

## A Novel Method of Voltage Sag and Swell Estimation for Power System Applications

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### ABSTRACT

This study presents four simple mathematical models to build the voltage shape with the knowledge of few parameters and detects voltage sag and swells accurately. Fourier series, cubic polynomial, Rational functions and Sum of Sin functions are used in this study to model the voltage profile. A new algorithm is developed to detect sag or swell in all the four models. The voltage estimation due to cubic polynomial has more voltage error and then rational function. The sum of sine function and Fourier series functions does accurate voltage estimation. Errors due to various functions are also tabulated for one electrical cycle. The performance of each method has been compared with other to know the effectiveness of different models and their results are presented.

**Keywords:** Dynamic Voltage Restorer (DVR), Voltage Sag and Swell, Fourier, Cubic Polynomial, Rational and Sum of Sine Functions

### 1. INTRODUCTION

Power quality has become the main interest by the power systems research community. Voltage sags and swells are mainly causing severe damage and down time in the power systems. Voltage Sag (dip) a decrease to between 0.1 and 0.9 pu in rms voltage or current at the power frequency for durations of 0.5 cycles to 1 min and Swell is an increase to between 1.1 pu and 1.8 pu in rms voltage or current at the power frequency durations from 0.5-1 min (Bollen, 2000). **Table 1** highlights the Categories and Characteristics of Power Systems Electromagnetic Phenomena (Dugan *et al.*, 2003). These sags and swells mainly due to turn on/turn off operations of supply lines and flow of inrush currents during starting of different loads (Dugan *et al.*, 2003). These turn on/turn off can happen either from supply or load side. Sometimes Lightning strikes and EMI can cause momentary sags/swells acc. Numerous solutions have been identified and proposed to mitigate sags and swells wherein a dynamic voltage restorer is one which injects a voltage in series with the supply lines and when any sags and swells are detected. Experimental investigation of voltage sag mitigation by an advanced static VAR

compensator has been well discussed in (Wang *et al.*, 1998). Mostly, the rms voltage measurement method is used to detect the sag and swell before any mitigation is employed. The man drawback of the RMS voltage measurement is that the RMS voltage would be measured through voltage sensors and fed to the ADC of the microcontroller or DSP to convert them into digital and use the data based on the old data which is system dependent. The disadvantages associated with the rms method are discussed in (Xiangning *et al.*, 2000; Tunaboyle *et al.*, 1998).

**Table 1.** Typical duraion and magnitudes of diffrenet power disturbances

Categories		Typical duration	Typical magnitude
Instantaneous	Sag	0.5-30 cycles	0.1-0.9 pu
	Swell	0.5-30 cycles	1.1-1.8 pu
Momentary	Interruption	0.5-3 sec	<0.1 pu
	Sag	0.5-3 sec	0.1-0.9 pu
	Swell	0.5-3 sec	1.1-1.8 pu
	Interruption	3 sec-min	<0.1 pu
Temporary	Sag	3 sec-1 min	0.1-0.9 pu
	Swell	3 sec-1 min	1.1-1.8 pu

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Also power quality Surveys shows that voltage sags are considered as the dominant factor affecting power quality (Goldstein and Speranza, 1982). Also in (Naidoo and Pillay, 2007), authors have used rapid sag detection through the use of a nonlinear adaptive filter wherein authors claimed that the filter has an ability to track the amplitude of the sag in real time which would be highly useful for sag and swell mitigation. A novel sag detection method for the line-interactive Dynamic Voltage Restorer (DVR) has been presented in (Bae *et al.*, 2010). Here is the study which discusses different methods to make the voltage shape and then to detect the voltage sags and swells at different operating conditions.

The study is organized as follows: Firstly necessity of initial voltage measurement and existing methods of sag and swell detection are presented. Subsequently, a description mathematical modeling of Fourier, Gaussian, Cubic polynomial and Sum of sine functions are presented. The voltage sag and swell detection algorithm are presented in further. The results and discussions and its performance evaluation of different methods is presented next and finally Conclusion is presented.

