for λ. Using the asymptotic normality of the test statistic $Z = \frac{(\bar{X} - \lambda)}{\sqrt{\bar{X}/n}}$, the (1-a) 100% Confidence Interval (CI) for $\sqrt{\lambda}$ is $\left(\sqrt{\bar{X} - Z_{\alpha/2}\sqrt{\frac{\bar{X}}{n}}}, \sqrt{\bar{X} + Z_{\alpha/2}\sqrt{\frac{\bar{X}}{n}}}\right)$, where $\bar{X}$ denotes the sample mean and Z_c denotes the 1-c quantile of the standard normal distribution.

2.2. WaldB Method

We estimate $\sqrt{\frac{\bar{X}}{n}}$ from bootstrap sample and obtained the following bootstrap version of Wald (named as WaldB) (1-α)100% Confidence Interval (CI) for $\sqrt{\lambda}$ as, $\left(\sqrt{\bar{X} - Z_{\alpha/2}\sqrt{\frac{\bar{X}_B}{n}}}, \sqrt{\bar{X} + Z_{\alpha/2}\sqrt{\frac{\bar{X}_B}{n}}}\right)$.

2.3. WaldZ

Wald confidence interval is based on the assumption that $\bar{X}$ has an approximate normal distribution with mean λ and variance $\frac{\lambda}{n}$. However, for small sample size, this assumption does not work well. Instead of using percentile points from Z-table, we propose to use percentile points from bootstrap samples. The proposed Wald lower and upper confidence intervals for $\sqrt{\lambda}$ are $\left(\sqrt{\bar{x} + T^*_{\alpha/2}\sqrt{\frac{\bar{x}}{n}}}, \sqrt{\bar{x} + T^*_{1-\alpha/2}\sqrt{\frac{\bar{x}}{n}}}\right)$ respectively, where $T^*_{\alpha/2}$ and $T^*_{1-\alpha/2}$ are the $(\alpha/2)^{th}$ and $(1-\alpha/2)^{th}$ sample quintiles of $T^*_i = \frac{\bar{x}^*_i - \bar{x}_B}{\hat{\sigma}_B}$ where $\bar{x}^*_i$ is the ith bootstrap sample mean, $\hat{\sigma}_B = \sqrt{\frac{1}{B-1}\sum_{i=1}^{B}(\bar{x}^*_i - \bar{x}_B)^2}$ and $\bar{x}_B = \frac{1}{B}\sum_{i=1}^{B}\bar{x}^*_i$.

2.4. Wald with Continuity Correction (WCC)

Since Wald interval uses a continuous distribution (normal) to approximate a discrete distribution (Poisson), a continuity correction might make this approximation more accurate (Barker, 2002). The Wald interval with continuity correction is given by:

$$\left(\sqrt{\bar{X} - Z_{\alpha/2}\sqrt{\frac{\bar{X}+0.5}{n}}}, \sqrt{\bar{X} + Z_{\alpha/2}\sqrt{\frac{\bar{X}+0.5}{n}}}\right)$$

2.5. WccB Method

We estimate $\sqrt{\frac{\bar{X}}{n}}$ from bootstrap sample and obtained the following bootstrap version of Wcc method (named as WccB) (1−α) 100% Confidence Interval (CI) for $\sqrt{\lambda}$ as:

$$\left(\sqrt{\bar{X} - Z_{\alpha/2}\sqrt{\frac{\bar{X}_B+0.5}{n}}}, \sqrt{\bar{X} + Z_{\alpha/2}\sqrt{\frac{\bar{X}_B+0.5}{n}}}\right)$$

2.6. WccZ

Instead of using percentile points from Z-table, we propose to use percentile points from bootstrap samples. The proposed Wald with continuity correction the lower and upper confidence intervals for $\sqrt{\lambda}$ are, respectively:

$$\left(\sqrt{\bar{x} + T^*_{\alpha/2}\sqrt{\frac{\bar{x}+0.5}{n}}}, \sqrt{\bar{x} + T^*_{1-\alpha/2}\sqrt{\frac{\bar{x}+0.5}{n}}}\right)$$

2.7. Scores (S)

The Score confidence interval (Wilson, 1927) is on the basis of asymptotic normality of the test statistic $Z = \frac{(\bar{X}-\lambda)}{\sqrt{\lambda/n}}$. The end points of the 1-α score confidence interval are the roots of the quadratic equation $Z_{\alpha/2}{}^2 = [\frac{(\bar{X}-\lambda)}{\sqrt{\lambda/n}}]^2$ and are given by:

$$\left(\sqrt{\bar{X}+\frac{Z^2_{\alpha/2}}{2n} - Z_{\alpha/2}\sqrt{\frac{4\bar{X}+\frac{Z^2_{\alpha/2}}{n}}{4n}}}, \sqrt{\bar{X}+\frac{Z^2_{\alpha/2}}{2n} + Z_{\alpha/2}\sqrt{\frac{4\bar{X}+\frac{Z^2_{\alpha/2}}{n}}{4n}}}\right)$$

2.8. Freeman and Turkey (FT)

Freeman and Turkey (1950) showed that $\sqrt{n}([\sqrt{\bar{X}}+\sqrt{\bar{X}+1}]-[\sqrt{\lambda}+\sqrt{\lambda+1}])$ has an asymptotically standard normal distribution. Then following Barker (2002), the lower and upper confidence limits of $\sqrt{\lambda}$ are obtained as $\left[\frac{(\sqrt{\bar{X}}+\sqrt{\bar{X}+1}-Z_{\alpha/2}\frac{1}{\sqrt{n}})^2-1}{2(\sqrt{\bar{X}}+\sqrt{\bar{X}+1}-Z_{\alpha/2}\frac{1}{\sqrt{n}})}\right]$ and $\left[\frac{(\sqrt{\bar{X}}+\sqrt{\bar{X}+1}+Z_{\alpha/2}\frac{1}{\sqrt{n}})^2-1}{2(\sqrt{\bar{X}}+\sqrt{\bar{X}+1}+Z_{\alpha/2}\frac{1}{\sqrt{n}})}\right]$ respectively. We need the restriction, $\sqrt{\bar{X}}+\sqrt{\bar{X}+1}-Z_{\alpha/2}\frac{1}{\sqrt{n}})^2 > 1$ to be valid the above interval.

2.9. Variance Stabilizing (VS)

The variance stabilizing transformation (Bartlett, 1947) for a Poisson distribution is the square root transformation. The stabilized variance of the transformed variable is approximately 0.25. Hence, the quantity $\frac{\sqrt{\bar{X}}-\sqrt{\lambda}}{\sqrt{1/4n}}$ is asymptotically standard normal. This can be inverted into the interval:

$$\left(\sqrt{\bar{X}+\frac{Z^2_{\alpha/2}}{4n} - Z_{\alpha/2}\sqrt{\frac{\bar{X}}{n}}}, \sqrt{\bar{X}+\frac{Z^2_{\alpha/2}}{4n} + Z_{\alpha/2}\sqrt{\frac{\bar{X}}{n}}}\right)$$

2.10. Recentered Variance Stabilizing (RVS)

For any positive constant c, $\frac{\sqrt{\bar{X}+c}-\sqrt{\lambda+c}}{\sqrt{1/4n}}$ is asymptotically standard normal? Anscombe (1948) showed that, as λ tends to ∞, $\mathrm{var}\sqrt{(\bar{X}+c)} = \frac{1+\frac{3-8c}{8\lambda}}{4n} + o(1/\lambda)$. If c is 3/8, the variance of $\sqrt{\bar{X}+0.5}$ is $\frac{1}{4n}+O(1/\theta)$. By inverting the expression $\frac{\sqrt{\bar{X}+c}-\sqrt{\lambda+c}}{\sqrt{1/4n}}$ we get the following confidence interval for $\sqrt{\lambda}$:

$$\sqrt{\bar{X}+\frac{Z^2_{\alpha/2}}{4n} - Z_{\alpha/2}\sqrt{\frac{\bar{X}+\frac{3}{8}}{n}}}, \sqrt{\bar{X}+\frac{Z^2_{\alpha/2}}{4n} + Z_{\alpha/2}\sqrt{\frac{\bar{X}+\frac{3}{8}}{n}}}$$

2.11. Exact Method

This method is based on an exact relationship between Poisson and Chi-square distribution and the end points of the exact interval are obtained by equating the tail sums of null Poisson probabilities to $\alpha/2$ (Garwood 1936; Agresti and Coull, 1998). The exact method is designed to guarantee at least 100 (1-α)% coverage. The lower and upper limits of exact intervals are:

$$\sqrt{\frac{\chi^2_{\alpha/2,df_1}}{2n}},\sqrt{\frac{\chi^2_{1-\alpha/2,df_2}}{2n}}$$

Respectively, where $df_1 = 2*\sum X$ and where, $df_2 = 2(\sum X + 1)$.

