

New Approach of Electromagnetic Fields of the Lightning Discharge

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ABSTRACT

Despite the significant developments in the protection means of electrical and electronic systems against the lightning and its effects. With its unpredictability and aggress if character, the lightning is the most dangerous phenomenon for electrical systems, which requires more interest and greater effort by researchers and designers means of protection. We present in this study a new analytic model of transient electromagnetic fields radiated by the lightning channel. To better estimate our new model, we developed a program 'SIMLIGHTNING' with Matlab to simulated and approved all mathematic components proposed in this study. The results obtained were compared with other simulations and experiment already published have given a very appreciable similar and affinity.

Keywords: Lightning, Radiation, Electromagnetic Fields, Return Current Stroke, SIMLIGHTNING, Modeling, Simulation

1. INTRODUCTION

Knowledge of the characteristics of electric and magnetic fields produced by lightning discharges is needed for studying the effects of the potentially deleterious coupling of lightning fields to various electric systems. Sensitive electronic circuits are particularly vulnerable to such effects. The computation of lightning electric and magnetic fields requires the use of a model that specifies current as a function of time at all points along the radiating lightning channel. The computed fields can be used as an input to electromagnetic coupling models, the latter, in turn, being used for the calculation of lightning induced voltages and currents in various circuits and systems. For the purpose of simulating and interpreting the effects of a lightning flash to earth, it is helpful to have a simple mathematical expression describing the spatial-temporal distribution of lightning current along the channel and its associated

electromagnetic fields. In this study, we will briefly describe some established approaches to the electromagnetic fields associated with the return stroke phase of a lightning discharge. There is no intention to present an exhaustive literature survey and, in what follows, only those models of interest for engineering applications due to their relative simplicity, with small number of unknowns, will be treated.

The evaluation of electromagnetic effects associated with a lightning return stroke process generally includes automatically the following points:

- Characterization and representation of the return stroke channel base current;
- Specification of the spatial-temporal distribution of the return-stroke current along the channel (using return-stroke models)
- Calculation of radiated electromagnetic fields
- Modeling the coupling of electromagnetic fields to electrical systems. This last part is not covered in

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this thesis and the reader can find exhaustive analyses in the literature

We based on the famous analytical formulation of M. Uman and all Equation (1 to 3) published in 1975 and the current in the lightning Canal model by the engineers of MTLE Nucci and Rachidi (1989) and Cooray (1994), our analytical and numerical development with theoretical tools such as the parity symmetric integration, the Numerical integration of Trapez and method of the difference operator of first order has led to expressions only in the time domain substituting the other variable parameters witch render the initial expression complicated. To better estimate our new model of the electromagnetic field, we developed a program module under Matlab named SIMLIGHTNING for simulation and approving the proposed model

2. ELECTROMAGNETIC FIELD PRODUCED BY THE LIGHTNING DISCHARGE

Due to the current that flows in the lightning channel, an electromagnetic field is radiated from the lightning. The visible part of the spectrum of this field represents only a small part of the whole electromagnetic energy, dissipated by the lightning stroke. The instantaneous energy released from the stroke is so big that, even at the distance of 100 km, the amplitude of the electric field pulse wave amounts to several V/m. **Figure 1** shows a schematic representation of the lightning channel's assumed geometry and indicates also the observation point P where the fields will be calculated. The cylindrical coordinate system is adopted to represent the fields in this geometry.

Several authors have treated the complete problem of the electromagnetic radiation of a dipole over a finitely conducting half-space by determining the solution of Maxwell's equations for both media in accordance with the boundary conditions on the air-ground interface. The resulting equations are obtained in the frequency domain and are in terms of slowly converging integrals (Sommerfeld integrals). The problem is greatly simplified if one assumes a perfectly-conducting ground. In that case, the components of the electric and magnetic fields at the location $P(r, \phi, z)$ produced by a short vertical section of infinitesimal channel dz' at height z' carrying a time-varying current $i(z', t)$ can be computed in the time domain using the following relations (Heidler, 1985; Nucci and Rachidi, 1989; Rachidi *et al.*, 1998).

2.1. Playback on the Analytic Model of Electromagnetic Fields

2.1.1. Horizontal Electric Field

$$E_r(r, z, t) = \frac{1}{4\pi\epsilon_0} \left[\int_{-H}^H \frac{3r(z-z')}{R^5} \int_0^t i(z', \tau - R/c) d\tau dz' + \int_{-H}^H \frac{3r(z-z')}{cR^4} i(z', t - R/c) dz' + \int_{-H}^H \frac{r^2}{c^2R^3} \frac{\partial i(z', t - R/c)}{\partial t} dz' \right] \tag{1}$$

2.1.2. Vertical Electric Field

$$E_z(r, z, t) = \frac{1}{4\pi\epsilon_0} \left[\int_{-H}^H \frac{2(z-z')^2 - r^2}{R^5} \int_0^t i(z', \tau - R/c) d\tau dz' + \int_{-H}^H \frac{2(z-z')^2 - r^2}{cR^4} i(z', t - R/c) dz' - \int_{-H}^H \frac{r(z-z')}{c^2R^3} \frac{\partial i(z', t - R/c)}{\partial t} dz' \right] \tag{2}$$

2.1.3. Azimuthal Magnetic Field

$$B_\phi(r, z, t) = \frac{\mu_0}{4\pi} \left[\int_{-H}^H \frac{r}{R^3} i(z', t - R/c) dz' + \int_{-H}^H \frac{r}{cR^2} \frac{\partial i(z', t - R/c)}{\partial t} dz' \right] \tag{3}$$

and:

$$R = \sqrt{(z-z')^2 + r^2}; H = v(t - \frac{R}{c})$$

Where:

- $i(z', t)$ = The current carried by the dz' dipole at time t ;
- ϵ_0 = The permittivity of the vacuum
- μ_0 = The permeability of the vacuum
- c = The speed of light
- R = The distance from the dipole to the observation point
- r = The horizontal distance between the channel and the observation point

In Equation (1 and 2), the terms containing the integral of the current (charge transferred through dz') are called "electrostatic fields" and because of their $1/r^3$ distance dependence, they are the dominant field component close to the source. The terms containing the derivative of the current are called "radiation fields" and, due to their $1/r$ distance dependence, they are the dominant component far from the source.

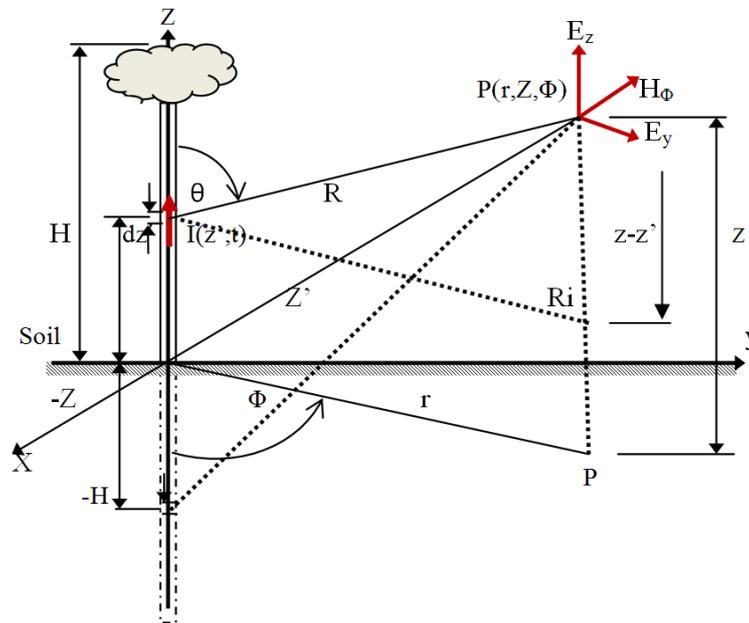


Fig. 1. Geometrical model used in calculating electromagnetic fields

The terms containing the current are called “induction fields”. In Equation (3), the first term is called “induction magnetostatic field” and is the dominant field component near the source and the second term is called “radiation field” and is the dominant field component at far distances from the source.

