

## Engineering of Ground for Liquefaction Mitigation Using Granular Columnar Inclusions: Recent Developments

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**Abstract: Problem statement:** Liquefaction was the most hazardous damage during an earthquake. Ground improvement techniques were employed to mitigate liquefaction hazards. Most common methods to improve engineering properties of soils are densification, reinforcement, grouting/mixing and drainage. Among various remedial measures available, installation of columnar granular inclusions is the most widely adopted method for liquefaction mitigation. **Approach:** Columnar granular inclusions function as drains and permit rapid dissipation of earthquake induced pore pressures by virtue of their high permeability. **Results:** One of the chief benefits of ground treatment with granular piles is the densification of *in situ* ground by which the in-situ properties of the ground get modified to mitigate liquefaction potential. Further, the very high deformation modulus and stiffness of the granular pile material provide reinforcement for the *in situ* soil and offer another mechanism to mitigate liquefaction. The study described briefly the phenomenon of liquefaction and the associated features. A short discussion on various ground improvement methods available for liquefaction mitigation was presented highlighting the importance of columnar inclusions. Construction methods of different granular columnar inclusions like sand compaction piles/ granular piles were discussed briefly. Recent developments in the research of columnar granular inclusions as liquefaction counter measures were presented in relation to physical, numerical and analytical model studies. **Conclusion/Recommendations:** Columnar granular inclusions were demonstrated to be very effective for liquefaction mitigation in different case studies and research investigations.

**Key words:** Liquefaction, mitigation, columnar inclusions, granular piles, stone columns, analytical/numerical studies

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### INTRODUCTION

Liquefaction and its associated ground displacements resulting from earthquake shaking are the major cause of damage in loose saturated granular soils. Many liquefaction induced failures or near-failures of foundations, buildings and infrastructure facilities like highway/railway embankments, port facilities and earth dams have been reported across the globe during various earthquakes. The 1995-Kobe earthquake emphasized the importance of foundation liquefaction as a potential source of damage. Liquefaction can be manifested either by the formation of boils and mud-spouts at the ground surface, by seepage of water through ground cracks or in some cases by the development of quicksand conditions over substantial areas<sup>[1]</sup>.

Ground improvement techniques like densification, reinforcement, grouting/mixing and drainage are

commonly employed to mitigate liquefaction hazards. Provision of columnar granular inclusions like gravel drains/granular piles/stone columns is the most commonly adopted ground treatment methodology for liquefaction mitigation which has proved its effectiveness in many instances<sup>[2]</sup>. Granular piles are the most widely preferred alternative all over the world, due to technical feasibility, low energy utilization and cost effectiveness. They improve the ground by reinforcement and densification of the surrounding soil apart from providing drainage. Different mechanisms operate in the function of gravel drains/granular piles in liquefaction mitigation. These mechanisms can be stated as drainage, storage, dilation, densification and reinforcement.

Ground improvement by means of granular piles/stone columns/geopiers, which is associated with partial substitution of the *in situ* soil, originated in the sixties. Stone columns generally use gravel or crushed

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stone as backfill. Effect of method of installation, cased and uncased holes, number of lifts and magnitude of compactive energy per lift given to granular piles and pile spacing were discussed by Madhav and Thiruselvam<sup>[3]</sup>. Consideration of granular piles/drains installation as a possible method of stabilizing a soil deposit, susceptible to liquefaction, started with the work by Seed and Booker<sup>[4]</sup>. They state that pore-water pressures generated by cyclic loading get dissipated almost as fast as they are generated through the system of gravel or rock drains. Since then, different types of columnar inclusions are used as liquefaction remedial measure, which basically provide the drainage facility to dissipate the excess porewater pressures during cyclic loading almost as fast as they are generated.

This study presents some of the recent developments in this very vital area of liquefaction and its counter-measures highlighting the importance of columnar granular inclusions.