2.12. Bootstrap Approach

Bootstrap is a commonly used computer-based non-parametric tool (introduced by Efron (1979; 1987), which requires no assumptions regarding the underlying population and can be applied to a variety of situations. An extensive array of different bootstrap methods is summarized as follows.

Let $X^{(*)} = X^{(*)}_{i1}, X^{(*)}_{i2}, \ldots, X^{(*)}_{in}$, where the ith sample is denoted by $X^*_{(i)}$ for I = 1, 2, B and B is the number of bootstrap samples. The number of bootstrap samples is typically between 1000 and 2000; because, the accuracy of the confidence interval depends on the size of the samples (Efron and Tibshirani, 1993). The bootstrap estimate of μ is $\bar{x}_B$ of the n estimates $\bar{x}^*_i$. Now, order the sample means of each bootstrap samples as follows: $\bar{x}^*_{(1)} \le \bar{x}^*_{(2)} \le \bar{x}^*_{(3)} \le \ldots \bar{x}^*_{(B)}$. Then CI for $\sqrt{\lambda}$ is obtained as:

$$L = \sqrt{\bar{x}^*_{[(\alpha/2)xB]}} \quad \text{and} \quad U = \sqrt{\bar{x}^*_{[(1-(\alpha/2))xB]}}$$

where, $(\alpha/2)\times B$ and $(1-\alpha/2)\times B$ quantiles of the bootstrap samples. This is known as non-parametric bootstrap confidence interval. In addition to these methods we want to propose the following two intervals.

2.13. Bayes Approach

The non-informative Jefferys prior plays a special role in the Bayesian analysis. The Jefferys prior for Poisson parameter λ is proportional to $\lambda^{-1/2}$ which is improper and the posterior is $\text{Gamma}(\text{shape} = \sum X + \frac{1}{2}, \text{rate} = n)$ (Cai, 2005). Hence the lower limit and upper limit of confidence interval for $\sqrt{\lambda}$ are:

$$\sqrt{G_{\alpha/2,\text{shape}=\sum X+0.5,\text{rate}=n}} \quad \text{and} \quad \sqrt{G_{1-\alpha/2,\text{shape}=\sum X+0.5,\text{rate}=n}}$$

Respectively, where G represents the cdf of a gamma distribution. All of the above interval estimators are summarized in **Table 1**.

2.14. Simulation Study

In this study we considered 13 different confidence intervals for estimating the population SNR of a Poisson distribution. Since a theoretical comparison is not possible, a Monte-Carlo simulation is conducted using the R version 2.14.0 software to compare the performance of the interval estimators. The performance of the estimators was considered for various SNR values, confidence level and sample sizes.

Table 1. Lower and upper critical values of poisson SNR based on proposed methods

Method	Lower critical value	Upper critical value
Wald	$\sqrt{\bar{X} - Z_{\alpha/2}\sqrt{\frac{\bar{X}}{n}}}$	$\sqrt{\bar{X} + Z_{\alpha/2}\sqrt{\frac{\bar{X}}{n}}}$
WaldB	$\sqrt{\bar{X} - Z_{\alpha/2}\sqrt{\frac{\bar{X}_B}{n}}}$	$\sqrt{\bar{X} + Z_{\alpha/2}\sqrt{\frac{\bar{X}_B}{n}}}$
Waldz	$\sqrt{\bar{x} + T^*_{\alpha/2}\sqrt{\frac{\bar{x}}{n}}}$	$\sqrt{\bar{x} + T^*_{1-\alpha/2}\sqrt{\frac{\bar{x}}{n}}}$
Wcc	$\sqrt{\bar{X} - Z_{\alpha/2}\sqrt{\frac{\bar{X}+0.5}{n}}}$	$\sqrt{\bar{X} + Z_{\alpha/2}\sqrt{\frac{\bar{X}+0.5}{n}}}$
WccB	$\sqrt{\bar{X} - Z_{\alpha/2}\sqrt{\frac{\bar{X}_B+0.5}{n}}}$	$\sqrt{\bar{X} + Z_{\alpha/2}\sqrt{\frac{\bar{X}_B+0.5}{n}}}$
Wccz	$\sqrt{\bar{x} + T^*_{\alpha/2}\sqrt{\frac{\bar{x}+0.5}{n}}}$	$\sqrt{\bar{x} + T^*_{1-\alpha/2}\sqrt{\frac{\bar{x}+0.5}{n}}}$
S	$\sqrt{\bar{X} + \frac{Z^2_{\alpha/2}}{2n} - Z_{\alpha/2}\sqrt{\frac{4\bar{X}+\frac{Z^2_{\alpha/2}}{n}}{4n}}}$	$\sqrt{\bar{X} + \frac{Z^2_{\alpha/2}}{2n} + Z_{\alpha/2}\sqrt{\frac{4\bar{X}+\frac{Z^2_{\alpha/2}}{n}}{4n}}}$
FT	$\left[\frac{(\sqrt{\bar{X}}+\sqrt{\bar{X}+1}-Z_{\alpha/2}\frac{1}{\sqrt{n}})^2-1}{2(\sqrt{\bar{X}}+\sqrt{\bar{X}+1}-Z_{\alpha/2}\frac{1}{\sqrt{n}})}\right]$	$\left[\frac{(\sqrt{\bar{X}}+\sqrt{\bar{X}+1}+Z_{\alpha/2}\frac{1}{\sqrt{n}})^2-1}{2(\sqrt{\bar{X}}+\sqrt{\bar{X}+1}+Z_{\alpha/2}\frac{1}{\sqrt{n}})}\right]$
VSS	$\sqrt{\bar{X} + \frac{Z^2_{\alpha/2}}{4n} - Z_{\alpha/2}\sqrt{\frac{\bar{X}}{n}}}$	$\sqrt{\bar{X} + \frac{Z^2_{\alpha/2}}{4n} + Z_{\alpha/2}\sqrt{\frac{\bar{X}}{n}}}$
RVS	$\sqrt{\bar{X} + \frac{Z^2_{\alpha/2}}{4n} - Z_{\alpha/2}\sqrt{\frac{\bar{X}+\frac{3}{8}}{n}}}$	$\sqrt{\bar{X} + \frac{Z^2_{\alpha/2}}{4n} + Z_{\alpha/2}\sqrt{\frac{\bar{X}+\frac{3}{8}}{n}}}$
Exact	$\sqrt{\frac{\chi^2_{\alpha/2,df_1}}{2n}}$	$\sqrt{\frac{\chi^2_{1-\alpha/2,df_2}}{2n}}$
Boot	$\sqrt{\bar{x}^*_{[(\alpha/2)xB]}}$	$\sqrt{\bar{x}^*_{[(1-(\alpha/2))xB]}}$
Bayes	$\sqrt{G_{\alpha/2,\text{shape}=\sum X+0.5,\text{rate}=n}}$	$\sqrt{G_{1-\alpha/2,\text{shape}=\sum X+0.5,\text{rate}=n}}$