## 2. MATERIALS AND METHODS

### 2.1. Necessity of Initial Voltage Measurement

Voltage measurement is an essential step to develop the mathematical model for the voltage profile. In this research, a 3 phase power quality analyzer was used to measure the voltage over one electrical cycle which would be used to model different mathematical equations.

The measured voltage for two electrical cycles is shown in **Fig. 1** and its corresponding data is tabulated in **Table 2** Also voltage was recorded for 5 min duration which is shown in **Fig. 2**.

### 2.2. Mathematical Modeling

Mathematical model is a description of any physical system using mathematical language. The process of developing mathematical equations for the systems is called as mathematical modeling. In this study four different mathematical models developed for a voltage profile which can be used to detect the Voltage sags and swells.

### 2.3. Fourier Series

The Fourier series is a sum of sine and cosine functions that is used to describe any periodic signal which can be represented in either the trigonometric form or the exponential form. Since the voltage signal is in the

form of periodic nature, here we used the trigonometric Fourier series form shown Equation 1:

$$y = a_0 + \sum_{i=1}^n a_i \cos(n\omega x) + \sum_{i=1}^n b_i \sin(n\omega x) \tag{1}$$

Where:

$a_0$  = A constant (intercept) term

$\omega$  = The fundamental frequency of the signal

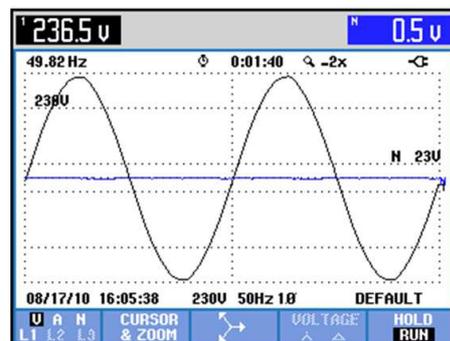
N = The number of terms (harmonics) in the series

Real time acquired value in **Fig. 1** were used to model the Fourier series equation for the voltage and the same is presented as Equation 2:

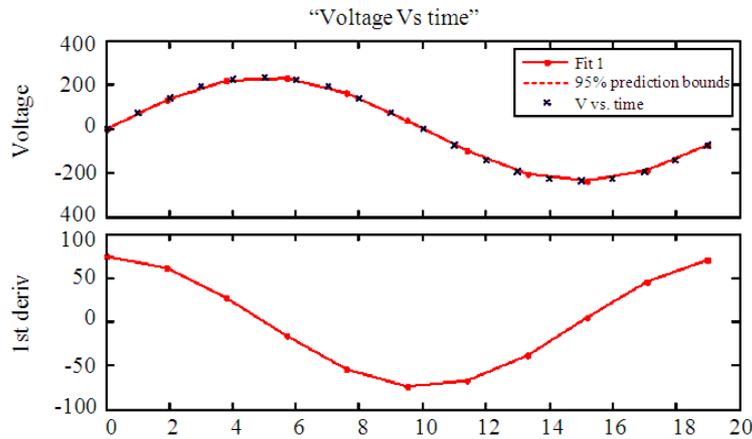
$$y(x) = 1.19e^{-7} - 2.152e^{-6} \cos(0.3142x) + 236 \sin(0.3142x) \tag{2}$$

**Table 2.** Voltage from real time measurement

Time in m sec	Voltage in volts
0	0.0000
1	72.9280
2	138.7173
3	190.9280
4	224.4493
5	236.0000
6	224.4493
7	190.9280
8	138.7173
9	72.9280
10	0.0000
11	-72.9280
12	-138.7173
13	-190.9280
14	-224.4493
15	-236.0000
16	-224.4493
17	-190.9280
18	-138.7173
19	-72.9280



**Fig. 1.** Voltage for two electrical cycles



**Fig. 2.** Voltage profile for one electrical cycle for real time measurement and fourier series function (Top) and its 1st derivative (bottom)

The coefficients of the equation are as follows:

- a0 = 1.196e-007 (-5.848e-006, 6.088e-006)
- a1 = -2.152e-006 (-1.743e-005, 1.312e-005)
- b1 = 236 (236, 236)
- w = 0.3142 (0.3142, 0.3142)

The Fourier series model developed using equation is plotted along with Voltage profile for one electrical cycle for real time measurement.

