American Journal of Engineering and Applied Sciences

Original Research Paper

Three Dimensional Investigation of Mode I Stress Intensity Factor Variations in Crack Front Using Finite Element Method

Mirsayar, M.M.

Zachry Department of Civil Engineering, Texas A&M University, College Station, TX, USA

Article history
Received: 30-11-2014
Revised: 10-12-2014
Accepted: 26-01-2015

Abstract: Overall failure may occur in many engineering structures due to the initiation and growth of cracks. Stress intensity factors are extensively used to investigate crack growth in Linear Elastic Fracture Mechanics (LEFM). In many cracked specimens, the stress singularity varies through the thickness and therefore, investigation of stress intensity factor variation in crack front and determination of its maximum value becomes important. In this study, mode I stress intensity factor (KI) variation in different sections of crack front is investigated using finite element method for three typical fracture test specimens. Then, these variations were compared with the result of two-dimensional analytical values. The results show that the stress intensity factor variations are independent with the thickness for all the specimens. Marked discrepancies are shown between the maximum values of stress intensity factors in crack front and those of obtained from two-dimensional analysis.

Keywords: Crack, Finite Element Method, Stress Intensity Factors

Introduction

Many failures of engineering structures are attributed to the consequences of pre-existing cracks or crack-like discontinuities which may occur during manufacturing or service life conditions. Originally, most structural engineering applications of fracture mechanics are limited to in-service assessment of structures. The first step in investigation of the failure in cracked structures is to study the stress field parameters around crack tip. The Linear Elastic Fracture Mechanics (LEFM) concept is a valid assumption for studying failure behavior of many engineering materials like brittle and quasi-brittle materials. For these kind of materials, the stress intensity factors and sometimes higher order terms of the Williams (1952) series expansion are considered as the fracture parameters to characterize elastic stress field around crack tip. The coefficients of the Williams series expansion are calculated so far by many researchers to study the effect of each term on the stress distribution adjacent the crack tip at both homogeneous and bi-material media (Ayatollahi *et al*., 2010; 2011; 2013; Mirsayar, 2013; 2014a; 2014b; Mirsayar *et al*., 2014; Ravichandran and Ramesh, 2005). However, it is well-known that the singular stress field, first and the second term of the stress field, are often more important than higher order terms at the crack tip regions. Therefore, it is very important to calculate the variation of the stress intensity factors around the crack tip.

In many engineering applications, experimental studies on cracked structures are often very expensive and difficult because of the complicated geometry and boundary conditions. Therefore, the numerical analysis using the finite element method is often employed to investigate the fracture parameters of these structures. In many situations straight crack has considerable thickness; therefore, investigation of stress intensity factor variations in crack front is important. The purpose of this research is the three-dimensional investigation of mode I stress intensity factor (KI) variations in different sections of crack front for three typical specimens. Then, these variations are compared with the results of two-dimensional analytical values.

In LEFM, the elastic stresses around the crack tip under mode I loading can be written as an infinite series expansion (Williams, 1952) Equation 1:

$$\begin{aligned}
\sigma_{xx}(r,\theta,z) &= \frac{K_I(z)}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)+H.O.T \\
\sigma_{yy}(r,\theta,z) &= \frac{K_I(z)}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)+H.O.T \\
\sigma_{xy}(r,\theta,z) &= \frac{K_I(z)}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}+H.O.T
\end{aligned} \tag{1}$$

where, r and θ are the conventional crack tip co-ordinates and $\sigma_{xx}(r, \theta, z)$, $\sigma_{yy}(r, \theta, z)$ and $\sigma_{xy}(r, \theta, z)$ are the stresses in the Cartesian co-ordinates system (Fig. 1). $K_I(z)$ is the stress intensity factor corresponding to mode I (opening mode). In the present study, the effect of higher order terms in the series expansion O $(r^{1/2})$ are considered to be negligible near the crack front. $K_I(z)$ in the three-dimensional cracked specimen depends on the z-coordinate and varies along the crack front.

Finite Element Modeling

To investigate the stress intensity factor variations in different sections of crack front three typical specimens were modelled using finite element method; (a) Single Edge-Cracked plate loaded in Tension (SECT) (b) Center-Cracked plate loaded in Tension (CCT) (c) Three Point Bend specimen (3PB). The schematic of each specimen as well as the corresponding boundary conditions are shown in Fig. 2.