To see the behavior of small and large sample sizes, we used n = 5, 10, 15, 20, 30, 50, 100. A random sample was generated with specific parameters (λ = 1.0, 2.0, 5.0 and 10) from Poisson distribution Eq. 1:

$$p(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}, x = 0,1,2,\ldots \quad (1)$$

If the data are from a symmetric distribution (or n is large), the coverage probability will be exact or close to 1-α. So the coverage probability would be a useful criterion for evaluating the confidence interval. Another criterion is the width of the confidence interval. A smaller width gives a more reliable confidence interval. When the coverage probabilities for different methods are the same, a smaller width indicates that the method is appropriate for the specific sample. In the bootstrap calculations we used 1000 bootstrap samples (Efron, 1979; 1987). The below and above rates of a confidence interval is the fraction out of 5,000 samples that resulted in an interval that lies entirely above and below the true value of the population mean. The coverage probability is found as the sum of the lower rate and upper rate and then subtracted from total probability 1. The simulated results for λ = 1.0, 2.0, 5.0 and 10.0 are presented in **Table 1-5** for 90% confidence intervals and in **Table 6-9** for 95% confidence intervals.

Table 2. The coverage probabilities and average widths for lambda = 1 and 1−α = 0.90

N		Wald	WaldB	Waldz	Wcc	WccB	Wccz	S	FT	VSS	RVS	Exact	Boot	Bayes
5	Cover	0.861	0.861	0.749	0.962	0.961	0.875	0.903	0.955	0.939	0.939	0.939	0.665	0.903
	Lower	0.012	0.013	0.080	0.005	0.006	0.045	0.065	0.012	0.028	0.028	0.028	0.115	0.065
	Upper	0.127	0.127	0.171	0.033	0.033	0.080	0.033	0.033	0.033	0.033	0.033	0.220	0.033
	Width	0.807	0.807	0.673	0.960	0.960	0.784	0.729	0.881	0.729	0.906	0.819	0.736	0.718
10	Cover	0.903	0.877	0.824	0.955	0.956	0.903	0.920	0.940	0.883	0.940	0.920	0.778	0.883
	Lower	0.032	0.031	0.058	0.017	0.016	0.045	0.053	0.032	0.053	0.032	0.053	0.072	0.053
	Upper	0.065	0.092	0.117	0.028	0.028	0.052	0.028	0.028	0.065	0.028	0.028	0.150	0.065
	Width	0.544	0.544	0.491	0.689	0.689	0.619	0.520	0.620	0.520	0.633	0.567	0.491	0.516
15	Cover	0.896	0.894	0.855	0.950	0.950	0.926	0.906	0.941	0.896	0.941	0.930	0.822	0.906
	Lower	0.029	0.028	0.045	0.010	0.010	0.027	0.053	0.019	0.029	0.019	0.029	0.059	0.053
	Upper	0.075	0.077	0.099	0.041	0.041	0.048	0.041	0.041	0.075	0.041	0.041	0.118	0.041
	Width	0.436	0.436	0.408	0.550	0.551	0.512	0.425	0.503	0.425	0.509	0.457	0.408	0.423
20	Cover	0.903	0.900	0.870	0.951	0.951	0.940	0.911	0.944	0.903	0.944	0.931	0.845	0.911
	Lower	0.034	0.032	0.047	0.014	0.014	0.023	0.054	0.021	0.034	0.021	0.034	0.059	0.054
	Upper	0.063	0.068	0.083	0.035	0.035	0.037	0.035	0.035	0.063	0.035	0.035	0.096	0.035
	Width	0.375	0.375	0.358	0.468	0.468	0.447	0.368	0.434	0.368	0.438	0.392	0.358	0.367
30	Cover	0.885	0.885	0.871	0.947	0.947	0.936	0.914	0.939	0.896	0.939	0.914	0.851	0.896
	Lower	0.031	0.031	0.047	0.014	0.014	0.023	0.046	0.021	0.046	0.021	0.046	0.053	0.046
	Upper	0.084	0.084	0.082	0.039	0.039	0.041	0.039	0.039	0.058	0.039	0.039	0.096	0.058
	Width	0.304	0.304	0.293	0.377	0.377	0.363	0.300	0.354	0.300	0.356	0.316	0.293	0.300
50	Cover	0.904	0.900	0.884	0.945	0.950	0.945	0.894	0.939	0.892	0.939	0.906	0.872	0.894
	Lower	0.032	0.036	0.049	0.018	0.018	0.022	0.058	0.024	0.045	0.024	0.045	0.053	0.058
	Upper	0.064	0.064	0.067	0.037	0.033	0.032	0.049	0.037	0.064	0.037	0.049	0.075	0.049
	Width	0.234	0.234	0.229	0.289	0.289	0.282	0.233	0.273	0.233	0.274	0.242	0.229	0.232
100	Cover	0.906	0.906	0.898	0.955	0.959	0.953	0.906	0.952	0.906	0.948	0.914	0.892	0.906
	Lower	0.041	0.037	0.044	0.019	0.018	0.022	0.049	0.022	0.041	0.026	0.041	0.048	0.049
	Upper	0.054	0.057	0.058	0.026	0.023	0.025	0.045	0.026	0.054	0.026	0.045	0.060	0.045
	Width	0.165	0.165	0.163	0.203	0.203	0.200	0.164	0.193	0.164	0.193	0.169	0.163	0.164

Table 3. The coverage probabilities and average widths for lambda = 2 and 1−α = 0.90

N		Wald	WaldB	Waldz	Wcc	WccB	Wccz	S	FT	VSS	RVS	Exact	Boot	Bayes
5	Cover	0.901	0.881	0.774	0.916	0.916	0.834	0.918	0.940	0.880	0.901	0.918	0.722	0.880
	Lower	0.030	0.030	0.085	0.016	0.016	0.061	0.052	0.030	0.052	0.030	0.052	0.107	0.052
	Upper	0.068	0.089	0.141	0.068	0.068	0.105	0.030	0.030	0.068	0.068	0.030	0.171	0.068
	Width	0.769	0.769	0.649	0.877	0.877	0.734	0.735	0.820	0.735	0.818	0.802	0.658	0.730
10	Cover	0.906	0.903	0.840	0.920	0.932	0.882	0.918	0.934	0.906	0.934	0.934	0.812	0.918
	Lower	0.033	0.032	0.061	0.020	0.020	0.048	0.049	0.033	0.033	0.033	0.033	0.071	0.049
	Upper	0.061	0.065	0.099	0.061	0.049	0.071	0.032	0.032	0.061	0.032	0.032	0.116	0.032
	Width	0.530	0.530	0.484	0.599	0.599	0.546	0.520	0.576	0.520	0.571	0.554	0.484	0.518
15	Cover	0.888	0.889	0.863	0.923	0.924	0.899	0.921	0.933	0.902	0.933	0.921	0.841	0.902
	Lower	0.031	0.032	0.053	0.023	0.023	0.041	0.044	0.031	0.044	0.031	0.044	0.062	0.044
	Upper	0.081	0.079	0.084	0.054	0.053	0.060	0.036	0.036	0.054	0.036	0.036	0.096	0.054
	Width	0.430	0.430	0.405	0.484	0.484	0.455	0.425	0.470	0.425	0.465	0.447	0.405	0.424
20	Cover	0.896	0.894	0.868	0.934	0.930	0.906	0.898	0.924	0.896	0.924	0.913	0.850	0.898
	Lower	0.043	0.040	0.054	0.022	0.026	0.042	0.058	0.032	0.043	0.032	0.043	0.062	0.058
	Upper	0.061	0.065	0.077	0.044	0.044	0.052	0.044	0.044	0.061	0.044	0.044	0.088	0.044
	Width	0.371	0.371	0.354	0.417	0.417	0.398	0.368	0.406	0.368	0.402	0.385	0.354	0.367
30	Cover	0.907	0.904	0.885	0.929	0.933	0.923	0.897	0.936	0.913	0.936	0.913	0.872	0.913
	Lower	0.027	0.030	0.044	0.021	0.021	0.027	0.054	0.027	0.037	0.027	0.037	0.051	0.037
	Upper	0.066	0.066	0.070	0.050	0.046	0.050	0.050	0.037	0.050	0.037	0.050	0.077	0.050
	Width	0.302	0.302	0.292	0.339	0.339	0.328	0.300	0.331	0.300	0.328	0.312	0.292	0.300
50	Cover	0.903	0.902	0.894	0.937	0.935	0.926	0.904	0.931	0.903	0.931	0.913	0.884	0.904
	Lower	0.040	0.036	0.043	0.025	0.024	0.029	0.048	0.031	0.040	0.031	0.040	0.050	0.048
	Upper	0.057	0.061	0.062	0.038	0.041	0.044	0.048	0.038	0.057	0.038	0.048	0.067	0.048
	Width	0.233	0.233	0.229	0.261	0.261	0.256	0.233	0.256	0.233	0.254	0.240	0.229	0.232
100	Cover	0.895	0.898	0.893	0.931	0.932	0.928	0.904	0.926	0.896	0.926	0.910	0.887	0.896
	Lower	0.044	0.044	0.050	0.030	0.031	0.032	0.051	0.034	0.051	0.034	0.044	0.053	0.051
	Upper	0.061	0.058	0.057	0.040	0.038	0.040	0.046	0.040	0.053	0.040	0.046	0.060	0.053
	Width	0.165	0.165	0.163	0.184	0.184	0.182	0.164	0.181	0.164	0.179	0.168	0.163	0.164