**2.4. Cubic Polynomial**

Generally the polynomial models for any physical systems are represented by Equation 3:

$$y = \sum_{i=1}^{n+1} p_i x^{n+1-i} \tag{3}$$

where, n + 1 is the order of the polynomial n is the degree of the polynomial. The order gives the number of coefficients to be fit and the degree gives the highest power of the predictor variable. In this research, polynomials are third-degree (cubic) polynomial is used and same is given by the linear model as Equation 4:

$$y = 0.6819x^3 - 20.28x^2 + 137.7x - 33.75 \tag{4}$$

The Coefficients of the equation are as follows:

- p1 = 0.6819 (0.6241, 0.7397)
- p2 = -20.28 (-21.96, -18.61)
- p3 = 137.7 (124.2, 151.1)
- p4 = -33.75 (-62.52, -4.971)

Polynomials are often used when a simple empirical model is required. The model can be used for

interpolation or extrapolation, or it can be used to characterize data using a global fit.

The Cubic Polynomial model developed using equation is plotted along with Voltage profile through real time measurement for one electrical cycle and its 1st derivative (bottom) is shown in **Fig. 3**.

**2.5. Rational function**

Rational models are defined as ratios of polynomials and are generally represented as Equation 5:

$$y = \frac{\sum_{i=0}^{n+1} p_i x^{n+1-i}}{x^m + \sum_{i=1}^m q_i x^{m-i}} \tag{5}$$

Where:

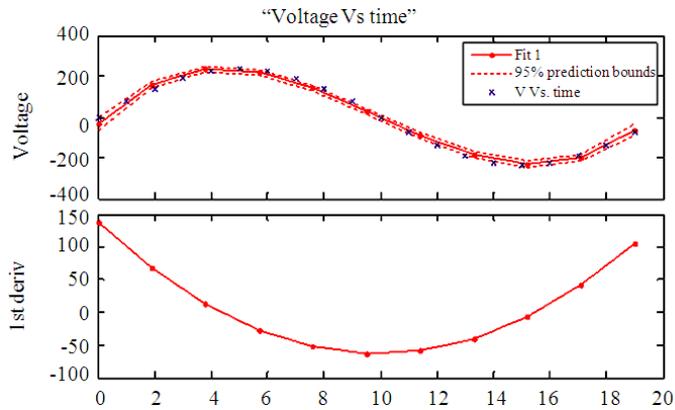
- n = The degree of the numerator polynomial
- m = The degree of the denominator polynomial

In this research, rational function is a function which is termed as degree of the numerator to the degree of the denominator. For example, a quadratic/cubic rational equation is given by Equation 6:

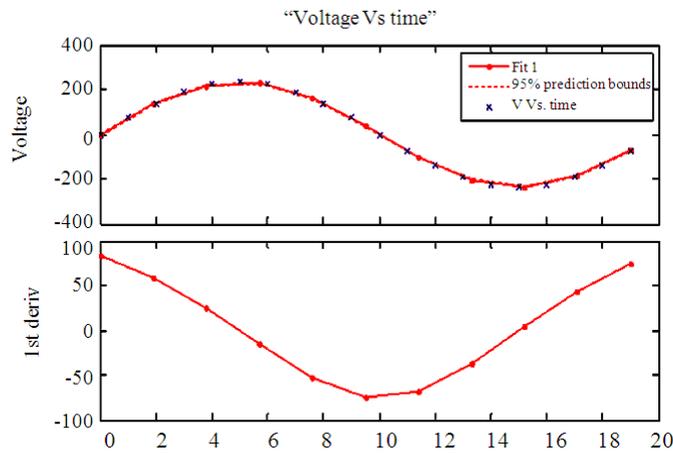
$$y = \frac{95.28x^3 - 2854x^2 + 19040x - 613.4}{x^3 - 20.2x^2 + 227x} \tag{6}$$