In these equations the presence of the perfectly conducting ground is taken into account by replacing the ground by an equivalent image as shown in Fig. 1. The total fields produced by the return stroke current at the observation point are obtained by integration of the previous equations along the channel and its image.

For distance ranges beyond several kilometers, the propagation over a ground of finite conductivity results in a noticeable attenuation of high frequency components of the fields (Dib *et al.*, 2007; Cooray, 2001).

At distances from the lightning channel not exceeding about one kilometer, the vertical component of the electric field and the azimuthal magnetic field can be calculated with reasonable approximation assuming the ground as a perfect conductor (Cooray, 1994; Rachidi *et al.*, 1998). However, the horizontal (radial) component of the electric field radiated by lightning is more affected by a finite ground conductivity. Although at some meters above ground its intensity is much smaller than that of the vertical component, within the context of certain field-to-transmission line coupling models (Cooray, 2003; Diendorfer and Uman, 1990;

Guerrieri *et al.*, 1996; Agrawal *et al.*, 1980), the horizontal electric field plays an important role and thus, its calculation requires the use of the rigorous expressions or at least reasonable approximations of those. Of the many approximations proposed in the literature, the Cooray-Rubinstein formula (Cooray, 1994; 2001; Guerrieri *et al.*, 1998) represent an efficient tool and it allows the computation of the horizontal electric field above a finitely-conducting ground with reasonable accuracy.

The calculation of the electromagnetic field requires the knowledge of the spatial-temporal distribution of the current along the channel, $i(z', t)$. This distribution is specified using a return stroke current model. In the following table we can summarize the existing models on the space-temporal distribution of the current return stroke of the lightning.

2.2. Current at the Base of the Channel Lightning

This is the only measurable characteristic, it is an important contribution in the spatio-temporal modeling of arc current back along the lightning channel. Different analytical expressions can be used to simulate the lightning current of these, found in the double exponential function, used by a number of authors who have the advantage of having analytical Fourier transforms, which allows a direct analysis in the frequency domain (Dib *et al.*, 2007).

Fonction 1 Equation (4):

$$i(0, t) = I_{01} \cdot (e^{-\alpha t} - e^{-\beta t}) + I_{02} \cdot (e^{-\gamma t} - e^{-\delta t}) \tag{4}$$

I_{01} , I_{02} , α , β , γ and δ : are the parameters which determine the exponential wave form (Guerrieri *et al.*, 1996).

In more recent publications concerning the simulation of the subsequent return stroke, a more appropriate analytical expression was proposed. The expression consists of the sum of two so-called Heidler (1985) functions.

Fonction 2:

$$i(0, t) = \frac{I_{01}}{\eta_1} \frac{(t / \tau_{11})^n}{1 + (t / \tau_{11})^n} \exp(-t / \tau_{12}) + \frac{I_{02}}{\eta_2} \frac{(t / \tau_{21})^n}{1 + (t / \tau_{21})^n} \exp(-t / \tau_{22}) \tag{5}$$

$$\eta_i = \exp \left[- \left(\frac{\tau_{i1}}{\tau_{i2}} \right) \left(n \frac{\tau_{i2}}{\tau_{i1}} \right)^{\frac{1}{n}} \right]$$

And:

- $i = 1, 2$
- I_{0i} = Amplitude parameter of the channel base current
- τ_{i1} = Front time constant
- τ_{i2} = Decay time constant
- η = Amplitude correction factor
- n = Exponent (2 ... 10)

Parameters of Equation (5) for the typical current waveform at the base of a lightning are given in **Table 1**.

The current waveform at the base of lightning channel calculated with parameters of case 1 is represented in **Fig. 2**. On this Figure are also plotted the double exponential function of Heidler which compose the Equation (5). It should be and **Fig. 3** that the analytical expressions for the current waveforms computed by Verbanov (2004) with the parameters of **Table 1** case 1. We can observe a great similarity between our simulation code SIMLIGHTNING, Verbanov (2004) and that measured experimentally and published by (Heidler and Hopf, 1994; Nucci and Rachidi, 1989; Rachidi *et al.*, 1992).

The function (5) is preferred to the double exponential function (4), because it allows one to easily change return stroke current amplitude, maximum current derivative and the electrical charge transferred to the ground by simply exchanging I_{0i} , τ_{i1} and τ_{i2} respectively. Equation (5) also has a time derivative equal to zero at $t = 0$, which is in agreement with the measured wave shapes of return stroke currents.

2.3. The MTLE, Modified Transmission Line Model

Published by Guerrieri *et al.* (1998), the model MTLE in Equation (6) corrects the defects of the TL model while keeping its simplicity by allowing an easy use in the coupling computation, based on this formulation of the space-temporal distribution along the channel of the current $i(z', t)$, defined by:

$$\begin{aligned} i(z', t) &= i(0, t - z'/v) \exp(-z'/\lambda) & z' \leq vt \\ i(z', t) &= 0 & z' > vt \end{aligned} \tag{6}$$

Where:

- λ = The decay constant which allows the current to reduce its amplitude with height
- v = The speed of propagation of the return stroke wave front

3. DEVELOPMENT OF A NEW MODEL OF ELECTROMAGNETIC FIELDS RADIATED BY LIGHTNING

The development of a new model of electromagnetic fields requires an adequate choice of the transient current propagating in the lightning Canal and therefore also that at the base of the lightning Canal. Through our reading in published research, the formulation that have demonstrated their effectiveness and validated by several authors are those of MTLE by F. Rachidi crossing the channel from the ground to the cloud and Heidler model to the current the base of the channel.

The theorem of parity symmetric integrals:

$$\int_{-H}^H f(z) dz = \int_0^H f(z) dz = \int_0^H f(-z) dz$$

We set Equation (7-9):

$$\begin{aligned} t_0 &= 0, t_{j+1} - t_j = k = \frac{t}{n} \forall j = 1, \dots, n-1, t_n = t \text{ avec } 0 < k \ll 1 \\ R &= \sqrt{r^2 + (z + z')^2} \text{ ou } \tilde{R} = \sqrt{r^2 + (H + Z')^2} \\ R_m &= \sqrt{r^2 + (z + mh)^2} \text{ ou } \tilde{R}_m = \sqrt{r^2 + (H + mh)^2} \end{aligned}$$

$$i\left(z', t - \frac{R}{c}\right) = I_0 \left\{ \begin{aligned} &\exp - \alpha \left(t - \frac{R}{c} - \frac{z'}{v} \right) - \\ &\exp - \beta \left(t - \frac{R}{c} - \frac{z'}{v} \right) \end{aligned} \right\} \exp\left(\frac{-z'}{\lambda}\right) \tag{7}$$