**Liquefaction and counter measures:** Liquefaction is the state when saturated sandy soil loses its shear strength due to increased pore pressure and consequent reduction in effective stresses. Terzaghi<sup>[5]</sup> originally introduced the term liquefaction into the engineering community in the classical book *Erdbaumechanik*<sup>[6]</sup> and Casagrande<sup>[7]</sup>, in 1936, used the term to explain the massive soil failures at Fort Peck Dam. The concept of liquefaction gathered worldwide attention in the 1960's, when in 1964 large magnitude earthquakes located near Anchorage, Alaska and Niigata, Japan caused massive structural damage through ground failure. Significant amount of work on this topic has been performed in the last few decades since these earthquakes, resulting in several state-of-the-art papers relating to the study, evaluation and remediation of liquefaction<sup>[8-19]</sup>.

As a consequence of the applied cyclic stresses, the structure of the cohesionless soil tends to become more compact but with a resulting transfer of stresses to the porewater and a reduction in the effective stresses on the soil grains. As a result, the soil grain structure rebounds to the extent required to keep the volume constant and this interplay of volume reduction and soil structure rebound determines the magnitude of the increase in porewater pressure in the soil<sup>[10]</sup>. The basic phenomenon is illustrated (Fig. 1) schematically by Seed<sup>[11]</sup>. The mechanism can be quantified so that the pore pressure increases due to any given sequence of stress applications can be computed from knowledge of the stress-strain and the volume change characteristics of the sand under cyclic strain conditions and the rebound characteristics of the sand due to stress reduction. Relationships between cyclic stress ratio and pore

pressure ratio, number of cycles required to cause liquefaction and critical stress ratio with relative density were presented by Seed *et al.*<sup>[20]</sup> based on the study on porewater pressure changes during soil liquefaction. The effect of seismic history on liquefaction characteristics of saturated sands was studied by<sup>[21]</sup> with a concluding remark that the deposits of sand subjected to low magnitude earthquakes, which were not sufficiently strong to cause liquefaction, will develop an increased resistance to liquefaction in subsequent earthquakes even though, there may not be a significant change in density. Seed<sup>[11]</sup> developed a method to estimate liquefaction potential for sand under level ground conditions using standard penetration test data. This method was based on field performance data from sites, which either had or had not experienced liquefaction due to earthquake loading. Similar such research works on the liquefaction and its evaluation are extensively reviewed and presented<sup>[14,15]</sup>. The recent review on this very interesting topic is the work by Sawicki and Mierczynski<sup>[18]</sup>, wherein the authors reviewed historical developments of mechanics of saturated granular soils in relation to the liquefaction phenomenon, development of theoretical approaches to liquefaction-related problems and modeling. In the recent years studies on the micro mechanical behavior of granular assemblies, in relation to liquefaction and associated mechanisms, are carried by several researchers<sup>[16, 18, 22, and 23]</sup>.

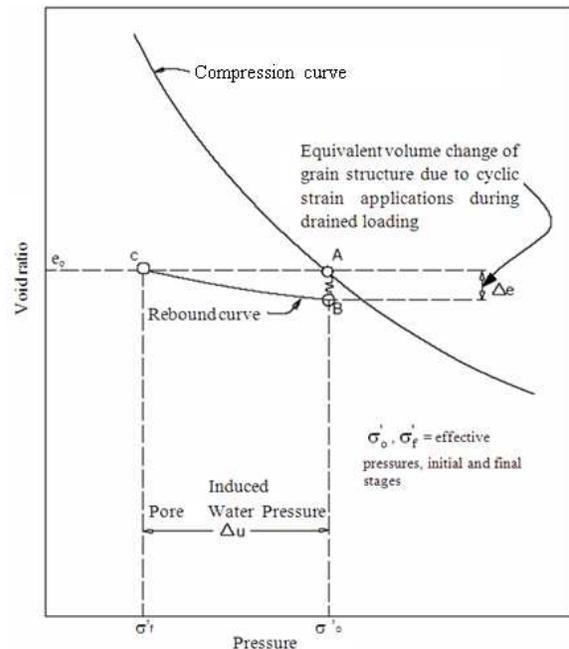


Fig. 1: Schematic illustration of mechanism of pore pressure generation during cyclic loading<sup>[11]</sup>

Table 1: Various ground improvement methods and their locations considered by Mitchell and Wentz<sup>[2]</sup>