Coefficients of the rational function are as follows:

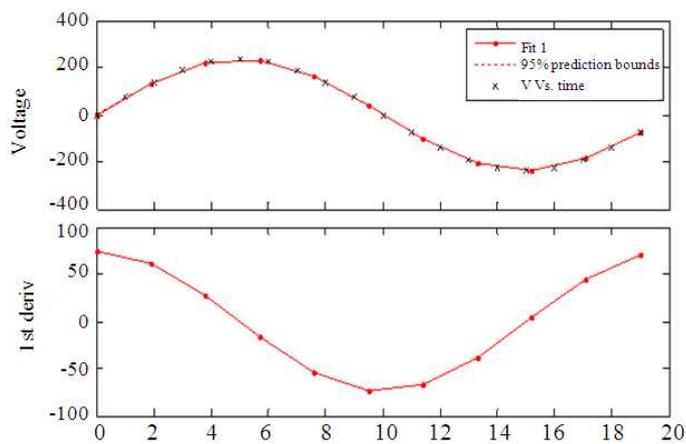
- p1 = 95.28 (88.91, 101.6)
- p2 = -2854 (-3042, -2665)
- p3 = 1.907e+004 (1.779e+004, 2.036e+004)
- p4 = -613.4 (-1308, 80.69)
- q1 = -20.2 (-20.52, -19.89)
- q2 = 227 (216.8, 237.1)



**Fig. 3.** Voltage profile for one electrical cycle for real time measurement and cubic polynomial function (Top) and its 1st derivative (bottom)



**Fig. 4.** Voltage profile for one electrical cycle for real time measurement and rational (Top) and its 1st derivative (bottom)



**Fig. 5.** Voltage profile for one electrical cycle for real time measurement and Sum of sines function (Top) and its 1st derivative (bottom)

