

Calculation of Total Cost, Tolerance Based on Taguchi's, Asymmetric Quality Loss Function Approach

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Abstract: Problem statement: Current world market force the manufacturing sectors to develop high quality product and process design with minimum possible cost. About 80% of problems in production units may be attributed to 20% of design tolerance causes. While design typically represents the smallest actual cost elements in products (around 5%), it leverages the largest cost influence (around 70%). So design engineers continuously stumble upon problem of design for high quality performance with lower cost. Objectives of this study where to: (i) simultaneous selection of design and manufacturing tolerance (ii) minimization of total cost (sum of the manufacturing cost and Taguchi's asymmetric quality cost) (iii) minimum cost and its machining tolerance. **Approach:** Rotor key base assembly was considered as case study to optimize the minimization of assembly total cost and machining tolerance. New global nonlinear optimization techniques called pattern search algorithm had been implemented to find optimal tolerance allocation and total cost. **Results:** In this study minimum cost arrived was 45.15 Cr and its corresponding tolerances for machining process turning, drilling, face milling, face milling and drilling where 0.063, 0.0508, 0.2127, 0.2127, 0.2540 mm respectively at worst case conditions. **Conclusion:** Results indicated that optimization by integer programming, sequential quadratic programming and exhaustive search, nonlinear programming, genetic algorithm, simulated annealing, fuzzy logic, number set theory and Monte Carlo simulation did not give much least total cost and also predicted that pattern search algorithm was robust method. Second the method, generally termed as concurrent tolerance synthesis was well suited for engineering environment, where high quality products with low total cost were designed and manufactured.

Key words: Dimension, optimization, tolerance, rotor key assembly, quality loss

INTRODUCTION

Tolerance is defined as the system of specifying the extent of permissible variations (or) In other words the amount of variation permitted for the basic size is called tolerance. It can specify in to three ways: Bilateral, Unilateral and Unbalanced. Bilateral tolerances are most common in industry, but more and more product designs are using unbalanced tolerances^[1].

In general, losses will be much grater than the cost of manufacture and none of the expense will necessarily recoup the loss of the company reputation. Quality is a virtue of design. The "robustness" of products is more a function of good design than of on-line control; however it is a stringent, manufacturing process. Indeed though not nearly so obvious an inherent lack of robustness in product design is the primary driver of superfluous manufacturing expenses. Furthermore,

research has shown that robust design practices can lead to low cost, improvements in quality, manufacturability and reliability. Taguchi's introduction of robust design resulted in significant improvements in the manufacturing processes and product quality of several major American industrial firms^[11].

At present, the assignment of design tolerances is performed largely on a trial and error basis using the tolerance analysis method. Several key tolerances are specified based upon the given design requirements. Other tolerances are determined by a designer based on his/her design experience and manufacturing knowledge, or assigned with default values. To change this undesirable situation in tolerance assignment and to accomplish the two objectives of tolerance design, tolerance synthesis (or tolerance optimization, optimal tolerance design) has been studied^[6]. In this article, they introduce a systematic design framework for process

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quality that embeds Taguchi's method and other robustness criteria within a stochastic optimization formulation.

Literature review: The allocation of the dimensional tolerances to a product highly affects their quality and manufacturing cost. In most cases, tighter tolerances realize smaller variations in the product performances and hence higher quality. On the other hand, tighter tolerances require precision machine tools and often longer process time, hence causing higher production cost^[3,4]. Since tolerances of some dimensions affect the quality and cost more than the other, it is desirable to allocate tight tolerances only to the dimensions that have high influences to the quality, to attain an optimal balance between the quality and cost^[10].

Tolerance design is a very important issue in product development. Conventionally tolerance design is carried out in two steps, CAD and CAPP, in a sequential manner. The approach suffers from several drawbacks such as more time consumption, sub optimality and an unhealthy working atmosphere. To overcome the drawbacks of this approach, an attempt was made at the simultaneous selection of optimal design and manufacturing tolerances. The methodology was demonstrated with the help of a simple linear assembly considering different tolerance stack-up conditions. Apart from the traditional worst case and RSS criteria, two non-traditional approaches, Spotts' modified and Greenwood and Chase's estimated mean shift stack-up conditions were explored^[13,14,15].