Sr. No.	Name	Location	Soil conditions	Method	Year of treatment
1	Medical/dental clinic	Treasure Island	Hydraulic sand fill	Stone columns	1989
2	Office building No. 450	Treasure Island	Hydraulic sand fill	Sand compaction piles	1967
3	Facilities 487-489	Treasure Island	Hydraulic sand fill	Vibrocompactin	1972
4	Approach area, pier 1	Treasure Island	Hydraulic sand fill	Stone columns	1984
5	Building 453	Treasure Island	Hydraulic sand fill	Non-structural timber piles	1969
6	Esplanade extension	Richmond	Silty, sandy and gravelly fill	Stone columns	1986
7	East bay park condominiums	Emeryville	Silty sand fill	Vibrocompactin	1981
8	Harbor bay business park	Alameda	Hydraulic sand fill	Deep dynamic compactin	1985
9	Hanover properties	Union city	Silty sand fill	Deep dynamic compactin	1988
10	Kaiser hospital	South San Francisco	Hydraulic sand fill	Compaction grout	1978
11	Riverside avenue bridge	Santa Cruz	Sands and gravels	Chemical Grout	1986
12	Adult detention facility	Santa Cruz	Silty, sandy fill	Deep dynamic compactin	1978

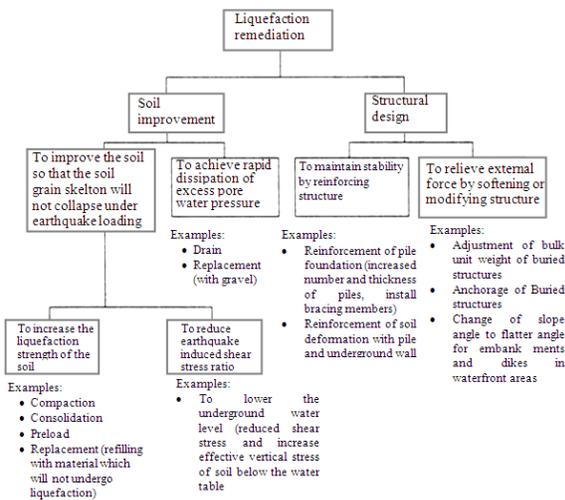


Fig. 2 Strategy for liquefaction remediation<sup>[25]</sup>

Various ground improvement methods that can be used as liquefaction counter measures can be classified into two broad categories<sup>[24]</sup>: (i) Prevention of liquefaction and (ii) Reduce the damage to structures due to liquefaction. The former one can be achieved by increasing the undrained cyclic strength as well as by improving the resistance to deformation or by dissipation of pore water pressure. The second one, reducing the damage, could be attained by strengthening the foundation of the structures and the ground supporting the structures to avoid reduction in bearing capacity or making the structures more flexible so that it can deform in accordance with the ground movement in case of buried structures<sup>[24]</sup>. Resistance to liquefaction can be improved by increasing the density, modifying the grain size distribution, stabilizing the soil fabric, reducing the degree of saturation, dissipation of the excess pore pressures generated and intercepting the propagation of excess pore pressures. PHRI<sup>[25]</sup> summarize (Fig. 2) the basic strategies for liquefaction remediation.

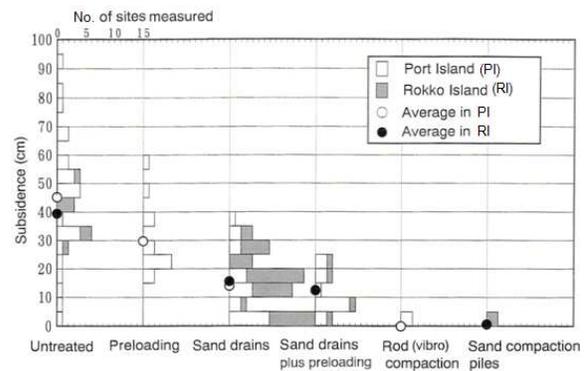


Fig. 3: Comparison of ground subsidence in zones treated with different methods<sup>[26]</sup>

The 1989 Loma Prieta earthquake experience provided the first opportunity to evaluate the behavior of treated ground that has been actually subjected to significant seismic shaking<sup>[2]</sup>. Twelve sites treated with different improvement methods prior to the earthquake were evaluated comprehensively by Mitchell and Wentz<sup>[2]</sup> (Table 1). They conclude that the procedures used for prediction of liquefaction were reliable and the ground improvement was very effective in mitigating liquefaction. Provision of gravel drains/granular piles/stone columns was the most commonly adopted ground treatment methodology for liquefaction mitigation which has proved its effectiveness in many instances.