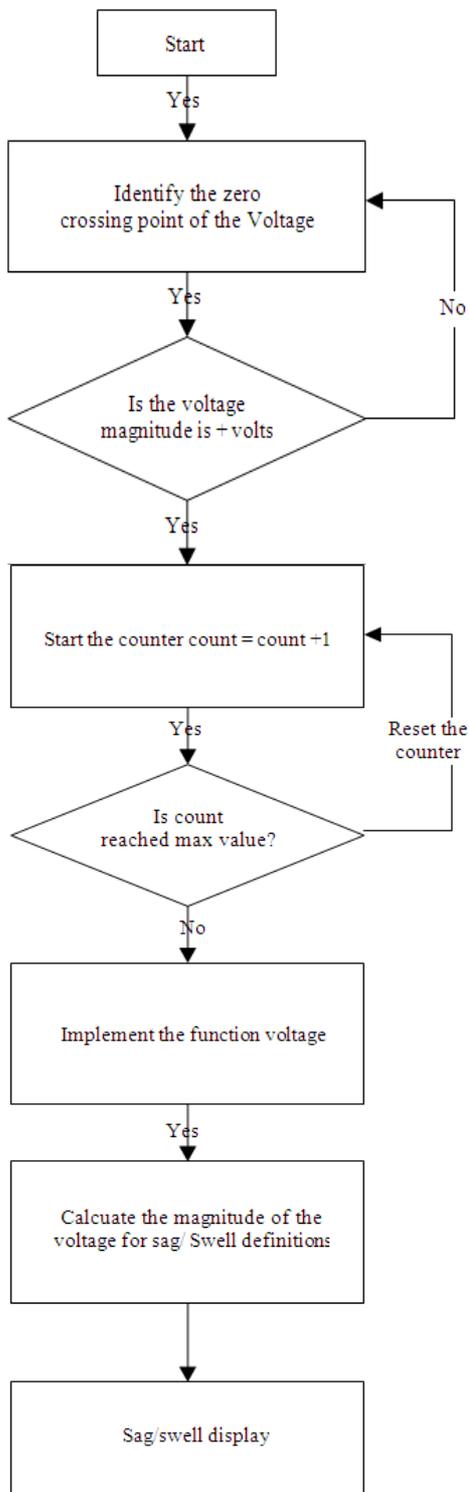


Fig. 6. Sag/Swell detection algorithm

Like polynomials function, rational function are often used in physical system modeling when a simple empirical models are required. The main advantage of rational is their flexibility with data that has complicated structure and the main disadvantage is that they become unstable when the denominator is around zero. The Rational function model developed using equation is plotted along with Voltage profile through real time measurement for one electrical cycle and its 1<sup>st</sup> derivative (bottom) is shown in Fig. 4.

### 2.6. Sum of sin function

The sum of sin function model is used for fitting periodic functions and is represented by the Equation 7:

$$y = \sum_{i=1}^n a_i \sin(b_i x + c_i) \quad (7)$$

Where:

- a = The amplitude
- b = The frequency
- c = The phase constant for each sine wave term
- n = The number of terms in the series

This equation is closely related to the Fourier series that has been described previously in this study. The main difference is that the sum of sin function equation includes the phase constant and does not include a constant (intercept) term Equation 8:

$$y = 236 \sin(0.3142x - 8.99e^{-9}) \quad (8)$$

The coefficients of the Sum of Sin Function are as follows:

$$\begin{aligned}
 A &= 236 \quad (236, 236) \\
 b1 &= 0.3142 \quad (0.3142, 0.3142) \\
 c1 &= -8.99e-009 \quad (-7.118e-008, 5.32e-008)
 \end{aligned}$$

The Sum of Sin Function developed using equation is plotted along with Voltage profile through real time measurement for one electrical cycle and its 1<sup>st</sup> derivative (bottom) is shown in Fig. 5.

#### Algorithm:

- Identify the zero crossing point of the voltage
- If the voltage magnitude is +ve, move to step number 3 else go to step 1
- Start the counter count=count+1

- If the counter reached max value, reset the counter and move to step 3 else go to step 5
- Implement the any one of the below functions
- Fourier series
- Cubic functions
- Rational functions
- Sum of sins
- Calculate the sag/swell as per **Table 1** and display the results