Table 4. The estimated coverage probabilities and average widths for lambda = 5 and confidence coefficient 0.90

N		Wald	WaldB	Waldz	Wcc	WccB	Wccz	S	FT	VSS	RVS	Exact	Boot	Bayes
5	Cover	0.878	0.889	0.786	0.918	0.914	0.822	0.910	0.905	0.889	0.905	0.910	0.749	0.889
	Lower	0.035	0.035	0.091	0.023	0.026	0.075	0.052	0.035	0.052	0.035	0.052	0.106	0.052
	Upper	0.087	0.076	0.124	0.059	0.059	0.103	0.038	0.059	0.059	0.059	0.038	0.145	0.059
	Width	0.747	0.747	0.633	0.786	0.786	0.666	0.736	0.771	0.736	0.765	0.779	0.633	0.734
10	Cover	0.898	0.894	0.845	0.898	0.904	0.865	0.891	0.903	0.888	0.903	0.903	0.829	0.891
	Lower	0.034	0.038	0.065	0.034	0.034	0.056	0.057	0.044	0.044	0.044	0.044	0.071	0.057
	Upper	0.067	0.068	0.090	0.067	0.062	0.079	0.052	0.052	0.067	0.052	0.052	0.100	0.052
	Width	0.524	0.524	0.484	0.550	0.550	0.508	0.520	0.545	0.520	0.540	0.542	0.484	0.519
15	Cover	0.886	0.895	0.853	0.910	0.910	0.870	0.907	0.918	0.892	0.918	0.918	0.844	0.892
	Lower	0.037	0.036	0.059	0.030	0.030	0.051	0.047	0.037	0.047	0.037	0.037	0.064	0.047
	Upper	0.077	0.069	0.088	0.060	0.060	0.079	0.046	0.046	0.060	0.046	0.046	0.093	0.060
	Width	0.427	0.427	0.401	0.448	0.448	0.421	0.425	0.444	0.425	0.441	0.439	0.401	0.424
20	Cover	0.898	0.893	0.865	0.905	0.910	0.882	0.899	0.909	0.898	0.909	0.909	0.852	0.899
	Lower	0.042	0.040	0.054	0.034	0.034	0.048	0.052	0.042	0.042	0.042	0.042	0.060	0.052
	Upper	0.060	0.067	0.082	0.060	0.056	0.070	0.049	0.049	0.060	0.049	0.049	0.089	0.049
	Width	0.369	0.369	0.352	0.387	0.387	0.369	0.368	0.385	0.368	0.382	0.379	0.352	0.368
30	Cover	0.899	0.901	0.881	0.915	0.917	0.897	0.910	0.918	0.901	0.918	0.910	0.871	0.901
	Lower	0.041	0.042	0.056	0.035	0.036	0.049	0.049	0.041	0.049	0.041	0.049	0.061	0.049
	Upper	0.060	0.057	0.063	0.050	0.047	0.055	0.041	0.041	0.050	0.041	0.041	0.068	0.050
	Width	0.301	0.301	0.292	0.316	0.316	0.306	0.300	0.314	0.300	0.311	0.308	0.292	0.300
50	Cover	0.902	0.901	0.892	0.914	0.917	0.907	0.909	0.915	0.902	0.915	0.909	0.886	0.902
	Lower	0.046	0.047	0.053	0.040	0.040	0.046	0.052	0.046	0.052	0.046	0.052	0.056	0.052
	Upper	0.052	0.052	0.055	0.046	0.043	0.046	0.039	0.039	0.046	0.039	0.039	0.058	0.046
	Width	0.233	0.233	0.229	0.244	0.244	0.240	0.233	0.243	0.233	0.241	0.237	0.229	0.233
100	Cover	0.899	0.899	0.889	0.912	0.912	0.905	0.898	0.912	0.900	0.908	0.900	0.885	0.900
	Lower	0.047	0.048	0.053	0.043	0.042	0.047	0.053	0.043	0.051	0.047	0.051	0.054	0.051
	Upper	0.054	0.053	0.058	0.045	0.046	0.048	0.049	0.045	0.049	0.045	0.049	0.060	0.049
	Width	0.165	0.165	0.163	0.173	0.173	0.171	0.164	0.172	0.164	0.171	0.167	0.163	0.164

Table 5. The estimated coverage probabilities and average widths for lambda = 10 and confidence coefficient 0.90