Dimitrellou^[5] presented a method for the automatic establishment and processing of machine shop and application specific cost-tolerance functions for the cost-optimum allocation of tolerances. The method can be readily integrated into a CAD environment and overcomes existing inefficiencies of other systems. The introduced tolerance elements are geometric, generic, machining process related and straightforwardly identifiable in conformance with the existing industrial understanding.

A method of synthesizing tolerances concurrently for both manufacturing cost and quality with position control in assembly has been presented^[12]. The method, generally termed as concurrent tolerance synthesis is well suited for engineering environment, where high quality products with low total cost are designed and manufactured. An integrated optimization model was presented^[2] to use the manufacturing and quality loss costs as the objective function and use process capability indices and quality loss constant to ensure the quality to a customer desired level.

Jeang^[9] discusses a set of models which determine optimal product tolerance and minimize combined

manufacturing and quality related costs. These models include the cases of the 'nominal-the-best', the 'smaller-the-better', the 'larger-the better' and 'asymmetric loss function'. The process capability index is applied to build the functional relationship between the product variability and product tolerance. Based on this relationship, the total cost of each model can be expressed as a function of product tolerance from which the optimal tolerance can be determined.

Huang^[8] has presented a robust optimization method in a concurrent tolerance environment. This method can determine multiple correlated critical tolerances and directly allocate them to process tolerances by using component process plans. In a concurrent environment, the product tolerance design and process tolerance design can be integrated into one stage. Tolerance design has been extended directly from the product design to the manufacturing stage. The necessity of redesign and rework between product tolerance design and process tolerance design has been eliminated, increasing the design efficiency. In a conventional tolerance design, the optimal model is established for two separate stages and the optimum solutions are for different stages but not for the entire product design process.

Tolerance and quality loss function: Quality loss function is a quadratic expression estimating the cost of the average then comparing it to the customer Identified target values and the variability of the product characteristic in term, of monetary loss due to product failure. There is very important concept of quality engineering inherent in the loss function. In the usual practice of manufacturing quality control the producer specifies a mean value of the performance characteristic and the tolerance interval around that value. With loss function as a definition of quality the emphasis is on achieving the target value of the performance characteristic and deviations from the target value the greater the quality loss^[2,7].

Types of loss function expressed as:

Normal-the-best is the:

$$L(y) = \frac{A}{\Delta^2} (y - m)^2 \quad (1)$$

Smaller-the-better is the:

$$L(y) = \frac{A}{\Delta^2} y^2 \quad (2)$$

Larger-the-better is the:

$$L(y) = A\Delta^2 \left(\frac{1}{y^2} \right) \quad (3)$$

In some situations, the quality loss resulting from deviation of quality characteristics in one direction is greater than the deviation in the other direction.

The asymmetric quality loss function:

$$L(y) = K_1(y - m)^2 y > m$$

$$L(y) = K_2(y - m)^2 y < m \tag{4}$$

Pattern search algorithms-an introduction: Pattern search methods belong to a class of optimization methods and a subclass of direct search algorithms. It is an evolutionary technique that is suitable to solve a variety of optimization problems that lie outside the scope of the standard optimization methods it was first introduced and analyzed by Torczon^[16] for unconstrained problems and extended by Lewis and Torczon to problems with bound constraints^[17] and a finite number of linear constraints. In all three results, convergence of a subsequence of iterates to a limit point satisfying first-order necessary conditions is proved.

These methods have a long and rich history in the scientific and engineering communities where they have been applied to numerous problems. The main attraction of direct search methods is their ability to find optimal solutions without the need for computing derivatives in contrast to the more familiar gradient-based methods.

A pattern is a set of vectors (v_i) that the pattern search algorithm uses to determine which points to search at each iteration. The set (v_i) is defined by the number of independent variables in the objective function, N and the positive basis set. Two commonly used positive basis sets in pattern search algorithms are the maximal basis, with $2N$ vectors and the minimal basis, with $N+1$ vector.