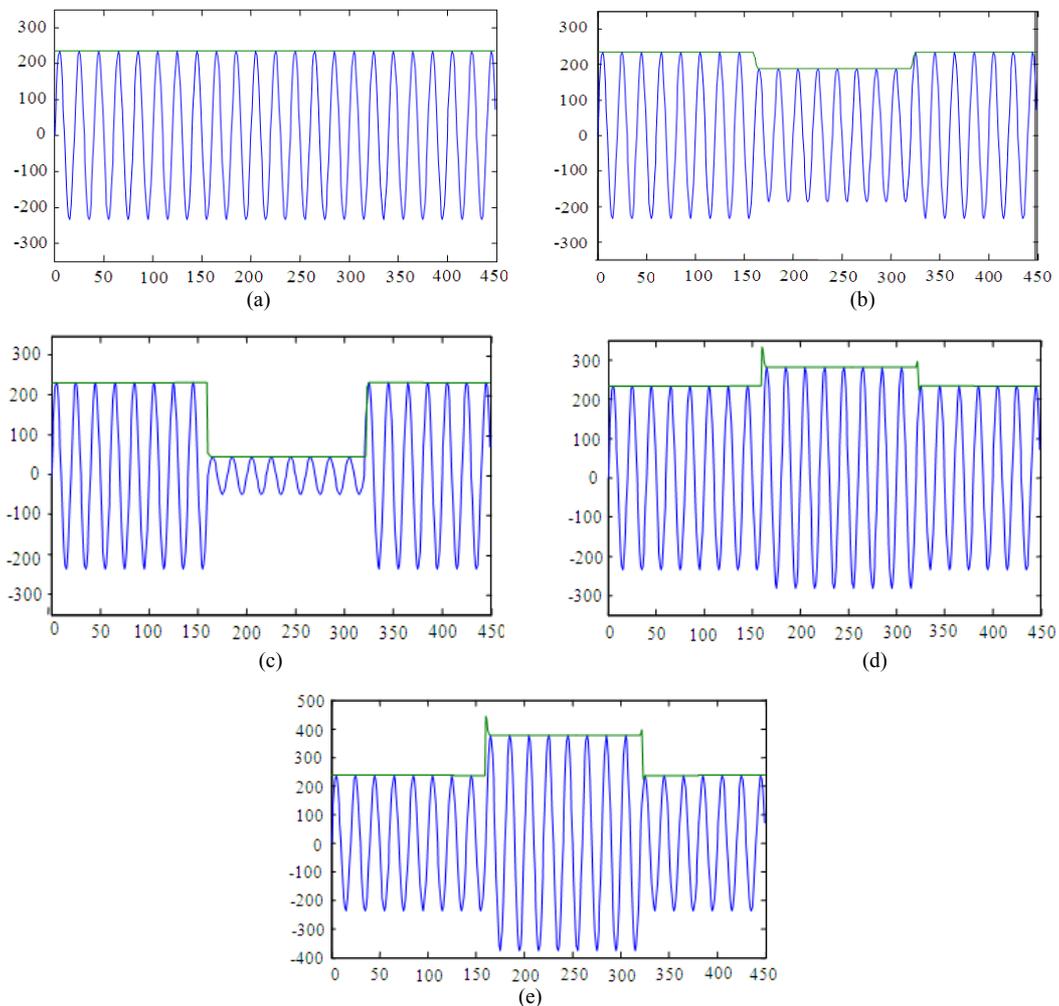
### 2.7. SAG/SWELL detection algorithm

The step by step procedure for detecting the sag and swell are presented below and its flow chart is given in **Fig. 6**.

### 3. RESULTS

The equations for Fourier, Cubic polynomial, Rational and Sum of sine functions for voltage profile has been developed through MATLAB/Simulink modeling Power Quality Analyzer.

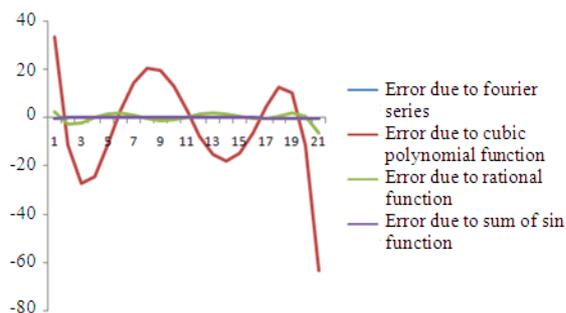
From the knowledge of zero crossing voltage information at each electrical cycle and time input, the Fourier, Cubic polynomial, Rational and Sum of sine functions implements the voltage waveform generation, which has been implemented through MATLAB/Simulink. **Figure 7** shows the Voltage Vs Time at (a). 236 Volts with no Sag/swell detection (b) 236 volts with 40% sag (c) 236 volts with 80% sag, (d) 236 volts with 20% swell and (e) 236 volts with 60% swell.



**Fig. 7.** Voltage Vs Time (a). 236 volts with no Sag/swell detection (b) 236 volts with 40% sag (c) 236 volts with 80% sag, (d) 236 volts with 20% swell and (e) 236 volts with 60% swell