These specimens are employed by many of the researchers in the past to study the failure mechanism at the crack tip (Kitsunai *et al*., 1998; Raju and Fichter, 1989). In order to study the fracture behavior in two and three dimensions, these specimens are analyzed using a large number of finite element models and calculated their stress intensity factor for pure mode I loading. For each models, the following geometry and loading conditions were considered: a = 1 mm, w = h = 4 mm, σ = 1 N/mm^2 (for CCT and SECT specimens), P = 1 N/mm (for 3 PB specimen). For all types of the specimens, the effect of thickness was studied by modeling in different normalized thicknesses (B/W, B is the thickness) as 0.1, 0.2, 0.3, 0.4 and 0.5. Due to the stress singularity at the crack tip, a very fine mesh was used around the crack tip. Because of symmetry conditions of these specimens, one-fourth (for SECT and 3pt) and one-eight (for CCT) were modelled. A typical configuration of the geometry, loading condition and finite element mesh for SECT specimen is shown in Fig. 3.

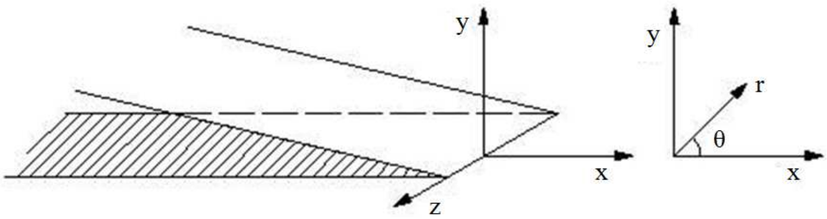
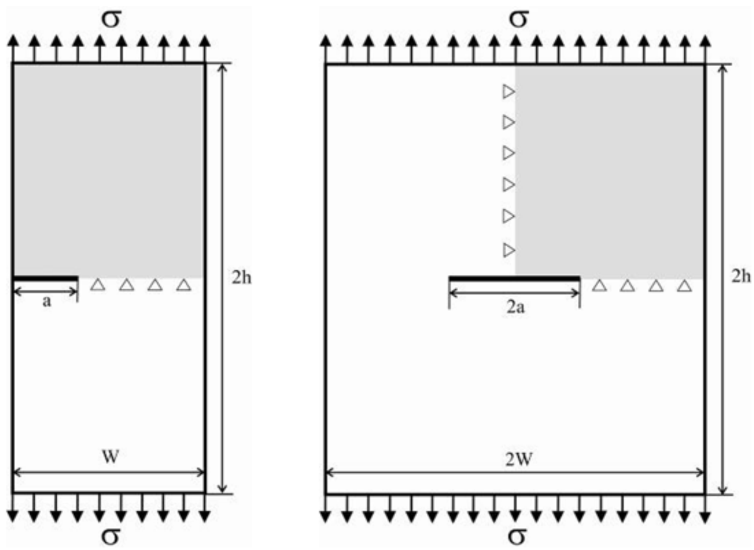
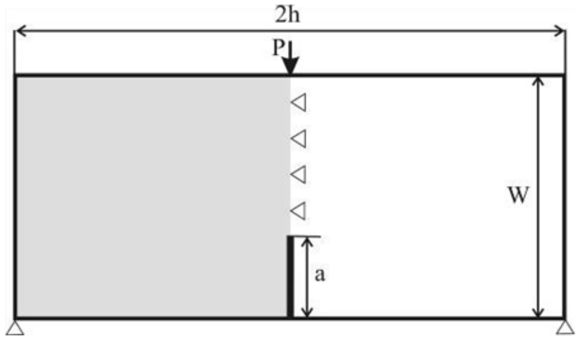


Fig. 1. Used coordinate around the crack tip



(a) (b)



(c)

Fig. 2. Configuration of the three type specimens: (a) Single Edge-Cracked plate loaded in Tension (SECT); (b) Center-Cracked plate loaded in Tension (CCT); (c) Three Point Bend specimen (3PB)