N		Wald	WaldB	Waldz	Wcc	WccB	Wccz	S	FT	VSS	RVS	Exact	Boot	Bayes
5	Cover	0.898	0.894	0.780	0.898	0.898	0.799	0.890	0.902	0.887	0.902	0.902	0.759	0.890
	Lower	0.037	0.041	0.097	0.037	0.038	0.089	0.060	0.048	0.048	0.048	0.048	0.108	0.060
	Upper	0.065	0.065	0.123	0.065	0.064	0.112	0.050	0.050	0.065	0.050	0.050	0.133	0.050
	Width	0.741	0.741	0.627	0.760	0.760	0.643	0.736	0.754	0.736	0.750	0.766	0.627	0.735
10	Cover	0.902	0.900	0.844	0.912	0.912	0.854	0.906	0.914	0.902	0.914	0.914	0.833	0.906
	Lower	0.042	0.039	0.066	0.032	0.034	0.062	0.049	0.042	0.042	0.042	0.042	0.070	0.049
	Upper	0.056	0.061	0.090	0.056	0.054	0.084	0.045	0.045	0.056	0.045	0.045	0.097	0.045
	Width	0.522	0.522	0.479	0.535	0.535	0.491	0.520	0.533	0.520	0.530	0.536	0.479	0.520
15	Cover	0.892	0.893	0.860	0.907	0.902	0.870	0.903	0.900	0.893	0.900	0.903	0.851	0.893
	Lower	0.041	0.041	0.060	0.034	0.037	0.056	0.048	0.041	0.048	0.041	0.048	0.064	0.048
	Upper	0.067	0.066	0.080	0.058	0.060	0.074	0.049	0.058	0.058	0.058	0.049	0.085	0.058
	Width	0.426	0.426	0.402	0.436	0.436	0.412	0.425	0.435	0.425	0.433	0.435	0.402	0.425
20	Cover	0.900	0.904	0.874	0.914	0.911	0.883	0.909	0.916	0.901	0.916	0.916	0.867	0.901
	Lower	0.041	0.041	0.058	0.035	0.038	0.054	0.049	0.041	0.049	0.041	0.041	0.062	0.049
	Upper	0.058	0.055	0.068	0.051	0.051	0.063	0.042	0.042	0.051	0.042	0.042	0.072	0.051
	Width	0.368	0.368	0.353	0.378	0.378	0.362	0.368	0.377	0.368	0.375	0.376	0.353	0.368
30	Cover	0.895	0.895	0.876	0.900	0.903	0.885	0.894	0.902	0.895	0.902	0.902	0.871	0.894
	Lower	0.051	0.049	0.060	0.046	0.046	0.056	0.059	0.051	0.051	0.051	0.051	0.063	0.059
	Upper	0.054	0.056	0.064	0.054	0.051	0.059	0.047	0.047	0.054	0.047	0.047	0.066	0.047
	Width	0.301	0.301	0.292	0.308	0.308	0.299	0.300	0.307	0.300	0.306	0.306	0.292	0.300
50	Cover	0.900	0.898	0.888	0.910	0.907	0.895	0.896	0.906	0.901	0.906	0.901	0.884	0.901
	Lower	0.045	0.047	0.053	0.041	0.043	0.049	0.055	0.045	0.050	0.045	0.050	0.054	0.050
	Upper	0.054	0.055	0.059	0.049	0.050	0.055	0.049	0.049	0.049	0.049	0.049	0.062	0.049
	Width	0.233	0.233	0.228	0.239	0.239	0.234	0.233	0.238	0.233	0.237	0.236	0.228	0.233
100	Cover	0.897	0.898	0.892	0.904	0.906	0.900	0.900	0.903	0.897	0.903	0.900	0.891	0.897
	Lower	0.050	0.049	0.055	0.046	0.046	0.051	0.052	0.050	0.052	0.050	0.052	0.056	0.052
	Upper	0.053	0.052	0.053	0.050	0.048	0.049	0.047	0.047	0.050	0.047	0.047	0.053	0.050
	Width	0.165	0.165	0.163	0.169	0.169	0.167	0.164	0.168	0.164	0.168	0.166	0.163	0.164

Table 6. The estimated coverage probabilities and average widths for lambda = 1 and confidence coefficient 0.95

N		Wald	WaldB	Waldz	Wcc	WccB	Wccz	S	FT	VSS	RVS	Exact	Boot	Bayes
5	Cover	0.868	0.868	0.761	0.966	0.966	0.880	0.972	0.995	0.955	0.995	0.988	0.681	0.939
	Lower	0.005	0.005	0.070	0.002	0.002	0.041	0.028	0.005	0.012	0.005	0.012	0.101	0.028
	Upper	0.127	0.127	0.168	0.033	0.033	0.079	0.000	0.000	0.033	0.000	0.000	0.218	0.033
	Width	1.003	1.003	0.757	1.166	1.166	0.907	0.869	1.051	0.869	1.059	0.952	0.813	0.852
10	Cover	0.925	0.925	0.881	0.968	0.969	0.945	0.960	0.981	0.955	0.981	0.974	0.845	0.940
	Lower	0.010	0.010	0.035	0.004	0.004	0.022	0.032	0.010	0.017	0.010	0.017	0.048	0.032
	Upper	0.065	0.065	0.085	0.028	0.027	0.034	0.009	0.009	0.028	0.009	0.009	0.107	0.028
	Width	0.664	0.664	0.589	0.845	0.846	0.736	0.620	0.741	0.620	0.758	0.665	0.593	0.614
15	Cover	0.916	0.921	0.907	0.978	0.978	0.961	0.952	0.972	0.941	0.972	0.963	0.886	0.963
	Lower	0.010	0.009	0.024	0.003	0.003	0.013	0.029	0.010	0.019	0.010	0.019	0.031	0.019
	Upper	0.075	0.069	0.069	0.019	0.019	0.026	0.019	0.019	0.041	0.019	0.019	0.082	0.019
	Width	0.527	0.527	0.485	0.671	0.671	0.611	0.506	0.601	0.506	0.610	0.537	0.485	0.503
20	Cover	0.951	0.942	0.924	0.974	0.978	0.970	0.945	0.981	0.944	0.981	0.958	0.909	0.958
	Lower	0.014	0.013	0.022	0.005	0.005	0.010	0.034	0.009	0.021	0.009	0.021	0.028	0.021
	Upper	0.035	0.045	0.054	0.021	0.017	0.019	0.021	0.011	0.035	0.011	0.021	0.062	0.021
	Width	0.451	0.451	0.427	0.566	0.566	0.534	0.438	0.518	0.438	0.523	0.462	0.427	0.436
30	Cover	0.928	0.935	0.924	0.983	0.982	0.970	0.945	0.977	0.939	0.977	0.955	0.912	0.955
	Lower	0.014	0.013	0.022	0.004	0.004	0.010	0.031	0.009	0.021	0.009	0.021	0.026	0.021
	Upper	0.058	0.052	0.054	0.014	0.014	0.020	0.024	0.014	0.039	0.014	0.024	0.061	0.024
	Width	0.364	0.364	0.348	0.454	0.454	0.433	0.358	0.422	0.358	0.425	0.374	0.348	0.357
50	Cover	0.945	0.943	0.933	0.983	0.980	0.976	0.942	0.977	0.939	0.977	0.950	0.926	0.950
	Lower	0.018	0.017	0.023	0.003	0.005	0.009	0.032	0.009	0.024	0.009	0.024	0.026	0.024
	Upper	0.037	0.040	0.044	0.013	0.015	0.015	0.026	0.013	0.037	0.013	0.026	0.048	0.026
	Width	0.280	0.280	0.273	0.346	0.346	0.337	0.277	0.326	0.277	0.327	0.287	0.273	0.277
100	Cover	0.947	0.949	0.947	0.986	0.985	0.981	0.954	0.977	0.952	0.980	0.958	0.943	0.958
	Lower	0.019	0.018	0.021	0.006	0.007	0.009	0.026	0.012	0.022	0.012	0.022	0.023	0.022
	Upper	0.034	0.033	0.032	0.008	0.009	0.009	0.020	0.011	0.026	0.008	0.020	0.034	0.020
	Width	0.197	0.197	0.194	0.242	0.242	0.238	0.196	0.230	0.196	0.231	0.201	0.194	0.196

Table 7. The estimated coverage probabilities and average widths for lambda = 2 and confidence coefficient 0.95