With Generalized Pattern Search algorithm (GPS), the collections of vectors that form the pattern are fixed-direction vectors. For example, if there are three independent variables in the optimization problem, the default for a $2N$ positive basis consists of the following pattern vectors:

$$V_1 = [1 \ 0 \ 0] \quad V_2 = [0 \ 1 \ 0]$$

$$V_3 = [0 \ 0 \ 1] \quad V_4 = [-1 \ 0 \ 0]$$

$$V_5 = [0 \ -1 \ 0] \quad V_6 = [0 \ 0 \ -1]$$

An $N+1$ positive basis consists of the following default pattern vectors:

$$V_1 = [1 \ 0 \ 0] \quad V_2 = [0 \ 1 \ 0] \quad V_3 = [0 \ 0 \ 1] \quad V_4 = [-1 \ -1 \ -1]$$

With Mesh Adaptive Search Algorithm (MADS), the collections of vectors that form the pattern are randomly selected by the algorithm. Depending on the poll method choice, the number of vectors selected will be $2N$ or $N+1$. As in GPS, $2N$ vectors consist of N vectors and their N negatives, while $N+1$.

Vectors consist of N vectors and one that is the negative of the sum of the others.

Meshes: At each step, the pattern search algorithm searches a set of points, called a mesh, for a point that improves the objective function. The GPS and MADS algorithms form the mesh by:

- Generating a set of vectors (v_i) by multiplying each pattern vector v_i by a scalar Δ_m . Δ_m is called the mesh size
- Adding the (d_i) to the current point-the point with the best objective function value found at the previous step

Example case studies: The proposed methodology has been explained with the help of rotor key assembly example problem. This is given as a case study in Al-Ansary^[1]. It is a simple linear mechanical assembly as shown in Fig. 1 involving only two components. The Optimal tolerance design and Asymmetric total cost problem can be formulated as follows.

Representation of variables: In Fig. 1 shows that the resultant dimension X_R as the principal design tolerance and the dimension chain for the X_R consists of the five dimensions X_{11} , X_{12} , X_{21} , X_{22} and X_{23} . For this the best suited machining process will be of turning, drilling, face milling, face milling and drilling associated with the five dimensions in the dimension chain for X_R , respectively.

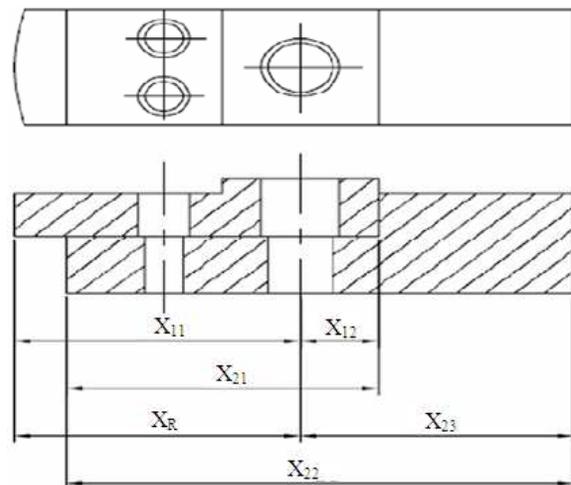


Fig. 1: Rotor key assembly

Table 1: Ranges of the principal machining tolerances for the rotor key assembly

Dimension tolerance in mm			

Machining process			
Sr. No.	with notation	Lower limit	Upper limit
1	Turning (δ_{11})	0.0635	0.762
2	Drilling (δ_{12})	0.0508	0.254
3	Face milling (δ_{21})	0.0254	0.508
4	Face milling (δ_{22})	0.0254	0.508
5	Drilling (δ_{23})	0.0508	0.254

Table 2: Coefficients for the five machining processes of an exponential model

Sr. No.	Cost-tolerance model	C ₀	C ₁
1	δ_{11}	66.43	2.738
2	δ_{12}	27.84	3.661
3	δ_{21}	431.50	17.640
4	δ_{22}	431.50	17.640
5	δ_{23}	27.84	3.661

Hence the design variables for the optimization problem include the design tolerance δ_R and the five machining tolerance δ_{11} , δ_{12} , δ_{21} , δ_{22} and δ_{23} . The ranges of tolerance for these five machining process are given in Table 1.

Objective function: Minimization of the total cost represents the objective function. Total cost is the summation of manufacturing cost with respect to the tolerance and asymmetric quality loss given in Eq. 4.