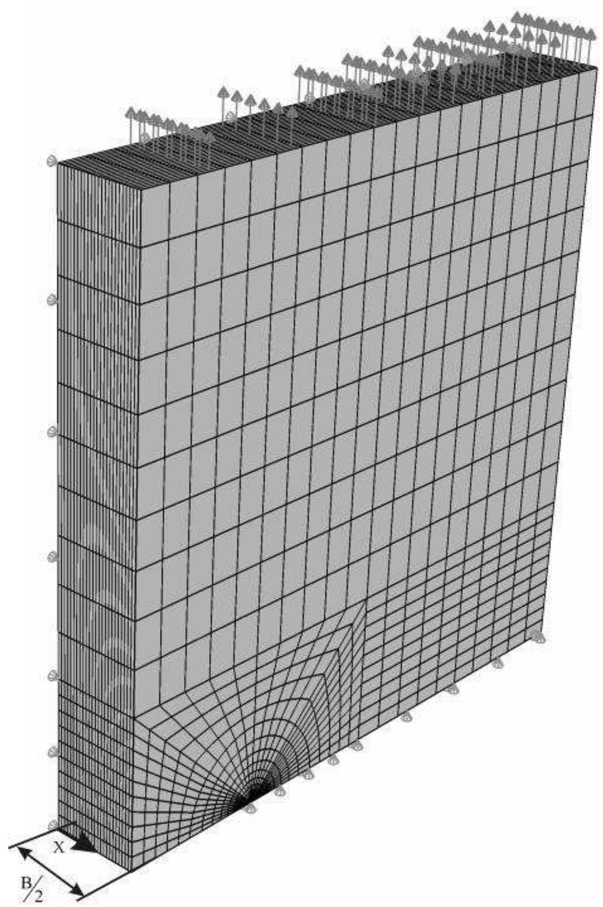


Fig. 3. Finite element model for SECT specimen

Result and Discussion

The mode I stress intensity factors (KI) were obtained using finite element software ABAQUS for different sections of crack front (2x/B = 0, 0.2, 0.4, 0.6, 0.8, 1). Figure 4-6, illustrate mode I stress intensity factor variations in different sections of crack front for three typical specimens in 2D and 3D analysis. It is seen that sequence variations of the stress intensity factors along the crack front for the three specimens are almost the same. While the mode I stress intensity data (KI) drop dramatically for the range 0.7<2X/<1, the FEM data become nearly constant for the range 2X/B<0.7. As shown in Fig. 4-6, it was found that the stress intensity factor variations of crack front for the three typical specimens are independent from the specimen thickness for the range 2X/B<0.7.

The results show clearly that the stress intensity factor values decrease towards the free surface and have the maximum values in its center for all specimens. That means the crack initiates to propagate at the center of the crack front for all of the specimens. The maximum difference between upper and lower KI values are around 10-15%.

The variation of the mode I stress intensity factor with the thickness is also compared with the 2D analytical predictions. For all types of the specimens, the 2D analytical prediction of the stress intensity factors could be represented as Equation 2:

$$K_I(z) = f(g)\sigma\sqrt{\pi a} \quad (2)$$

where, the $f(g)$ is a function of geometry for each specimen, σ is the far field stress perpendicular to the crack and a is the crack length for single edge crack and three point bending specimens and semi-crack length for center-cracked plate. The far field stress, σ, is a function of bending moment for three point bending specimen. For specimens with small values of a/W, the function $f(g)$ equals 1.12. It could be easily shown that using the geometries used in this study one can find $f(g) \approx 1.12$ for all the specimens (Anderson, 2005). It could be seen from Fig. 4-6 that the 2D results estimates the fracture resistance less than the same specimen in 3D configurations for 2X/B<0.7. However, the difference in KI values of center of the specimens in 3 and 2D results are 3-5%. Neglecting the small variations in stress intensity factors obtained from 3D analysis within the range 2X/B<0.7, one can obtain stress intensity factors via 2D analysis and estimate the 3D values using a correction factor. In other words, one can first find the stress intensity factors via a simple 2D analysis and multiply that with 1.03-1.05 correction factor to estimate the 3D simulation results. As shown in Fig. 4-6, the 2D and 3D calculations coincide at 2x/B ≈ 0.8 and higher 2x/B ratio, the 3D simulation predict lower values than 2D simulation. However, the maximum value of stress intensity factor do not significantly change with the thickness for 2X/B<0.7.