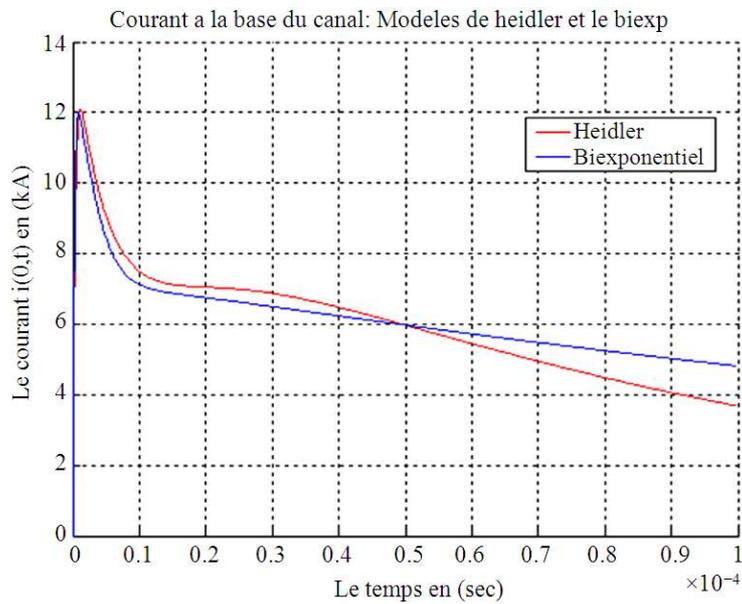


Fig. 2. Current at the base of Lightning channel with Heidler and double exponential models (SIMLIGHTNING 2012)

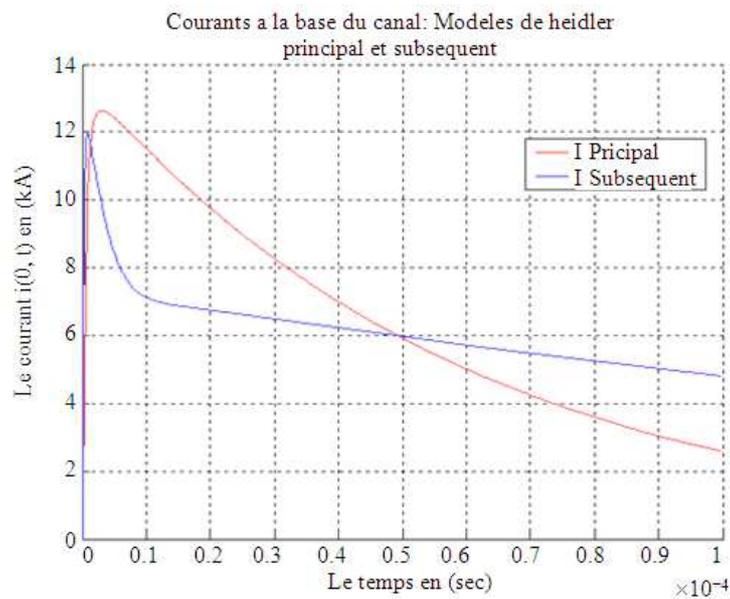


Fig. 3. Principal and subsequent strokes of the Current at the base of Lightning channel with Heidler model (SIMLIGHTNING 2012)

Table 1. Parameters for the typical current waveform at the base of a lightning used in Heidler model

	I1 kA	t11 ms	t12 ms	n1	I2 kA	t21 ms	t22 ms	n2	l km	v m/s
Case1	17.0	0.4	4.0	2	8	4	50	2	1.5	1.0×10^8
Case2	10.5	2.0	4.8	2	9	20	26	2	2.0	1.5×10^8
Case3	19.5	1.0	2.0	2	12	8	30	2	1.5	1.0×10^8

$$\int_0^t i\left(z', t - \frac{R}{c}\right) d\tau = \frac{I_0}{\alpha} (1 - e^{-\alpha t}) \exp\left[\frac{\alpha R}{c} + \left(\frac{\alpha}{v} - \frac{1}{\lambda}\right) z'\right] - \frac{I_0}{\beta} (1 - e^{-\beta t}) \exp\left[\frac{\beta R}{c} + \left(\frac{\beta}{v} - \frac{1}{\lambda}\right) z'\right] \tag{8}$$

$$\frac{\partial i}{\partial t} = I_0 \left\{ \begin{aligned} & -\alpha e^{-\alpha t} \exp\left[\frac{\alpha R}{c} + \left(\frac{\alpha}{v} - \frac{1}{\lambda}\right) z'\right] \\ & + \beta e^{-\beta t} \exp\left[\frac{\beta R}{c} + \left(\frac{\beta}{v} - \frac{1}{\lambda}\right) z'\right] \end{aligned} \right\} \tag{9}$$

Numerical approaches of trapez integration Equation 10 and 11:

$$\int_0^t i\left(z', \tau - \frac{R}{c}\right) d\tau \cong k \left[\frac{1}{2} i\left(z', \tau - \frac{R}{c}\right) + \sum_{j=1}^{j=n-1} i\left(z', jk - \frac{R}{c}\right) + \frac{1}{2} i\left(z', t - \frac{R}{c}\right) \right] \tag{10}$$

$$\int_0^t i\left(-z', \tau - \frac{\tilde{R}}{c}\right) d\tau \cong k \left[\frac{1}{2} i\left(z', -\frac{\tilde{R}}{c}\right) + \sum_{j=1}^{j=n-1} i\left(z', jk - \frac{\tilde{R}}{c}\right) + \frac{1}{2} i\left(-z', t - \frac{\tilde{R}}{c}\right) \right] \tag{11}$$

Numerical Method of the operator of first order differences Equation (12 and 13):

$$\frac{\partial i\left(z', t - \frac{R}{c}\right)}{\partial t} \Big|_{z_m} \cong \frac{i\left(z_m, t - \frac{R}{c}\right) - i\left(z_m, t - k - \frac{R}{c}\right)}{k} \tag{12}$$

3.1. Horizontal Electrical Field

$$E_r(r, z, t) = \frac{1}{4\pi\epsilon_0} \left[\begin{aligned} & \left[\int_0^H \frac{3r(z-z')}{R^5} \int_0^t i\left(z', \tau - \frac{R}{c}\right) d\tau dz' \right. \\ & \left. + \int_0^H \frac{3r(z+z')}{\tilde{R}^5} \int_0^t i\left(-z', \tau - \frac{\tilde{R}}{c}\right) d\tau dz' \right] \\ & + \left[\int_0^H \frac{3r(z+z')}{cR^4} i\left(z', t - \frac{R}{c}\right) dz' \right. \\ & \left. + \int_0^H \frac{3r(z+z')}{c\tilde{R}^4} i\left(-z', t - \frac{\tilde{R}}{c}\right) dz' \right] \\ & + \left[\int_0^H \frac{r^2}{c^2 R^3} \frac{\partial i\left(z', t - \frac{R}{c}\right)}{\partial t} dz' \right. \\ & \left. + \int_0^H \frac{r^2}{c^2 \tilde{R}^3} \frac{\partial i\left(z', t - \frac{\tilde{R}}{c}\right)}{\partial t} dz' \right] \end{aligned} \right]$$

$$E_r(r, z, t) = [S_1 + S_2 + S_3]$$

$$S_1 = \frac{1}{4\pi\epsilon_0} \left[\frac{3rk}{2} [I_1 + I_2 + \tilde{I}_1 + \tilde{I}_2] + 3rk \sum_{j=1}^{j=n-1} (L_j + \tilde{L}_j) \right] \tag{13}$$