Yasuda *et al.*<sup>[26]</sup> investigated the liquefied and not-liquefied subsoil conditions of two reclaimed islands in Kobe City after the 1995 Hyogoken-Nambu earthquake. Based on the study it was found that the non-liquefied zones had been improved by several methods, including sand compaction piles, rod (vibro) compaction, sand drains and preloading, before buildings had been constructed on them. Figure 3 depicts performance of different ground improvement methods in reducing the ground subsidence in the earthquake affected sites.

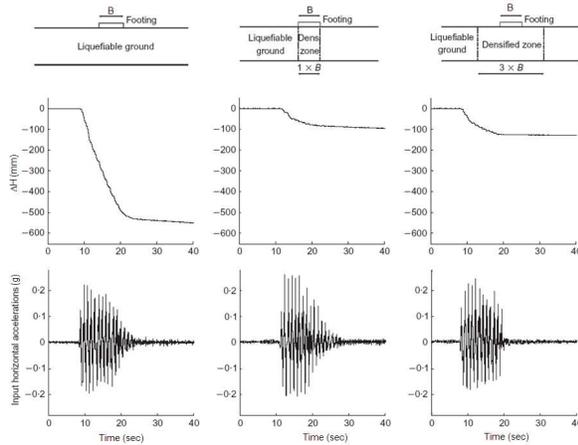


Fig. 4: Liquefaction-induced settlement of a bridge pier built with different in-situ densification widths<sup>[27]</sup>

It can be observed that the subsoil treated with columnar inclusions like sand compaction piles did not liquefy and nor subside even though the earthquake shaking was very strong. Madabhushi<sup>[27]</sup> discussed and showed the effectiveness of three different ground improvement methods, viz., in-situ densification, gravel drains and grouting, through dynamic centrifugal model tests. Figure 4 evidences the effectiveness of *in situ* densification and its extent.

Among the different ground improvement methods available, provision of columnar granular inclusions in the in-situ soil is considered to be the most effective method for liquefaction mitigation due to its ability to provide drainage facility in lowering the excess porewater pressures and strengthen the ground.

## MATERIALS AND METHODS

**Columnar granular inclusions as liquefaction counter-measure:** Columnar inclusions are of different types viz. sand drains, sand compaction piles; pre-fabricated vertical drains, granular piles or stone columns which are stiffer and stronger than the ambient soil, that can be installed in different arrays as shown in the Fig. 5. Installation of sand compaction piles in dynamic vibratory and static methods was discussed by Tsukamoto *et al.*<sup>[28]</sup> (Fig. 6). Theoretical background, analysis, design aspects and installation techniques for stone columns/granular piles were being developed since 1970s by various researchers and practitioners<sup>[29-34]</sup>. Granular piles are installed by vibro-compaction, vibro-replacement, cased borehole (rammed stone columns/rammed granular piles) or by simple auger methods<sup>[31,35]</sup>.

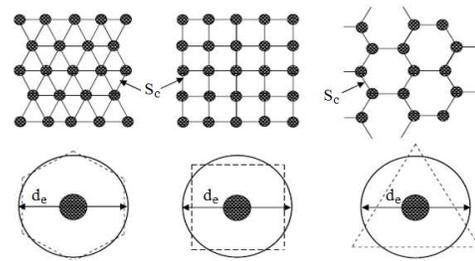


Fig. 5: Various arrangements of columnar inclusions and zones of influence; (a): Triangular; (b): Square; (c): Hexagonal