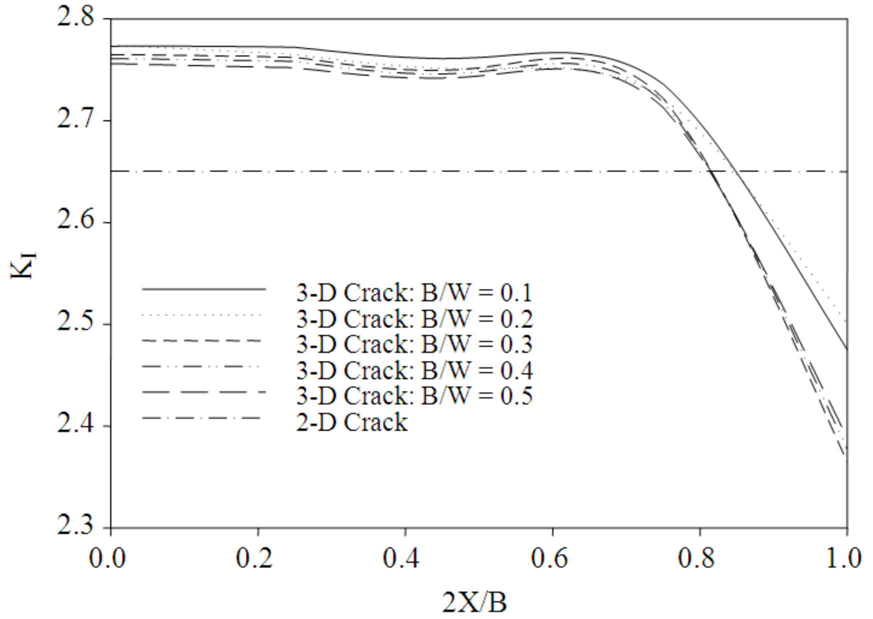


Fig. 4. Variations of the KI along the thickness for SECT specimen

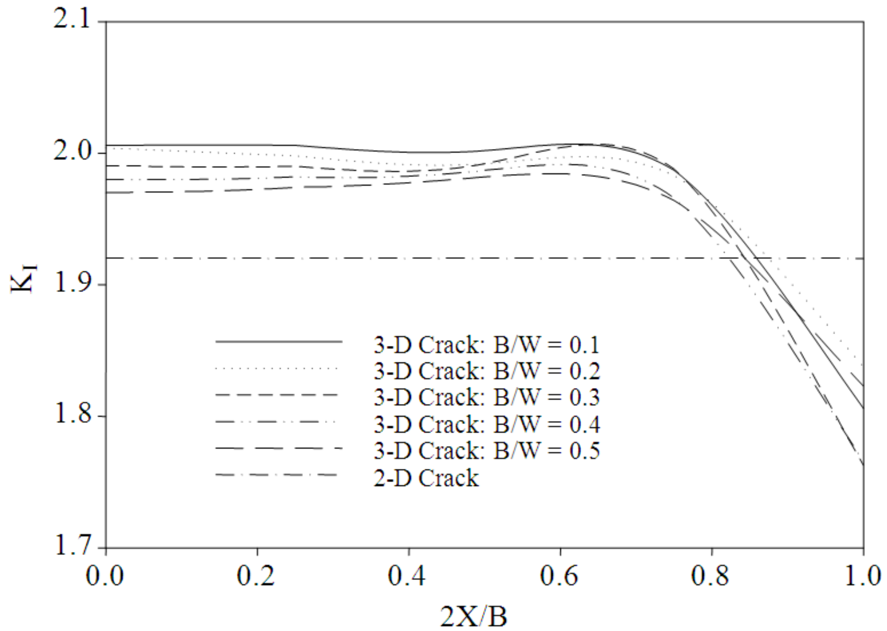


Fig. 5. Variations of the KI along the thickness for CCT specimen

DOI: 10.3844/ajeassp.2015.11.16

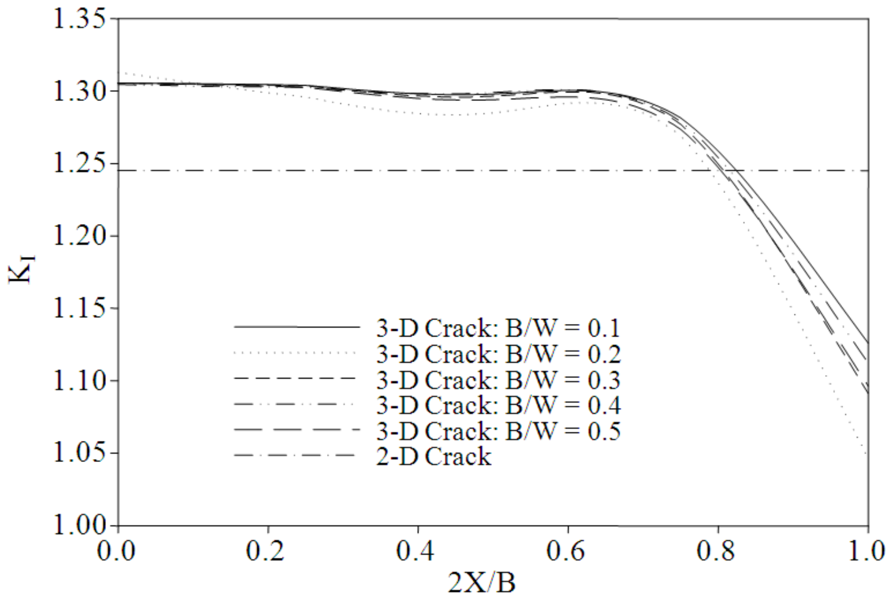


Fig. 6. Variations of the KI along the thickness for 3PB specimen

Conclusion

In this study, mode I stress intensity factor variations in different sections of crack front for three typical specimens were investigated using finite element method. The results showed that the stress intensity factor variations of crack front for the three typical specimens are independent from the specimen thickness within the range 2X/B<0.7. The maximum measured value for stress intensity factor occurs within this range and is independent with the thickness of the specimen. The comparison between 2 and 3D simulation showed that the 3D calculation for stress intensity factors could be estimated by multiplying the value obtained from a simple 2D analysis to a correction factor 1.03-1.05.

Funding Information

The authors have no support or funding to report.

Ethics

This article is original and contains unpublished material. The corresponding author confirms that all of the other authors have read and approved the manuscript and no ethical issues involved.

References

Anderson, T.L., 2005. Fracture Mechanics: Fundamentals and Applications. 3rd Edn., CRC Press, ISBN-10: 0849316561, pp: 640.

Ayatollahi, M.R., M. Dehghany and M.M. Mirsayar, 2013. A comprehensive photoelastic study for mode I sharp V-notches. Eur. J. Mechan., 37: 216-230. DOI: 10.1016/j.euromechsol.2012.07.001

Ayatollahi, M.R., M.M. Mirsayar and M. Nejati, 2010. Evaluation of first non-singular stress term in bi-material notches. Computat. Materials Sci., 50: 752-60. DOI: 10.1016/j.commatsci.2010.10.007