N		Wald	WaldB	Waldz	Wcc	WccB	Wccz	S	FT	VSS	RVS	Exact	Boot	Bayes
5	Cover	0.923	0.923	0.806	0.964	0.964	0.850	0.958	0.954	0.954	0.954	0.973	0.767	0.940
	Lower	0.008	0.008	0.069	0.005	0.006	0.051	0.030	0.016	0.016	0.016	0.016	0.085	0.030
	Upper	0.068	0.068	0.125	0.030	0.030	0.099	0.011	0.030	0.030	0.030	0.011	0.148	0.030
	Width	0.938	0.938	0.752	1.072	1.072	0.845	0.876	0.980	0.876	0.977	0.940	0.760	0.868
10	Cover	0.955	0.950	0.902	0.978	0.976	0.932	0.951	0.972	0.948	0.972	0.965	0.880	0.965
	Lower	0.013	0.011	0.033	0.006	0.005	0.025	0.033	0.013	0.020	0.013	0.020	0.042	0.020
	Upper	0.032	0.039	0.064	0.016	0.019	0.043	0.016	0.016	0.032	0.016	0.016	0.079	0.016
	Width	0.637	0.637	0.575	0.722	0.722	0.650	0.620	0.687	0.620	0.682	0.653	0.575	0.617
15	Cover	0.931	0.938	0.915	0.972	0.970	0.942	0.946	0.962	0.941	0.962	0.955	0.899	0.955
	Lower	0.015	0.015	0.029	0.006	0.007	0.023	0.031	0.015	0.023	0.015	0.023	0.036	0.023
	Upper	0.054	0.047	0.056	0.022	0.023	0.036	0.022	0.022	0.036	0.022	0.022	0.065	0.022
	Width	0.515	0.515	0.481	0.581	0.581	0.542	0.506	0.560	0.506	0.555	0.528	0.481	0.505
20	Cover	0.942	0.942	0.922	0.969	0.970	0.949	0.948	0.965	0.950	0.965	0.958	0.914	0.939
	Lower	0.015	0.015	0.029	0.011	0.009	0.020	0.032	0.015	0.022	0.015	0.022	0.034	0.032
	Upper	0.044	0.043	0.048	0.020	0.021	0.031	0.020	0.020	0.028	0.020	0.020	0.052	0.028
	Width	0.444	0.444	0.421	0.499	0.499	0.473	0.438	0.484	0.438	0.480	0.455	0.421	0.437
30	Cover	0.949	0.948	0.937	0.976	0.974	0.962	0.962	0.969	0.957	0.969	0.966	0.929	0.954
	Lower	0.014	0.014	0.021	0.007	0.008	0.014	0.021	0.014	0.017	0.014	0.017	0.022	0.021
	Upper	0.037	0.038	0.043	0.017	0.018	0.024	0.017	0.017	0.025	0.017	0.017	0.048	0.025
	Width	0.361	0.361	0.348	0.405	0.405	0.390	0.358	0.395	0.358	0.391	0.369	0.348	0.357
50	Cover	0.945	0.947	0.943	0.972	0.971	0.965	0.953	0.969	0.951	0.969	0.958	0.938	0.958
	Lower	0.017	0.017	0.021	0.011	0.010	0.013	0.025	0.014	0.021	0.014	0.021	0.023	0.021
	Upper	0.038	0.036	0.036	0.017	0.019	0.022	0.021	0.017	0.028	0.017	0.021	0.039	0.021
	Width	0.279	0.279	0.272	0.312	0.312	0.305	0.277	0.306	0.277	0.303	0.284	0.272	0.277
100	Cover	0.945	0.948	0.945	0.972	0.972	0.967	0.948	0.970	0.947	0.970	0.951	0.941	0.951
	Lower	0.021	0.021	0.025	0.013	0.012	0.015	0.030	0.015	0.026	0.015	0.026	0.026	0.026
	Upper	0.033	0.030	0.030	0.015	0.016	0.017	0.022	0.015	0.027	0.015	0.022	0.033	0.022
	Width	0.196	0.196	0.193	0.220	0.220	0.216	0.196	0.216	0.196	0.214	0.199	0.193	0.196

Table 8. The estimated coverage probabilities and average widths for lambda = 5 and confidence coefficient 0.95

N		Wald	WaldB	Waldz	Wcc	WccB	Wccz	S	FT	VSS	RVS	Exact	Boot	Bayes
5	Cover	0.947	0.944	0.832	0.953	0.953	0.858	0.944	0.965	0.939	0.965	0.956	0.812	0.956
	Lower	0.014	0.012	0.070	0.009	0.009	0.059	0.035	0.014	0.023	0.014	0.023	0.079	0.023
	Upper	0.038	0.044	0.098	0.038	0.038	0.083	0.021	0.021	0.038	0.021	0.021	0.109	0.021
	Width	0.895	0.895	0.736	0.943	0.943	0.775	0.877	0.919	0.877	0.912	0.918	0.736	0.874
10	Cover	0.946	0.941	0.898	0.951	0.953	0.911	0.942	0.958	0.938	0.958	0.950	0.887	0.950
	Lower	0.019	0.017	0.042	0.014	0.014	0.037	0.034	0.019	0.027	0.019	0.027	0.046	0.027
	Upper	0.035	0.042	0.060	0.035	0.034	0.052	0.023	0.023	0.035	0.023	0.023	0.067	0.023
	Width	0.626	0.626	0.574	0.658	0.658	0.603	0.620	0.649	0.620	0.644	0.641	0.574	0.619
15	Cover	0.951	0.946	0.910	0.951	0.956	0.924	0.942	0.955	0.949	0.955	0.949	0.902	0.949
	Lower	0.014	0.016	0.032	0.014	0.012	0.027	0.030	0.017	0.024	0.017	0.024	0.034	0.024
	Upper	0.036	0.038	0.058	0.036	0.032	0.049	0.028	0.028	0.028	0.028	0.028	0.064	0.028
	Width	0.509	0.509	0.476	0.535	0.535	0.500	0.506	0.530	0.506	0.525	0.520	0.476	0.506
20	Cover	0.942	0.944	0.921	0.955	0.956	0.933	0.948	0.958	0.945	0.958	0.953	0.915	0.953
	Lower	0.018	0.017	0.031	0.013	0.013	0.027	0.027	0.018	0.023	0.018	0.023	0.033	0.023
	Upper	0.040	0.039	0.047	0.032	0.031	0.040	0.025	0.025	0.032	0.025	0.025	0.051	0.025
	Width	0.440	0.440	0.418	0.462	0.462	0.439	0.438	0.459	0.438	0.455	0.449	0.418	0.438
30	Cover	0.953	0.948	0.934	0.963	0.959	0.943	0.958	0.966	0.951	0.954	0.958	0.929	0.951
	Lower	0.018	0.021	0.030	0.014	0.016	0.026	0.026	0.018	0.026	0.023	0.026	0.032	0.026
	Upper	0.029	0.031	0.036	0.023	0.025	0.032	0.016	0.016	0.023	0.023	0.016	0.039	0.023
	Width	0.359	0.359	0.347	0.377	0.377	0.364	0.358	0.374	0.358	0.371	0.365	0.347	0.358
50	Cover	0.952	0.952	0.943	0.963	0.963	0.954	0.947	0.962	0.951	0.962	0.953	0.942	0.951
	Lower	0.022	0.020	0.027	0.015	0.015	0.022	0.031	0.019	0.027	0.019	0.027	0.028	0.027
	Upper	0.026	0.028	0.030	0.022	0.022	0.024	0.022	0.019	0.022	0.019	0.019	0.030	0.022
	Width	0.278	0.278	0.272	0.291	0.291	0.285	0.277	0.290	0.277	0.287	0.282	0.272	0.277
100	Cover	0.946	0.946	0.941	0.958	0.957	0.953	0.946	0.956	0.947	0.956	0.949	0.940	0.949
	Lower	0.025	0.025	0.027	0.022	0.021	0.023	0.031	0.024	0.028	0.024	0.028	0.028	0.028
	Upper	0.029	0.028	0.032	0.021	0.022	0.024	0.023	0.021	0.025	0.021	0.023	0.032	0.023
	Width	0.196	0.196	0.193	0.206	0.206	0.203	0.196	0.205	0.196	0.203	0.198	0.193	0.196

Table 9. The estimated coverage probabilities and average widths for lambda = 10 and confidence coefficient 0.95