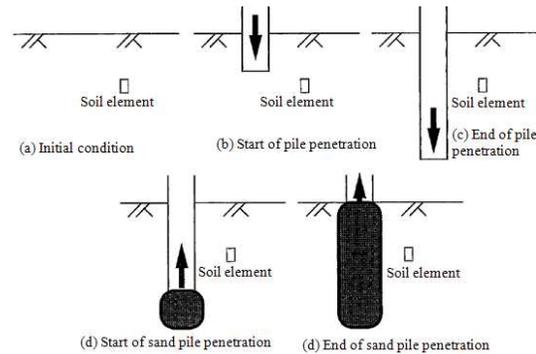


Fig. 6: Installation of sand compaction piles<sup>[28]</sup>

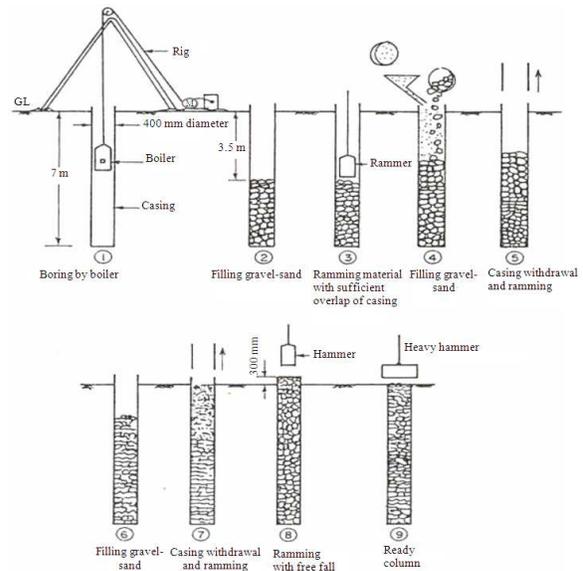


Fig. 7: Installation of stone columns-cased borehole method<sup>[36]</sup>

In cased borehole method<sup>[36]</sup>, granular piles are installed into the ground by full displacement methods and by ramming in stages, using a heavy falling weight, within a 'pre-bored casing' or 'driven closed end casing' and retracting the casing pipe stepwise (Fig. 7). In recent

years, use of encased columns as a ground improvement method is increasing. Granular inclusions are encapsulated in geosynthetic materials to increase the resistance to bulging<sup>[37-42]</sup>.

The pioneering work on gravel drains as a possible method to stabilize liquefaction susceptible soil deposit is reported by Seed and Booker<sup>[4]</sup>. Numerous publications<sup>[43-47]</sup> describe the use of stone columns for ground reinforcement and their potential to mitigate liquefaction.

Granular columnar inclusions (Granular piles) help in mitigating earthquake induced liquefaction effects through one or more of these functions or effects:

- Granular piles function as drains and permit rapid dissipation of earthquake induced pore pressures by virtue of their high permeability with the additional advantage that they tend to dilate as they get sheared during an earthquake event
- Pore water pressures generated by cyclic loading get dissipated almost as fast as they are generated due to significant reduction in the drainage path
- Granular piles density and reinforce the in-situ soil; improve the deformation properties of the ambient soil
- Granular piles, installed in to a very dense state, are not prone to liquefaction and replace a significant quantity of in-situ liquefiable soil
- Granular piles modify the nature of earthquake experienced by the *in situ* soil

### RESULTS

Adalier and Elgamal<sup>[48]</sup> reviewed the current state of stone column technologies as a liquefaction countermeasure. Sasaki and Tinaguchi<sup>[49]</sup> conducted large scale shaking table test on gravel drain system as a liquefaction counter-measure. Figure 8 shows different model configurations considered and typical distribution of the pore water pressures<sup>[49]</sup>. Adalier *et al.*<sup>[50]</sup> performed a series of highly instrumented dynamic centrifuge model tests (Fig. 9) to evaluate effectiveness of stone columns in mitigating the liquefaction potential of non-plastic silty deposits. Al-Homoud and Degen<sup>[47]</sup> present an introduction to earthquake-resistant design of marine stone columns. Similar studies on different types of granular columnar inclusions include<sup>[51,28,52]</sup> on sand compaction piles<sup>[53]</sup> on prefabricated vertical drains<sup>[27,54,55]</sup> on gravel drains.