And:

$$I_1 = \int_0^H \frac{(z-z')}{R^5} i\left(z', -\frac{R}{c}\right) dz' \cong h \left[\frac{1}{2} \frac{z}{R_0^5} i\left(0, -\frac{R_0}{c}\right) + \sum_{m=1}^{m=N-1} \frac{(z-mh)}{R_m^5} i\left(mh, -\frac{R_m}{c}\right) + \frac{1}{2} \frac{(z-H)}{R_N^5} i\left(H, -\frac{R_N}{c}\right) \right]$$

$$I_2 = \int_0^H \frac{(z-z')}{R^5} i\left(z', t - \frac{R}{c}\right) dz' \cong h \left[\frac{1}{2} \frac{z}{R_0^5} i\left(0, t - \frac{R_0}{c}\right) + \sum_{m=1}^{m=N-1} \frac{(z-mh)}{R_m^5} i\left(mh, t - \frac{R_m}{c}\right) + \frac{1}{2} \frac{(z-H)}{R_N^5} i\left(H, t - \frac{R_N}{c}\right) \right]$$

$$\tilde{I}_1 = \int_0^H \frac{(z+z')}{\tilde{R}^5} i\left(-z', -\frac{\tilde{R}}{c}\right) dz' \cong h \left[\frac{1}{2} \frac{z}{\tilde{R}_0^5} i\left(0, -\frac{\tilde{R}_0}{c}\right) + \sum_{m=1}^{m=N-1} \frac{(z+mh)}{\tilde{R}_m^5} i\left(-mh, -\frac{\tilde{R}_m}{c}\right) + \frac{1}{2} \frac{(z+H)}{\tilde{R}_N^5} i\left(-H, -\frac{\tilde{R}_N}{c}\right) \right]$$

$$\tilde{I}_2 = \int_0^H \frac{(z+z')}{\tilde{R}^5} i\left(-z', t - \frac{\tilde{R}}{c}\right) dz' \cong h \left[\frac{1}{2} \frac{z}{\tilde{R}_0^5} i\left(0, t - \frac{\tilde{R}_0}{c}\right) + \sum_{m=1}^{m=N-1} \frac{(z+mh)}{\tilde{R}_m^5} i\left(-mh, t - \frac{\tilde{R}_m}{c}\right) + \frac{1}{2} \frac{(z+H)}{\tilde{R}_N^5} i\left(-H, t - \frac{\tilde{R}_N}{c}\right) \right]$$

$$L_j = \int_0^H \frac{(z-z')}{R^5} i\left(z', jk - \frac{R}{c}\right) dz' \cong h \left[\frac{1}{2} \frac{z}{R_0^5} i\left(0, jk - \frac{R_0}{c}\right) + \sum_{m=1}^{m=N-1} \frac{(z-mh)}{R_m^5} i\left(-mh, jk - \frac{R_m}{c}\right) + \frac{1}{2} \frac{(z+H)}{R_N^5} i\left(-H, jk - \frac{R_N}{c}\right) \right]$$

$$\tilde{L}_j = \int_0^H \frac{(z+z')}{\tilde{R}^5} i\left(-z', jk - \frac{\tilde{R}}{c}\right) dz' \cong h \left[\frac{1}{2} \frac{z}{\tilde{R}_0^5} i\left(0, jk - \frac{\tilde{R}_0}{c}\right) + \sum_{m=1}^{m=N-1} \frac{(z+mh)}{\tilde{R}_m^5} i\left(-mh, jk - \frac{\tilde{R}_m}{c}\right) + \frac{1}{2} \frac{(z+H)}{\tilde{R}_N^5} i\left(-H, jk - \frac{\tilde{R}_N}{c}\right) \right]$$

$$S_1 = \frac{3rk}{4\pi\epsilon_0} \left[\frac{I_1 + I_2 + \tilde{I}_1 + \tilde{I}_2}{2} + \sum_{j=1}^{j=n-1} (L_j + \tilde{L}_j) \right]$$

Similarly find the expressions of S2 and S3 Equation (14):

$$S_2 \cong \frac{3rh}{4c\pi\epsilon_0} \left[\left[\frac{1}{2R_0^4} i \left(0, t - \frac{R_0}{c} \right) + \sum_{m=1}^{m=N-1} \frac{(z-mh)}{R_m^4} i \left(-mh, t - \frac{R_m}{c} \right) + \frac{1}{2} \frac{(z-H)}{R_N^4} i \left(H, t - \frac{R_N}{c} \right) \right] \right. \\ \left. + \left[\frac{1}{2\tilde{R}_0^4} i \left(0, t - \frac{\tilde{R}_0}{c} \right) + \sum_{m=1}^{m=N-1} \frac{(z+mh)}{\tilde{R}_m^4} i \left(-mh, t - \frac{\tilde{R}_m}{c} \right) + \frac{1}{2} \frac{(z+H)}{R_N^4} i \left(H, t - \frac{\tilde{R}_N}{c} \right) \right] \right]$$

$$S_3 \cong \frac{hr^2}{4c^2\pi\epsilon_0} \left[\left[\frac{1}{2R_0^3} \frac{i \left(0, t - \frac{R}{c} \right) - i \left(0, t - k - \frac{R}{c} \right)}{k} \right. \right. \\ \left. \left. + \sum_{m=1}^{m=N-1} \frac{1}{R_m^3} \frac{i \left(mh, t - \frac{R_m}{c} \right) - i \left(mh, t - k - \frac{R_m}{c} \right)}{k} + \frac{1}{2} \frac{1}{R_N^3} \frac{i \left(H, t - \frac{R_m}{c} \right) - i \left(H, t - k - \frac{R_m}{c} \right)}{k} \right] \right. \\ \left. \left[\frac{1}{2\tilde{R}_0^3} \frac{i \left(0, t - \frac{\tilde{R}}{c} \right) - i \left(0, t - k - \frac{\tilde{R}}{c} \right)}{k} \right. \right. \\ \left. \left. + \sum_{m=1}^{m=N-1} \frac{1}{R_m^3} \frac{i \left(mh, t - \frac{\tilde{R}_m}{c} \right) - i \left(mh, t - k - \frac{\tilde{R}_m}{c} \right)}{k} + \frac{1}{2} \frac{1}{R_N^3} \frac{i \left(H, t - \frac{\tilde{R}_m}{c} \right) - i \left(H, t - k - \frac{\tilde{R}_m}{c} \right)}{k} \right] \right]$$

$Er(r, z, t) \cong S_1 + S_2 + S_3$ (14)

3.2. Vertical Electrical Field

$$E_z(r, z, t) = \frac{1}{4\pi\epsilon_0} \left[\left[\int_0^H \frac{2(z-z')^2 - r^2}{R^5} \int_0^t i \left(z', \tau - \frac{R}{c} \right) d\tau dz' + \int_0^H \frac{2(z-z')^2 - r^2}{R^5} \int_0^t i \left(-z', \tau - \frac{\tilde{R}}{c} \right) d\tau dz' \right] \right. \\ \left. + \left[\int_0^H \frac{2(z-z')^2 - r^2}{c^2 R^4} i \left(z', t - \frac{R}{c} \right) dz' + \int_0^H \frac{2(z+z')^2 - r^2}{c\tilde{R}^4} i \left(-z', t - \frac{\tilde{R}}{c} \right) dz' \right] \right. \\ \left. + \left[\int_0^H \frac{\Gamma(z-z')}{c^2 R^3} \frac{\partial i \left(z', t - \frac{R}{c} \right)}{\partial t} dz' + \int_0^H \frac{\Gamma(z-z')}{c^2 \tilde{R}^3} \frac{\partial i \left(-z', t - \frac{\tilde{R}}{c} \right)}{\partial t} dz' \right] \right]$$