Thus the objective function is expressed as, manufacturing cost is expressed in exponential model as:

$$g(\delta) = c_0 \frac{1}{e^{c_1 \delta}} \tag{5}$$

Total manufacturing cost (C_m):

$$C_m = g_{11}(\delta_{11}) + g_{12}(\delta_{12}) + g_{21}(\delta_{21}) + g_{22}(\delta_{22}) + g_{23}(\delta_{23}) \tag{6}$$

The coefficients for these models (Eq. 5 and 6) for the various cost-tolerance functions are given in Table 2.

Constraints: The Design tolerances are framed by four stacked up conditions are Worst case, RSS, Spotts and estimated mean shift criteria. These stack-up conditions yield a set of design constraints as below:

Worst case criteria:

$$\delta_{11} + \delta_{12} + \delta_{21} + \delta_{22} + \delta_{23} \leq 1.016$$

RSS criteria:

$$\sqrt{\delta_{11}^2 + \delta_{12}^2 + \delta_{21}^2 + \delta_{22}^2 + \delta_{23}^2} \leq 1.016$$

Spotts criteria:

$$\frac{1}{2} \left[(\delta_{11} + \delta_{12} + \delta_{21} + \delta_{22} + \delta_{23}) + \sqrt{(\delta_{11}^2 + \delta_{12}^2 + \delta_{21}^2 + \delta_{22}^2 + \delta_{23}^2)} \right] \leq 1.016$$

Estimated mean shift criteria:

$$\frac{(m_1 \delta_{11} + m_2 \delta_{12} + m_3 \delta_{21} + m_4 \delta_{22} + m_5 \delta_{23}) + \frac{z}{3} \sqrt{\frac{(1-m_1)^2 \delta_{11}^2 + (1-m_2)^2 \delta_{12}^2 + (1-m_3)^2 \delta_{21}^2}{+(1-m_4)^2 \delta_{22}^2 + (1-m_5)^2 \delta_{23}^2}}}{3} \leq 1.016 \tag{7}$$

The total machining cost is optimized subject to the constraints and the ranges of the principal design and machining tolerance mentioned in Table 1. The pattern search algorithm method has been used as an optimization engine. In this an individual length of 20 bits with 100 generations are employed. The results of optimization using pattern search algorithm method are discussed as follows.

MATERIALS AND METHODS

The objective function is to minimize the total cost. Total cost is the summation of manufacturing cost with respect to the tolerance and asymmetric quality loss given in equation 4. Thus the Manufacturing cost is expressed in exponential model:

$$C_m = \sum_{ijk} g_{ijk}(\delta_{ijk}) \tag{8}$$

Assign different value (range from Table 1) of tolerance in to the above equation 8, accordingly it will give different values of total cost. The different values of total costs are optimized by using pattern search algorithm we will be getting the least total cost.

RESULTS

The optimization of total cost verses tolerance of the rotor key assembly, for each case of the design constraint was carried out for 100 generations on a P-IV personal computer using MATLAB 7.5.0 (R2007b) version the results are shown in Table 3-6 and also represented in Fig. 2 and 3.

Table 3: Optimal tolerances allocated using pattern search algorithm-worst case method

Dimensional tolerances		Least total cost (manufacturing cost + asymmetric quality loss)
δ_{11}	0.0635	45.15 C_r
δ_{12}	0.0508	
δ_{21}	0.2127	
δ_{22}	0.2127	
δ_{23}	0.2540	

Table 4: Optimal tolerances allocated using Pattern search algorithm-RSS method

Dimensional tolerances		Least total cost (manufacturing cost + asymmetric quality loss)
δ_{11}	0.0635	44.85 C_r
δ_{12}	0.0508	
δ_{21}	0.2127	
δ_{22}	0.2127	
δ_{23}	0.2540	

Table 5: Optimal tolerances allocated using pattern search algorithm-Spots method

Dimensional tolerances		Least total cost (Manufacturing cost + asymmetric quality loss)
δ_{11}	0.0635	44.35 C_r
δ_{12}	0.0508	
δ_{21}	0.2127	
δ_{22}	0.2127	
δ_{23}	0.2540	

Table 6: Optimal tolerances allocated using pattern search algorithm-greenwood and chases method

Dimensional tolerances		Least total cost (manufacturing cost + asymmetric quality loss)
δ_{11}	0.0635	45.08 C_r
δ_{12}	0.0508	
δ_{21}	0.2127	
δ_{22}	0.2127	
δ_{23}	0.2540	

Note: C_r is the reference cost for the cost-tolerance data.