**Analytical studies on columnar inclusions as liquefaction remedial measure:** Seed and Booker<sup>[4]</sup> were the first to propose an analytical model for the generation and dissipation of pore pressure in a soil

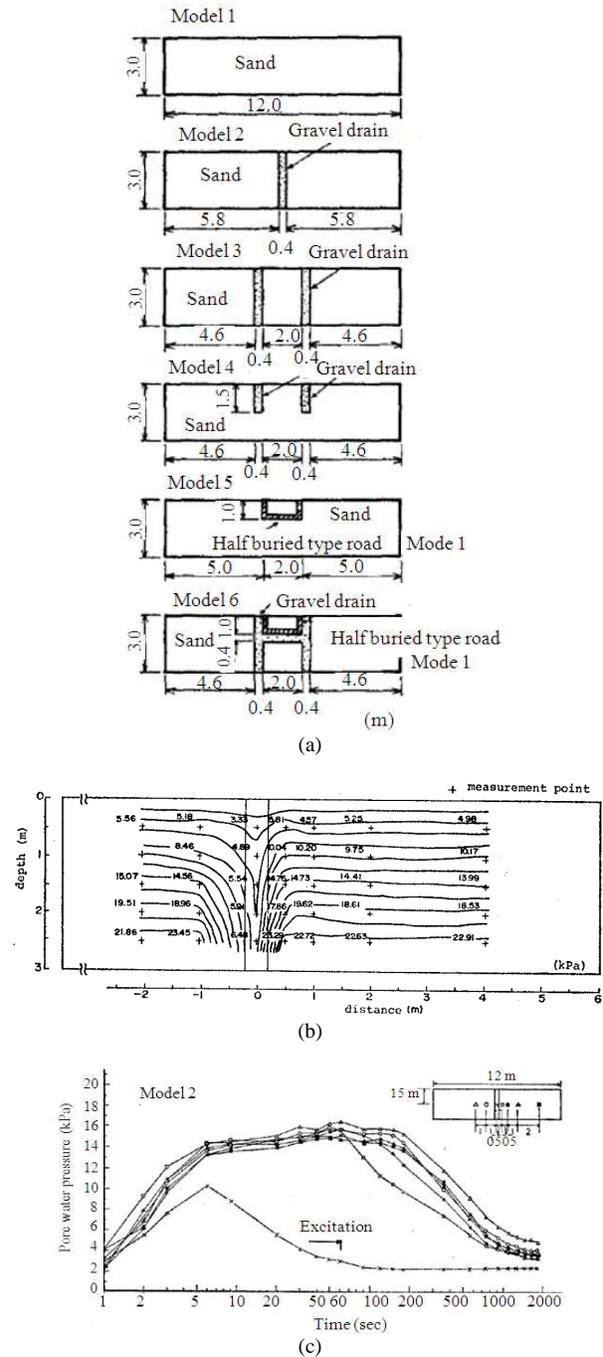


Fig. 8: (a): Different models used; (b): Distribution of pore-water pressure for model 2 after 20 sec; (c): Variation of generation and dissipation of pore water pressures for model 2<sup>[49]</sup>

deposit with vertical drains. Under the assumptions of purely radial drainage, constant coefficient of compressibility and infinite permeability of drains,

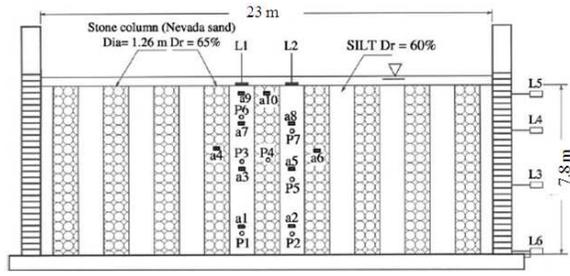


Fig. 9: Typical model configuration considered for centrifugal model studies<sup>[50]</sup>