$E_z(r, z, t)$ is the sum of three terms or field components Equation (15):

$$E_z(r, z, t) \cong S'_1 + S'_2 + S'_3$$

$$S'_1 = \frac{1}{4\pi\epsilon_0} \left[\frac{J_1 + J_2 + \tilde{J}_1 + \tilde{j}_2}{2} + \sum_{j=1}^{j=n-1} (M_j + \tilde{M}_j) \right]$$

$$\begin{aligned}
 S_2 &\cong \frac{h}{4c\pi\epsilon_0} \left[\left[\frac{1}{2} \frac{2z^2 - r^2}{R_0^4} i \left(0, t - \frac{R_0}{c} \right) + \sum_{m=1}^{m=N-1} \frac{2(z-mh)^2 - r^2}{R_m^4} i \left(mh, t - \frac{R_m}{c} \right) + \frac{1}{2} \frac{(z-H)^2}{R_N^4} i \left(H, t - \frac{R_N}{c} \right) \right] \right. \\
 &\quad \left. + \left[\frac{1}{2} \frac{2z^2 - r^2}{\tilde{R}_0^4} i \left(0, t - \frac{\tilde{R}_0}{c} \right) + \sum_{m=1}^{m=N-1} \frac{2(z-mh)^2 - r^2}{\tilde{R}_m^4} i \left(mh, t - \frac{\tilde{R}_m}{c} \right) + \frac{1}{2} \frac{2(z-mh)^2 - r^2}{\tilde{R}_N^4} i \left(-H, t - \frac{\tilde{R}_N}{c} \right) \right] \right] \\
 S_3 &\cong \frac{hr}{4c^2\pi\epsilon_0} \left[\left[\frac{1}{2} \frac{z}{R_0^3} \frac{i \left(0, t - \frac{R}{c} \right) i \left(0, t - k - \frac{R}{c} \right)}{k} + \sum_{m=1}^{m=N-1} \frac{z-mh}{R_m^3} \frac{i \left(mh, t - \frac{R}{c} \right) - i \left(mh, t - k - \frac{R}{c} \right)}{k} + \frac{1}{2} \frac{z-H}{R_N^3} \frac{i \left(H, t - \frac{R}{c} \right) - i \left(H, t - k - \frac{R}{c} \right)}{k} \right] \right. \\
 &\quad \left. + \left[\frac{1}{2} \frac{z}{\tilde{R}_0^3} \frac{i \left(0, t - \frac{\tilde{R}}{c} \right) - i \left(0, t - k - \frac{\tilde{R}}{c} \right)}{k} + \sum_{m=1}^{m=N-1} \frac{z+mh}{\tilde{R}_m^3} \frac{i \left(-mh, t - \frac{\tilde{R}_N}{c} \right) - i \left(mh, t - k - \frac{\tilde{R}_N}{c} \right)}{k} \right] \right. \\
 &\quad \left. + \left[\frac{1}{2} \frac{z}{\tilde{R}_0^3} \frac{i \left(-H, t - \frac{\tilde{R}_N}{c} \right) i \left(-H, t - k - \frac{\tilde{R}_N}{c} \right)}{k} \right] \right] \\
 E_z(r, z, t) &\cong S_1' + S_2' + S_3' \tag{15}
 \end{aligned}$$

- S1 and S₁' = The electrostatic component in the Electrical field
- S2 and S₂' = The electric radiation component in the Electrical field
- S3 and S₃' = The magnetic induction component in the Electrical field

3.3. Azimutal Magnetic Field Equation (16)

$$\begin{aligned}
 B\phi(r, z, t) &= \frac{\mu_0}{4\pi} \left[\int_0^H \frac{r}{R^3} i \left(z', \tau - \frac{R}{c} \right) dz' + \int_0^H \frac{r}{\tilde{R}^3} i \left(-z', \tau - \frac{\tilde{R}}{c} \right) dz' \right] + \left[\int_0^H \frac{r}{cR^2} \frac{\partial i \left(z', t - \frac{R}{c} \right)}{\partial t} dz' + \int_0^H \frac{r}{c\tilde{R}^2} \frac{\partial i \left(-z', t - \frac{\tilde{R}}{c} \right)}{\partial t} dz' \right] \\
 S_1'' &\cong \frac{h\mu_0 r}{4\pi} \left[\left[\frac{1}{2} \frac{1}{R_0^3} i \left(0, t - \frac{R_0}{c} \right) + \sum_{m=1}^{m=N-1} \frac{1}{R_m^3} i \left(mh, t - \frac{R_m}{c} \right) + \frac{1}{2} \frac{1}{R_N^3} i \left(H, t - \frac{R_N}{c} \right) \right] \right. \\
 &\quad \left. + \left[\frac{1}{2} \frac{1}{\tilde{R}_0^3} i \left(0, t - \frac{\tilde{R}_0}{c} \right) + \sum_{m=1}^{m=N-1} \frac{1}{\tilde{R}_m^3} i \left(-mh, t - \frac{\tilde{R}_m}{c} \right) + \frac{1}{2} \frac{1}{\tilde{R}_N^3} i \left(H, t - \frac{\tilde{R}_N}{c} \right) \right] \right] \\
 S_1''' &\cong \frac{\mu_0 r}{4c\pi} \left[\left[\frac{1}{2} \frac{z}{R_0^3} \frac{i \left(0, t - \frac{R}{c} \right) - i \left(0, t - k - \frac{R}{c} \right)}{k} + \sum_{m=1}^{m=N-1} \frac{1}{R_m^2} \frac{i \left(mh, t - \frac{R}{c} \right) - i \left(mh, t - k - \frac{R}{c} \right)}{k} + \frac{1}{2} \frac{1}{R_N^2} \frac{i \left(H, t - \frac{R}{c} \right) - i \left(H, t - k - \frac{R}{c} \right)}{k} \right] \right. \\
 &\quad \left. + \left[\frac{1}{2} \frac{z}{\tilde{R}_0^2} \frac{i \left(0, t - \frac{\tilde{R}}{c} \right) - i \left(0, t - k - \frac{\tilde{R}}{c} \right)}{k} \right. \right. \\
 &\quad \left. \left. + \sum_{m=1}^{m=N-1} \frac{1}{\tilde{R}_m^2} \frac{i \left(-mh, t - \frac{\tilde{R}_N}{c} \right) - i \left(-mh, t - k - \frac{R_N}{c} \right)}{k} + \frac{1}{2} \frac{1}{\tilde{R}_N^2} \frac{i \left(-H, t - \frac{\tilde{R}_N}{c} \right) - i \left(-H, t - k - \frac{R_N}{c} \right)}{k} \right] \right]
 \end{aligned}$$

$$B_{\phi}(r,z,t) = S''_1 + S''_2 \quad (16)$$

S''_1 = The electric radiation component in the magnetic field

S''_2 = The magnetic induction component in the magnetic field

4. SIMLIGHTNING CODE

After having developed a program with over 350 lines in Fortran, it has been implemented as a module (code) into Matlab in Version 2011 considering all the parameters electrical, magnetic and geometric expressions involved in the proposed new model of electromagnetic fields and all approximations required by the mathematical tools used. This program named SIMLIGHTNING code and his principal function is to simulate the current in lightning channel and the lightning electromagnetic fields with the following variables:

- The current at the base of the channel by lightning two types of functions
- The return current in the channel model by the lightning MTLE
- The 08 components of the three fields radiated by lightning

- The 03 vectors of electromagnetic fields radiated by the lightning Canal: Electric vertical, horizontal electric and azimuth magnetic

4.1. Results of SIMLIGHTNING Code Show in Fig. 2, 3, 4, 6, 7, 9, 10 and 11

4.1.1. Analysis Results

The results obtained by simulation are compared with those obtained by experimental measurements and other results published by other authors cited in references:

- For electric currents generated by lightning, the results illustrated in (Fig. 2, 3, 6 and 7) by SIMLIGHTNING have high concordance with those figures obtained by measurements (Fig. 5) in (Heidler and Hopf, 1994; Nucci and Rachidi, 1989; Rachidi *et al.*, 1992) or published (Fig. 8) in (Cooray, 1994; 1998)
- For the results of the electromagnetic field obtained by SIMLIGHTNING for its three components on the basis of the proposed new model in (Fig. 9-14) the agreement was satisfactory compared to measurements and results published in (Cooray, 1994; 1998; 2001) particular for the electric field where agreement was better than that of the magnetic field

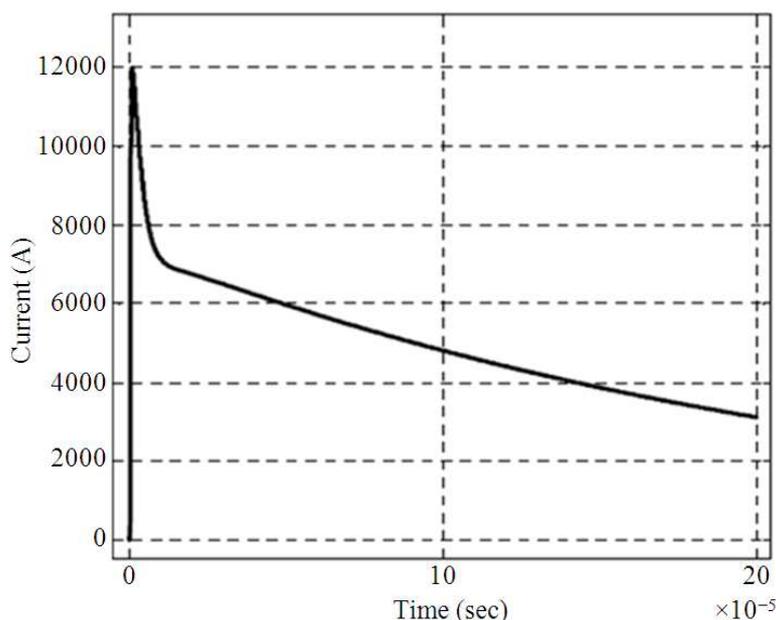


Fig. 4. Current at the base of Lightning channel with Heidler model (Verbanov, 2004)

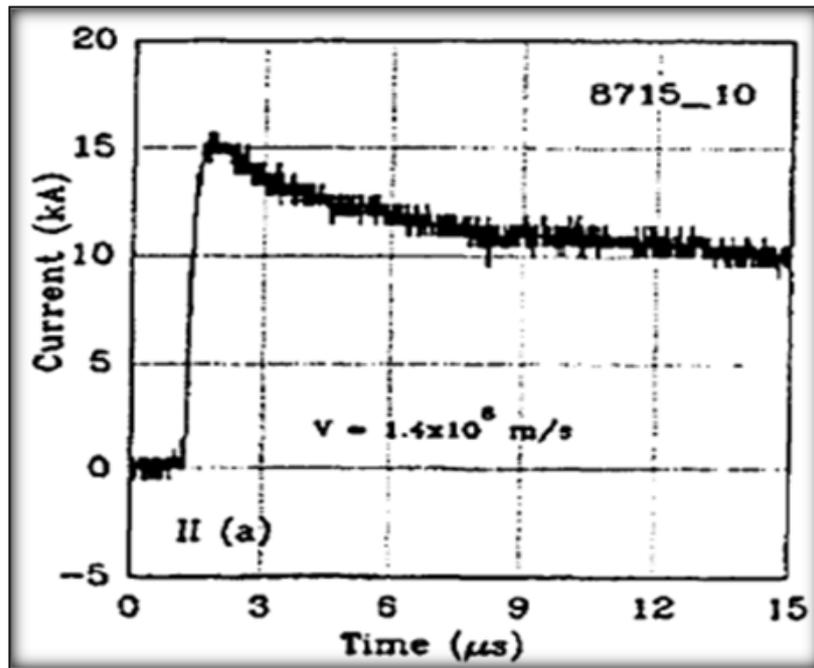


Fig. 5. Current at the base of Lightning channel Experiment measurement (Nucci and Rachidi, 1989; Rachidi *et al.*, 1992; 1998)

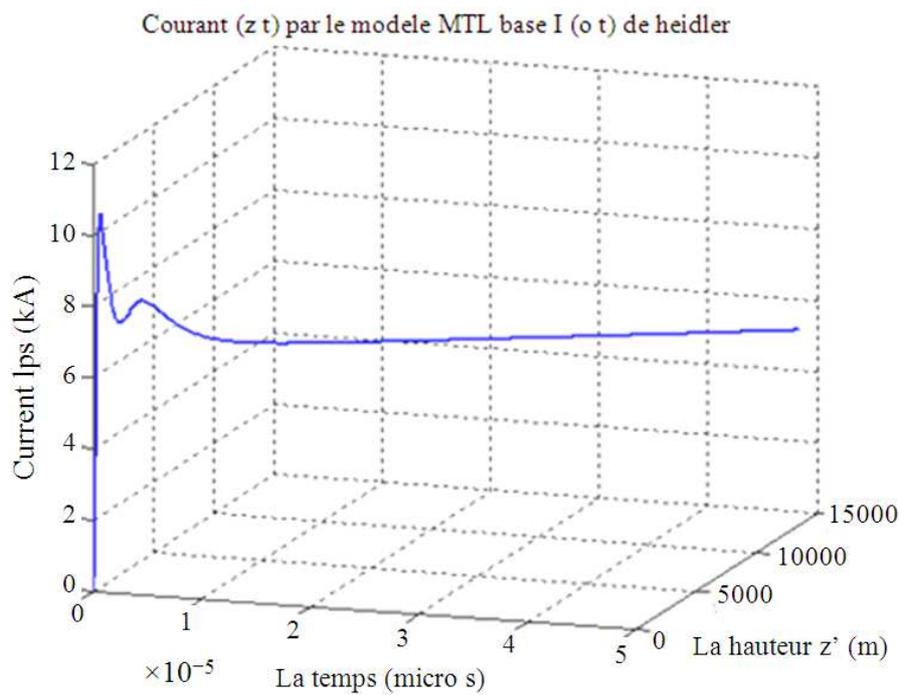


Fig. 6. Current in lightning channel of the Model MTLE with $i(0, t)$ of Heidler case1 (SIMLIGHTNING, 2012)