N		Wald	WaldB	Waldz	Wcc	WccB	Wccz	S	FT	VSS	RVS	Exact	Boot	Bayes
5	Cover	0.945	0.941	0.834	0.950	0.948	0.849	0.935	0.954	0.936	0.954	0.945	0.819	0.945
	Lower	0.018	0.017	0.072	0.013	0.014	0.064	0.037	0.018	0.027	0.018	0.027	0.079	0.027
	Upper	0.037	0.042	0.094	0.037	0.038	0.087	0.028	0.028	0.037	0.028	0.028	0.102	0.028
	Width	0.885	0.885	0.729	0.908	0.908	0.747	0.877	0.898	0.877	0.893	0.907	0.729	0.875
10	Cover	0.946	0.948	0.897	0.958	0.955	0.908	0.953	0.960	0.952	0.957	0.957	0.889	0.957
	Lower	0.017	0.017	0.043	0.014	0.015	0.039	0.025	0.017	0.020	0.020	0.020	0.047	0.020
	Upper	0.037	0.035	0.060	0.028	0.029	0.053	0.023	0.023	0.028	0.023	0.023	0.064	0.023
	Width	0.623	0.623	0.567	0.639	0.639	0.582	0.620	0.635	0.620	0.632	0.635	0.567	0.619
15	Cover	0.953	0.950	0.916	0.953	0.954	0.924	0.956	0.955	0.951	0.955	0.956	0.911	0.951
	Lower	0.016	0.018	0.036	0.016	0.016	0.031	0.024	0.020	0.024	0.020	0.024	0.038	0.024
	Upper	0.031	0.033	0.049	0.031	0.030	0.044	0.020	0.025	0.025	0.025	0.020	0.051	0.025
	Width	0.508	0.508	0.478	0.520	0.520	0.490	0.506	0.518	0.506	0.516	0.516	0.478	0.506
20	Cover	0.951	0.953	0.930	0.959	0.959	0.938	0.954	0.960	0.952	0.960	0.957	0.926	0.957
	Lower	0.019	0.019	0.030	0.016	0.016	0.027	0.025	0.019	0.022	0.019	0.022	0.032	0.022
	Upper	0.030	0.029	0.039	0.025	0.025	0.035	0.021	0.021	0.025	0.021	0.021	0.042	0.021
	Width	0.439	0.439	0.419	0.450	0.450	0.430	0.438	0.449	0.438	0.446	0.446	0.419	0.438
30	Cover	0.945	0.944	0.930	0.948	0.949	0.936	0.943	0.947	0.943	0.947	0.945	0.929	0.945
	Lower	0.027	0.027	0.034	0.024	0.024	0.031	0.031	0.027	0.029	0.027	0.029	0.035	0.029
	Upper	0.028	0.029	0.036	0.028	0.027	0.033	0.026	0.026	0.028	0.026	0.026	0.036	0.026
	Width	0.358	0.358	0.347	0.367	0.367	0.356	0.358	0.366	0.358	0.365	0.363	0.347	0.358
50	Cover	0.947	0.947	0.937	0.954	0.952	0.943	0.945	0.953	0.948	0.953	0.950	0.937	0.950
	Lower	0.022	0.022	0.029	0.021	0.021	0.026	0.030	0.022	0.025	0.022	0.025	0.028	0.025
	Upper	0.031	0.031	0.034	0.025	0.027	0.031	0.025	0.025	0.027	0.025	0.025	0.035	0.025
	Width	0.277	0.277	0.271	0.284	0.284	0.278	0.277	0.284	0.277	0.282	0.280	0.271	0.277
100	Cover	0.950	0.951	0.943	0.956	0.956	0.948	0.946	0.955	0.949	0.955	0.949	0.943	0.949
	Lower	0.025	0.024	0.028	0.021	0.021	0.025	0.031	0.023	0.029	0.023	0.029	0.028	0.029
	Upper	0.025	0.026	0.029	0.023	0.023	0.027	0.023	0.022	0.023	0.022	0.022	0.029	0.023
	Width	0.196	0.196	0.194	0.201	0.201	0.198	0.196	0.201	0.196	0.200	0.198	0.194	0.196

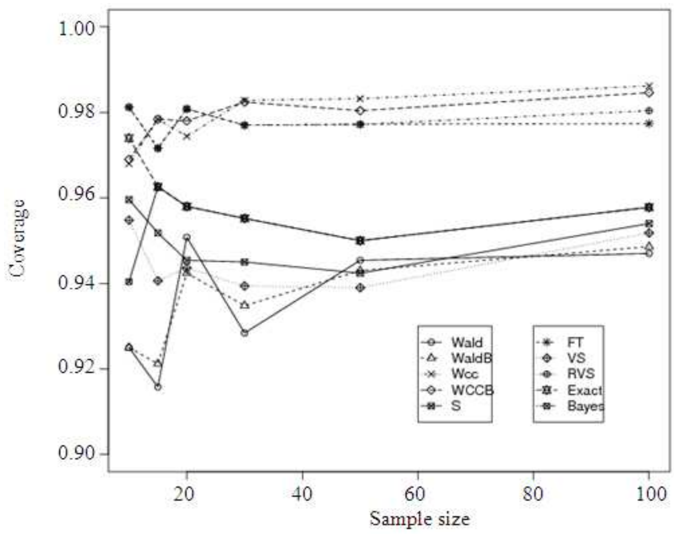

Fig. 1. Coverage vs n, λ = 1, confidence coefficient = 0.95

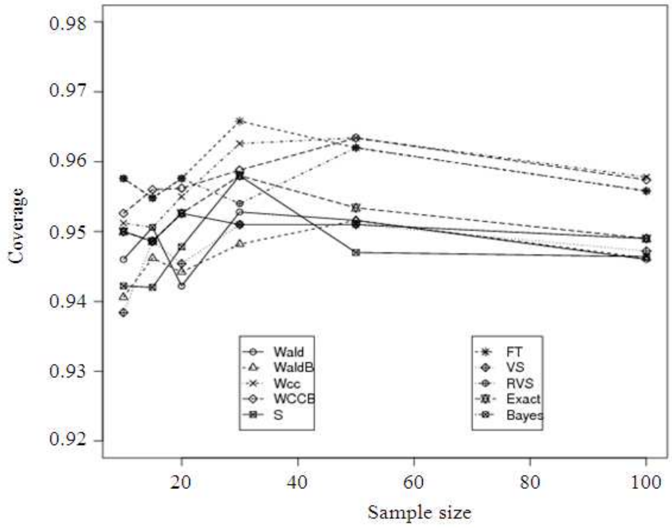

Fig. 2. Coverage vs n, λ = 5, confidence coefficient = 0.95

The coverage probability vs sample size for λ = 1 and λ = 5 are presented in **Fig. 1 and 2** respectively. The average width vs sample size for λ = 1 and λ = 5 are presented in **Fig. 3 and 4** respectively. The coverage probability vs λ and average width vs λ for n = 30 are presented in **Fig. 5 and 6** respectively.

3. RESULTS

From **Table 2-9** and **Fig. 1-6**, we can see a general pattern is that as the sample size increase the nominal sizes for all estimators also increase and converge to 0.90 (**Table 2-5**) and 0.95 (**Table 6-9**). Results also indicated that there are not much fundamental differences among the interval estimators when both sample and SNR are large. However, they performed differently for small sample size and small SNR. For all sample sizes, the exact method performed the best when the SNR is large in the sense of attaining nominal size and smaller width. It also performed the best in the sense of the symmetry of lower and upper error rates. For small samples, the Waldz, WccZ, bootstrap methods are performing poorly. A possible explanation for this is the sampling distribution of the bootstrapped statistics is frequently not symmetric. Thus, the size (α) of the critical region for bootstrap t is not actually divided equally. In fact, the main contribution comes from the upper tail of the test statistic. We also note that the bootstrap version of Wald (named as WaldB) and Wcc (named as WccB) performed better than the corresponding Wald and Wcc methods. However, the performnace of WaldB is better than WccB.