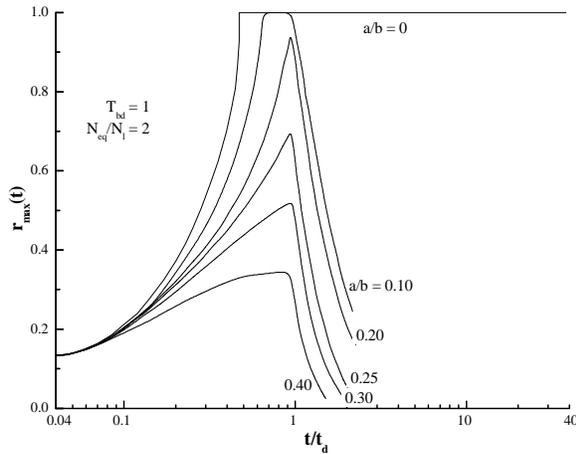


Fig. 10: Effect of drain diameter and drain spacing on maximum pore pressure ratio<sup>[4]</sup>

design charts (Fig. 10) were developed to evaluate the effects of drain diameter and spacing for the expected earthquake loading on excess pore pressure ratio. For flow into a gravel drain, assuming pure radial flow and constant coefficients of permeability ( $k_h$ ) and volume compressibility ( $m_v$ ), the governing equation can be written as<sup>[4]</sup>:

$$\frac{k_h}{\gamma_w \cdot m_v} \left( \frac{1}{r} \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial r^2} \right) = \frac{\partial u}{\partial t} - \frac{\partial u_g}{\partial N} \cdot \frac{\partial N}{\partial t} \quad (1)$$

Where:

- $u$  = The excess pore pressure at a radial distance,  $r$ , from the centre
- $t$  = Time,  $\gamma_w$  - the unit weight of water
- $u_g$  = Peak excess hydrostatic porewater pressure generated by the earthquake

The rate of generation of pore pressure during an earthquake event is defined by:

$$\frac{\partial u_g}{\partial N} = \frac{\sigma'}{a\pi N_1} \frac{1}{\sin^{2a-1} \left( \frac{\pi}{2} r_u \right) \cos \left( \frac{\pi}{2} r_u \right)} \quad (2)$$

Where:

- $r_u = u/\sigma'_o$  = The pore pressure ratio
- $\sigma'_o$  = The initial mean bulk effective stress for axi-symmetric conditions or the initial vertical effective stress for simple shear conditions
- $N_1$  = The number of cycles required to cause liquefaction
- $\alpha$  = An empirical constant which is a function of the soil properties with a typical average value of 0.7

The irregular cyclic loading induced by an earthquake is converted<sup>[56]</sup> to an equivalent number,  $N_{eq}$ , of uniform cycles at an amplitude of 65% of the peak cyclic shear stress, i.e.,  $\tau_{cyc} = 0.65\tau_{max}$ , occurring over a duration of time,  $t_d$  and:

$$\frac{\partial N}{\partial t} = \frac{N_{eq}}{t_d} \quad (3)$$

Tokimatsu and Yoshimi<sup>[57]</sup>, Sasaki and Taniguchi<sup>[49]</sup> and Onoue<sup>[58]</sup>, report results similar to those of Seed and Booker<sup>[4]</sup> taking into consideration additional factors such as well resistance (finite permeability of gravel drain) and drain slenderness ratio (slenderness ratio:  $L/r$ , where  $L$  is the length and  $r$  the radius of the gravel drain). Pestana *et al.*<sup>[59,60]</sup> analyzed the provision of a reservoir to minimize the drain resistance to flow in to the drain. Poorooshasb *et al.*<sup>[61]</sup> propose an equivalent coefficient permeability,  $k_{eq} = k_{untr} \cdot t_{50}$  (for untreated ground)/ $t_{50}$  (for treated ground), for the treated soil, in terms of the permeability,  $k_{untr}$ , of untreated ground and  $t_{50}$  values for the untreated and granular pile treated ground are the times for 50% degree of consolidation based on one dimensional and radial consolidation theories respectively. Dilation effect on the drainage function of granular piles was studied by Madhav and Arlekar<sup>[62]</sup>. The densification effect of granular piles in improving deformation properties of the ambient soil was studied by Murali Krishna and Madhav<sup>[63]</sup> and Murali Krishna *et al.*<sup>[64,65]</sup>.

Murali Krishna *et al.*<sup>[64]</sup> incorporated the densification effect of granular piles, with respect to variation of flow parameters from the centre of the granular pile, in the analysis of pore pressure generation and dissipation that was originally developed by<sup>[4]</sup>.