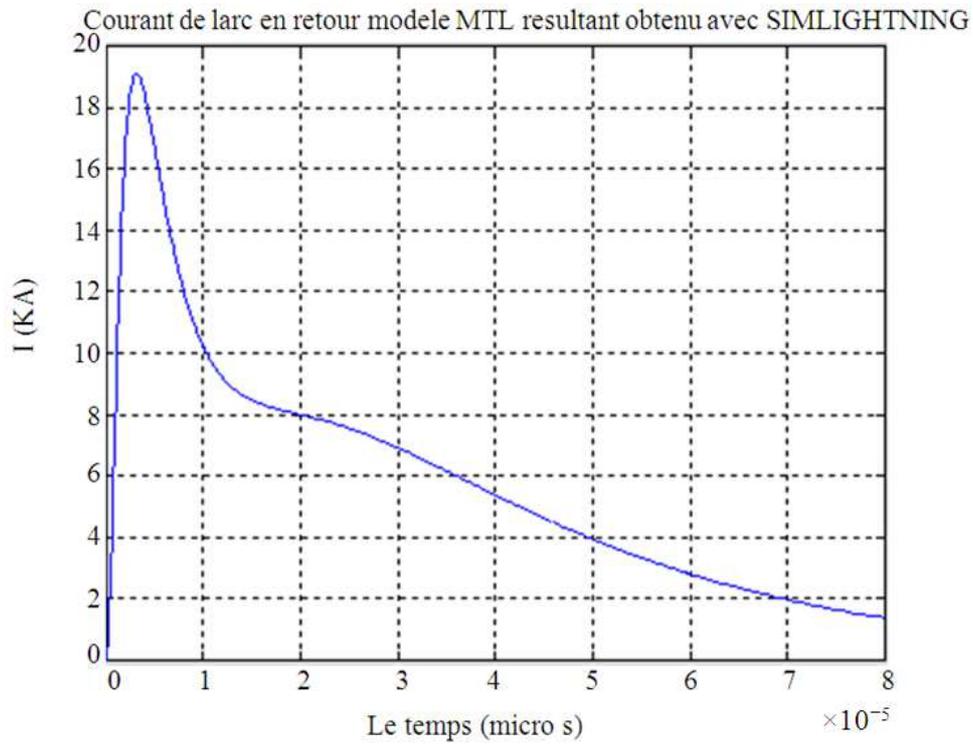


Fig. 7. Current in lightning channel of the Model MTL with $i(0, t)$ of Heidler case2 (SIMLIGHTNING 2012)

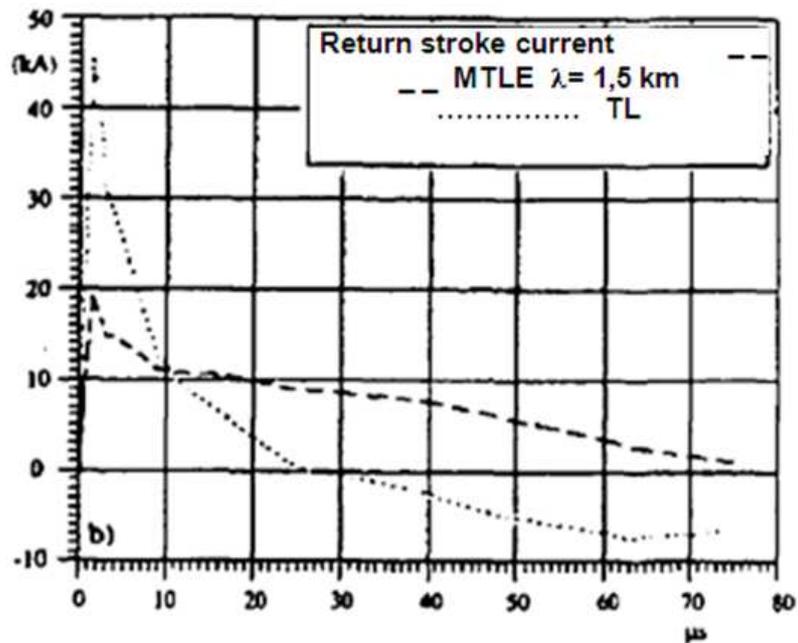


Fig. 8. Return stroke Current in lightning channel, MTL and TL Model (Cooray, 1994; 1998)

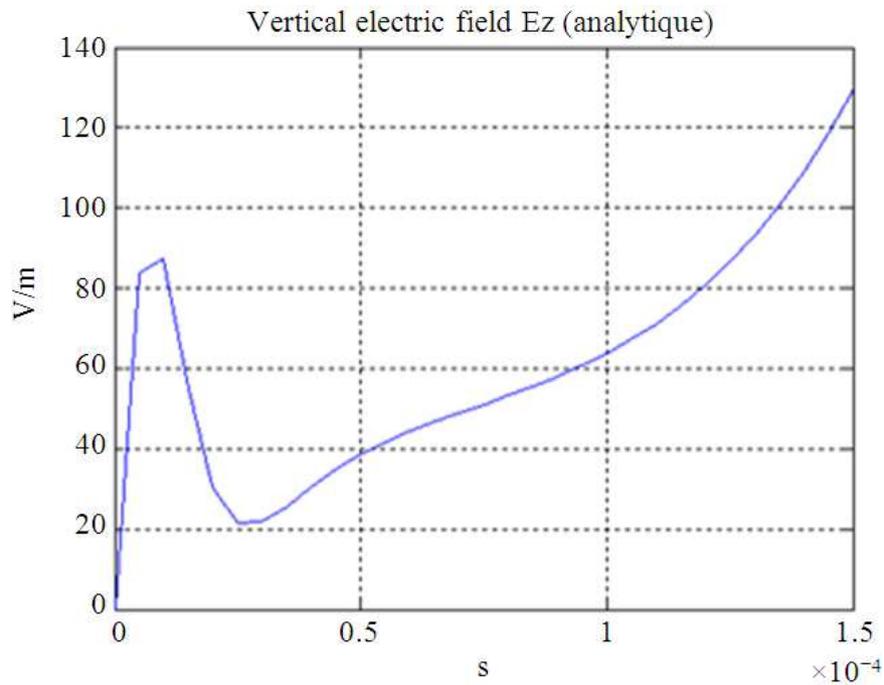


Fig. 9. Vertical Electric fields calculated by the new model (SIMLIGHTNING, 2012) MTLE with $i(0, t)$ of Heidler

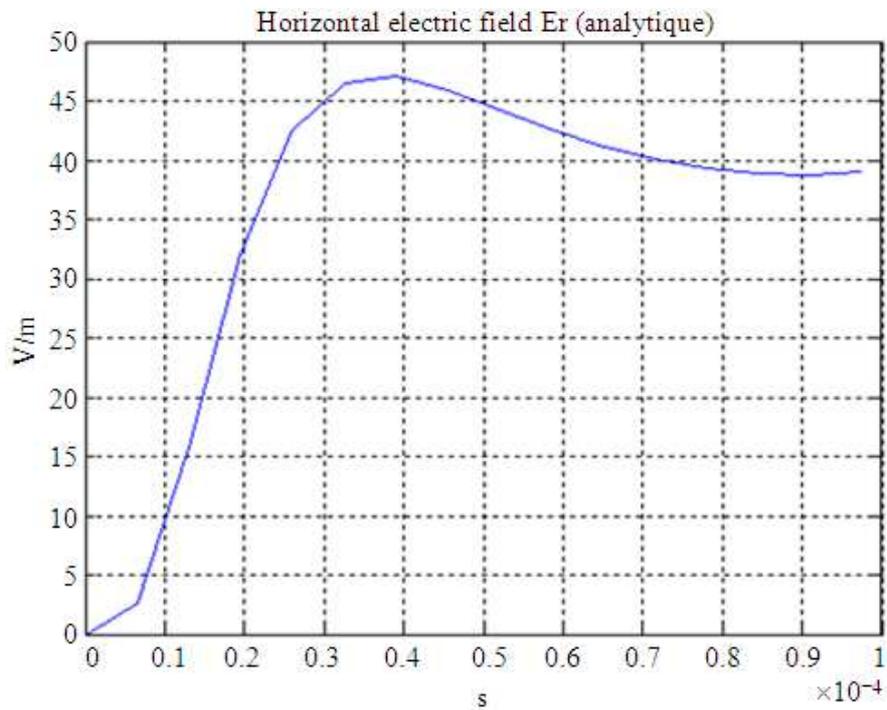


Fig. 10. Horizontal Electric fields calculated by the new model (SIMLIGHTNING, 2012) MTLE with $i(0, t)$ of Heidler