**Table 3.** Voltage error due to Fourier, cubic polynomial, rational and sum of sine functions

Time in M sec	Measured voltage	Fourier Series Function (FSF)	Error due to FSF	Cubic Polynomial Function (CPF)	Error due to CPF	Rational Function (RF)	Error due to RF	Sum of Sin Function (SSF)	Error due to SSF
0.0010	0.0000	0.0741	0.0741	-33.6123	33.6123	-2.6184	2.6184	0.0741	-0.0741
1.0000	72.9280	72.9372	-0.0092	84.3519	-11.4239	75.5432	-2.6152	72.9372	-0.0092
2.0000	138.7173	138.7329	-0.0156	165.9852	-27.2679	140.9908	-2.2735	138.7329	-0.0156
3.0000	190.9280	190.9450	-0.0170	215.2413	-24.3133	190.8960	0.0320	190.9450	-0.0170
4.0000	224.4493	224.4612	-0.0119	236.2116	-11.7623	222.5679	1.8814	224.4612	-0.0119
5.0000	236.0000	236.0000	0.0000	232.9875	3.0125	233.7523	2.2477	236.0000	0.0000
6.0000	224.4493	224.4315	0.0178	209.6604	14.7889	223.1529	1.2964	224.4315	0.0178
7.0000	190.928	190.8885	0.0395	170.3217	20.6063	191.0226	-0.0946	190.8884	0.0396
8.0000	138.7173	138.6551	0.0622	119.0628	19.6545	139.6751	-0.9578	138.6551	0.0622
9.0000	72.9280	72.8457	0.0823	59.9751	12.9529	73.7062	-0.7782	72.8457	0.0823
10.000	0.0000	-0.0961	0.0961	-2.8500	2.8500	-0.7062	0.2672	-0.0961	0.0961
11.000	-72.9280	-73.0286	0.1006	-65.3211	-7.6069	-74.4016	1.4736	-73.0286	0.1006
12.000	-138.7173	-138.8106	0.0933	-123.3468	-15.3705	-140.7897	2.0724	-138.8106	0.0933
13.0000	-190.9280	-191.0014	0.0734	-172.8357	-18.0923	192.6480	1.7200	-191.0014	0.0734
14.0000	-224.4493	224.4909	0.0416	-209.6964	-14.7529	-225.1718	0.7225	-224.4909	0.0416
15.0000	-223.0000	-236.0000	0.0000	-229.8375	-6.1625	-235.8617	-0.1383	-236.0000	0.0000
16.0000	-224.4493	-224.4018	-0.0475	-229.1676	4.7183	-224.3462	-0.1031	-224.4018	-0.0475
17.0000	-190.9280	-190.8319	-0.0961	-203.5953	12.6673	-191.8816	0.9536	-190.8319	-0.0961
18.0000	-138.7173	-138.5773	-0.1400	-149.0292	10.3119	-140.7494	2.0321	-138.5773	-0.1400
19.0000	-72.9280	-72.7543	-0.1737	-61.3779	-11.5501	-73.7115	0.7835	-72.7543	-0.1737
20.0000	0.0000	0.1923	-0.1923	63.1923	-63.4500	6.3973	-6.3973	-0.1923	-0.1923



**Fig. 8.** Voltage Error due to fourier, cubic polynomial, rational and sum of sine functions Vs time for one electrical cycle

#### 4. DISCUSSION

It has been observed from the **Fig. 8** that sag/swell detector exactly detects the sag and swell information and conveys the message that there is sag and swell. Exists for different operating conditions **Fig. 8** shows that Voltage Error due to Fourier, Cubic polynomial, Rational and Sum of sine functions vs. time for one electrical cycle. It has been observed from the **Fig. 8** that the voltage estimation due cubic polynomial has more voltage error and then rational function. The sum of sine function and Fourier series functions does accurate voltage estimation. Errors due to various functions are also tabulated in **Table 3** for one electrical cycle. It has been observed that the sum of sine function, Fourier series and rational functions are performing better than cubic polynomial. Since these are

simple mathematical models, the time consumption implement them in real time is very simple. Also these models can be very well extended to any supply system to achieve the higher reliability and repeatability in sag and swell detections. Its interesting to note that these model requires only few input parameters such as time, voltage and frequency magnitude with zero crossing information. It was found that the developed algorithms detects the sag and swell accurately and quickly within 2 m sec.

#### 5. CONCLUSION

Four new mathematical models have been developed and have been presented and applied to the voltage sag and swell detection of sags in a single phase power system. Also each developed method has been compared with others to find their effectiveness with respect to others. This study conveys that the ability of the each algorithm to detect voltage sags and swells very quickly than the than available methods.

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