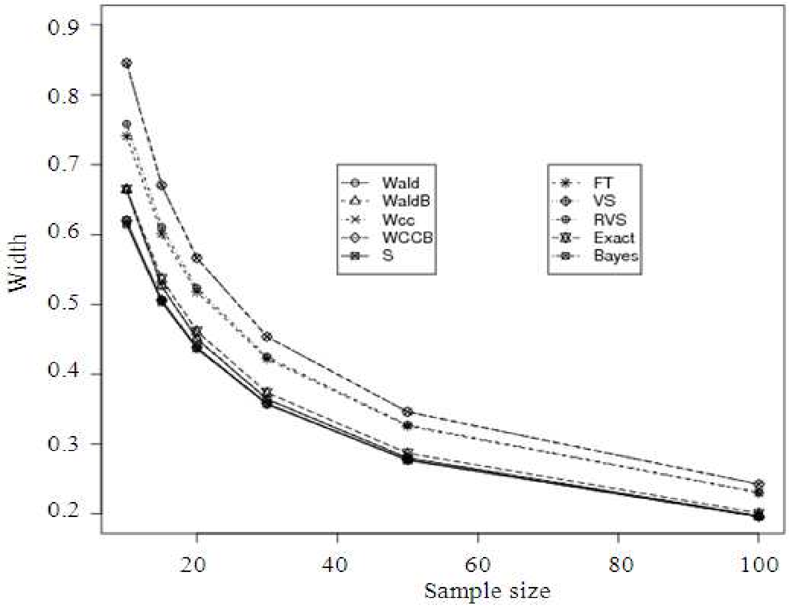

Fig. 3. Width vs n, $\lambda = 1$, confidence coefficient = 0.95

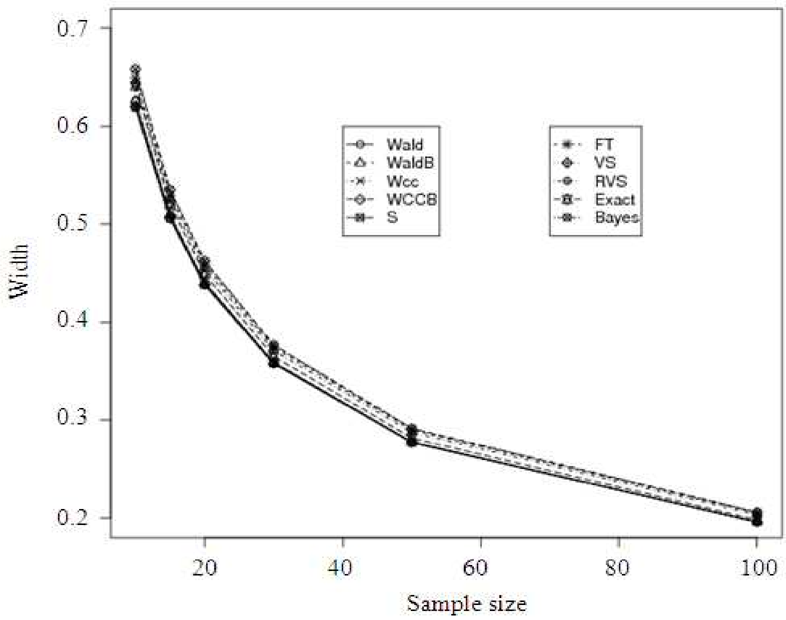

Fig. 4. Width vs n, $\lambda = 5$, confidence coefficient = 0.95

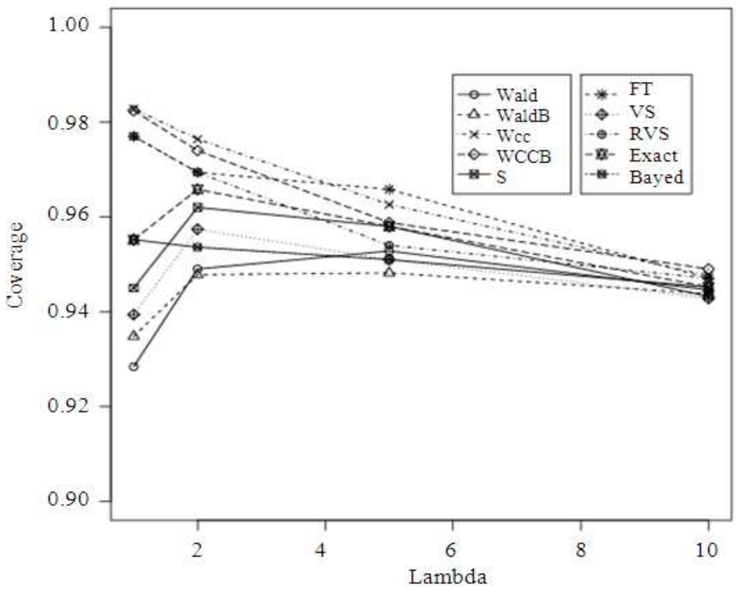

Fig. 5. Coverage vs λ, n = 30, confidence coefficient = 0.95

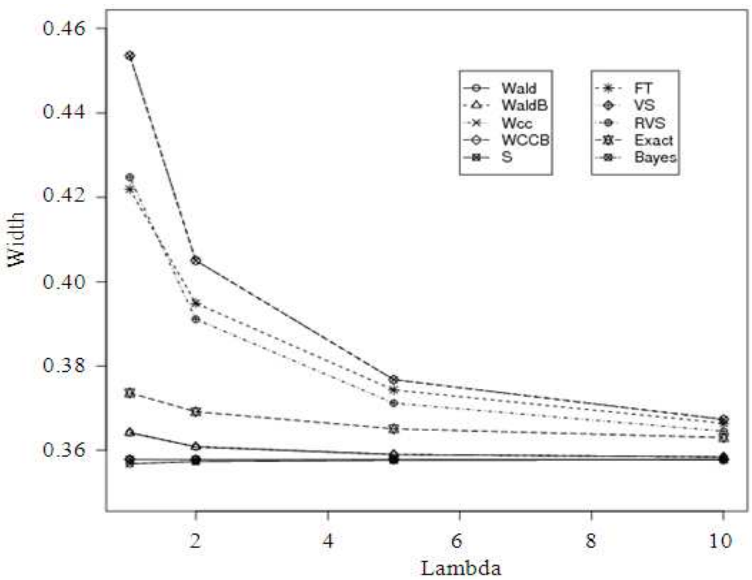

Fig. 6. Width vs λ, n = 30, confidence coefficient = 0.95

4. DISCUSSION

From **Table 2-9** and **Fig. 1-6**, we can see a general pattern is that as the sample size increase the nominal sizes for all estimators also increase and converge to 0.90 (**Table 1-4**) and 0.95 (**Table 6-9**). Results also indicated that there are not much fundamental differences among the interval estimators when both sample and SNR are large. However, they performed differently for small sample size and small SNR. For all sample sizes, the exact method performed the best when the SNR is large in the sense of attaining nominal size and smaller width. It also performed the best in the sense of the symmetry of lower and upper error rates. For small samples, the Waldz, WccZ, bootstrap methods are performing poorly. A possible explanation for this is the sampling distribution of the bootstrapped statistics is frequently not symmetric.

Thus, the size (α) of the critical region for bootstrap t is not actually divided equally. In fact, the main contribution comes from the upper tail of the test statistic. We also note that the bootstrap version of Wald (named as WaldB) and Wcc (named as WccB) performed better than the corresponding Wald and Wcc methods. However, the performance of WaldB is better than WccB.

5. CONCLUSION

This study considered thirteen interval estimators for estimating the population Signal to Noise Ratio (SNR) for a Poisson distribution. Coverage probability, the symmetry of lower and upper error rates and average width are considered as a criterion of a good estimator. We observed that the performance of the estimators depend on the sample size and the values of the SNR. Overall, most of the proposed intervals except methods Wald, Waldz and bootstrap are performing well in the sense of attaining nominal size. The exact method performed the best followed by VSS, Wald B and Bayes in the sense of attaining nominal size and shorter width when the SNR is large. We believe that the findings of the study will be useful for the researcher that is mentioned in the introduction.

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