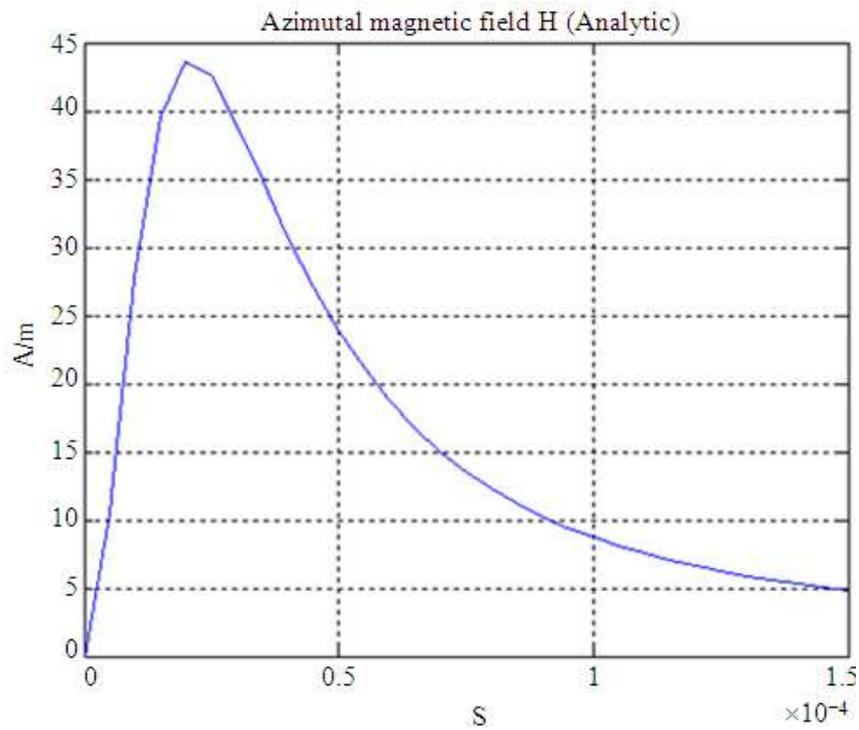


Fig. 11. Azimuth magnetic fields calculated by the new model (SIMLIGHTNING, 2012) MTLE with $i(0, t)$ of Heidler

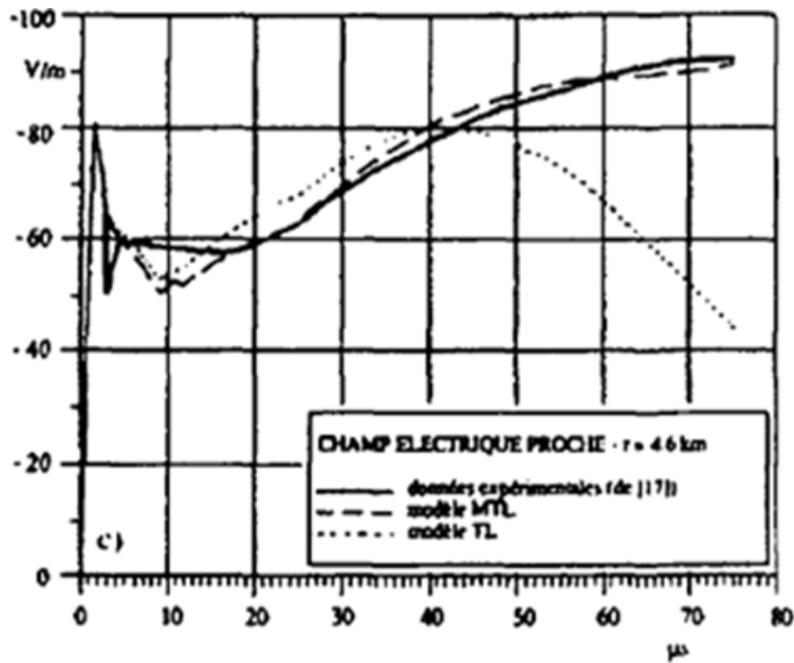


Fig. 12. Vertical Electric fields: Experiment and calculated with MTL and TL model of return stroke Current (Guerrieri *et al.*, 1996; 1998; Rachidi *et al.*, 1992; 1998)

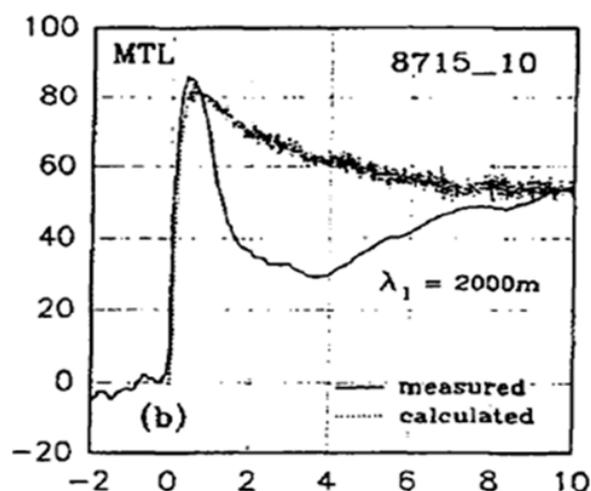


Fig. 13. Horizontal Electric fields: Experiment and calculated with MTL model of return stroke Current (Guerrieri *et al.*, 1996; 1998; Rachidi *et al.*, 1992; 1998)

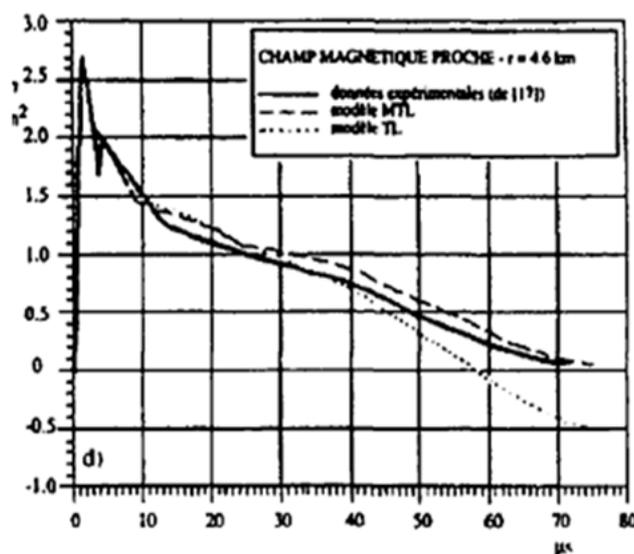


Fig. 14. Magnetic fields: Experiment and calculated with MTL and TL models of return stroke Current (Guerrieri *et al.*, 1996; 1998; Rachidi *et al.*, 1992; 1998)

5. CONCLUSION

In this study, we have proposed new equations for electromagnetic fields Radiated by the lightning channel at a point-in-space above-the ground. The analytical development proceeded That We Have Allowed us to models in the time domain by substituting Other variable parameters, Kinds of Space, Geometrical and electrical Such as the current in the lightning channel, Considered the main source of electromagnetic radiation transient.

The theorem of parity symmetric integrals and numerical approaches of Trapez integration and the operator of first order differences, was the various techniques used in the development of new equations. The development of a module named SIMLIGHTNING under Matlab for the simulation of new electromagnetic fields models allows us to have very interesting results of all quantities involved in this development in the lightning current at the canal base, the return current in the lightning canal and the various components of

electromagnetic fields: Vertical electric field, horizontal electric field and azimuth magnetic field, this is compared with experimental measurements and simulation results published by several authors (Diendorfer and Uman, 1990; Guerrieri *et al.*, 1996; Nucci and Rachidi, 1989; Rachidi *et al.*, 1992; 1998; Dib *et al.*, 2007) and have given a very appreciable similar and affinity.

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