Ayatollahi, M.R., M.M. Mirsayar and M. Dehghany, 2011. Experimental determination of stress field parameters in bi-material notches using photoelasticity. Materials Design, 32: 4901-8. DOI: 10.1016/j.matdes.2011.06.002

Kitsunai, Y., M. Tanaka and E. Yoshihisa,1998. Influence of residual stresses and loading frequencies on corrosion fatigue crack growth behavior of weldments. Metallurgical Materials Trans. A, 29: 1289-1298. DOI: 10.1007/s11661-998-0255-2

Mirsayar, M.M., 2013. Calculation of stress intensity factors for an interfacial notch of a bi-material joint using photoelasticity, Eng. Solid Mechan., 1: 149-153. DOI: 10.5267/j.esm.2013.09.006

Mirsayar, M.M., 2014a. On fracture of kinked interface cracks-the role of *T*-stress. Materials Design, 61: 117-123. DOI: 10.1016/j.matdes.2014.04.074

Mirsayar, M.M., 2014b. A new mixed mode fracture test specimen covering positive and negative values of T-stress. Eng. Solid Mechan., 2: 67-72. DOI: 10.5267/j.esm.2014.2.006

Mirsayar, M.M., M.R.M. Aliha and A.T. Samaei, 2014. On fracture initiation angle near bi-material notches-effects of first non-singular stress term. Eng. Fracture Mechan., 119: 124-131. DOI:10.1016/j.engfracmech.2014.02.017

Ravichandran, M. and K. Ramesh, 2005. Evaluation of stress field parameters for an interface crack in a bimaterial by digital photoelasticity. J. Strain Anal. Eng. Des., 40: 327-43. DOI: 10.1243/030932405X16034

Raju, I.S. and W.B. Fichter, 1989. A finite-element alternating method for two dimensional mode I crack configurations. Eng. Fracture Mechan., 33: 525-540. DOI: 10.1016/0013-7944(89)90037-4

Williams, M.L., 1952. Stress singularities resulting from various boundary conditions. J. Applied Mechan., 19: 526-528.