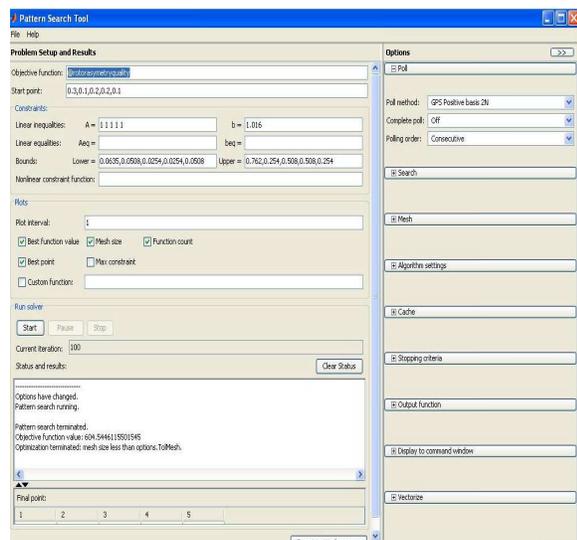


Fig. 2: Work setup for pattern

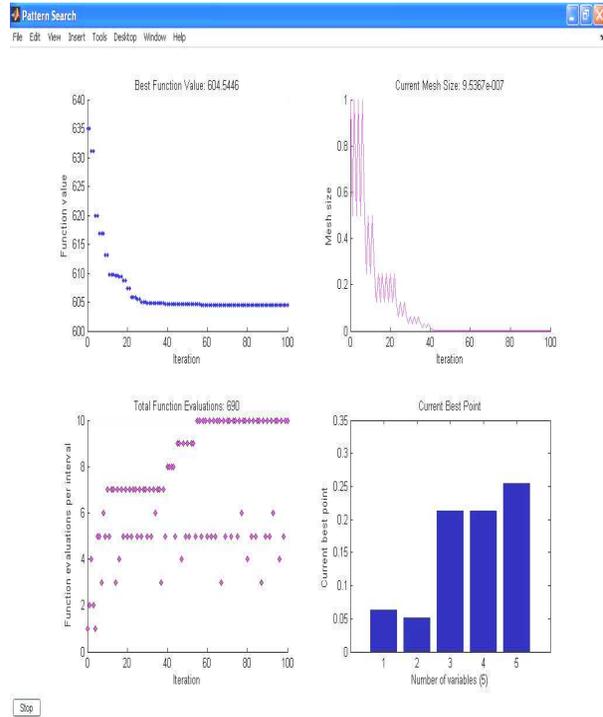


Fig. 3: Performance characteristics graph of cost and tolerance for an exponential model

DISCUSSION

A method of synthesizing tolerances concurrently for both manufacturing cost and quality with position control in assembly has been presented. The method, generally termed as concurrent tolerance synthesis is well suited for engineering environment, where high quality products with low total cost are designed and manufactured.

In Fig. 2 shows the general behavior and problem setup. From these Fig. 2 and 3 it is clearly understood that the optimum total cost as determined from the pattern search algorithm model will be 45.15 C_r using worst case model. The result indicates that the minimum total cost of the assembly is lowest with Greenwood and Chase Method and highest with RSS method.

In Fig. 3 there will be four graphs. In first graph plotted between iteration with cost in this the best value will be \$604.54 and its corresponding cost in C_r will be 45.15. Second graph will be between iteration with mesh size in this mesh size drastically decreasing up to 40 iterations and further it will remain constant. Third graph plotted between iteration vs. intervals in this the values are scattered. Finally fourth graph will be represented the best tolerance values of the five machining process.

The total cost obtained in my study is high when compared with study done by Al-Ansary^[1] since I have included asymmetric quality loss cost with machining cost. It is clearly shows from the observation of number of trials by changing the mean shift in different positions that when moving the mean shift towards Lower Consumer Tolerance (LCT) the total cost keeps on increasing.

CONCLUSION

These results indicate that the optimized value of tolerance using Pattern search algorithm has given least total cost for this Rotor key assembly. Pattern search method is one of the most popular classes of methods to minimize functions compare with other exact algorithms and heuristics optimization techniques. The concurrent method is comparatively simple. A better model may be approached for different assembly problems to find the Optimized cost using different optimization techniques.

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