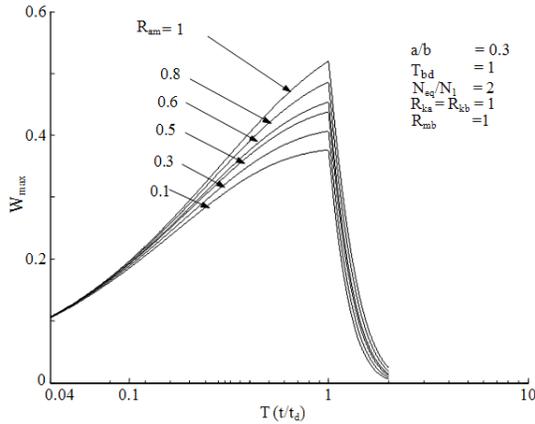


Fig. 11: Effect of  $R_{ma}$  on  $W_{max}$  [64]

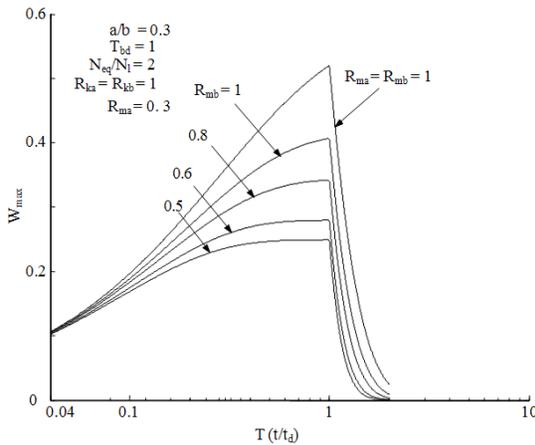


Fig. 12: Effect of  $R_{mb}$  on  $W_{max}$  [64]

The modified form of the governing Eq. 1 with the inclusion effects of densification is:

$$\frac{k_h(r)}{\gamma_w \cdot m_v(r)} \left( \frac{1}{r} \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial r^2} \right) + \frac{1}{\gamma_w \cdot m_v(r)} \cdot \frac{\partial(k_h(r))}{\partial r} \cdot \frac{\partial u}{\partial r} = \frac{\partial u}{\partial t} - \frac{\partial u_g}{\partial N} \cdot \frac{\partial N}{\partial t} \quad (4)$$

In this case coefficients of permeability,  $k_h(r)$  and volume change,  $m_v(r)$ , are functions of radial distance,  $r$ , from the point of densification and degree of densification. Murali Krishna *et al.* [64] studied the densification effect with respect to the coefficients of permeability and volume change at the near and at the farthest ends of the granular pile, individually and together, on maximum pore pressure variations during an earthquake event. Figure 11 and 12 show the densification effect on maximum pore pressure ratio with respect to coefficient of volume change at the near and farthest ends respectively.  $R_{ma}$  and  $R_{mb}$  are normalized coefficients of volume change due to densification at the near and farthest ends respectively.

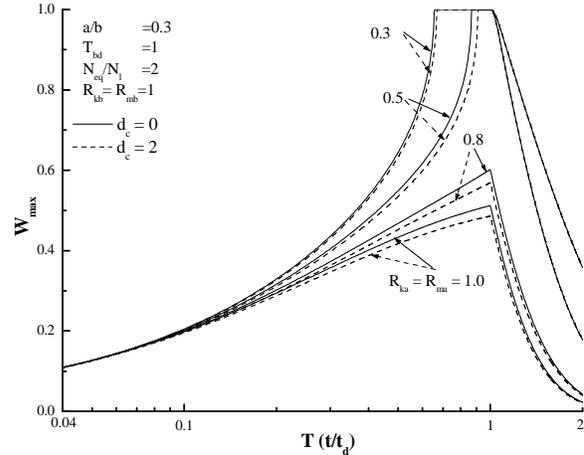


Fig. 13: Effect of densification with respect to  $R_{ka}$  and  $R_{ma}$  and dilation on  $W_{max}$  [66]

Murali Krishna and Madhav [66] combined both the densification and dilation effects and incorporated them in the analysis of pore pressure generation and dissipation. They also verified the effect of variation of permeability with distance on maximum pore pressure ratios and concluded that the pore pressures ratios are not sensitive to the type of variation of permeability with distance. Figure 13 shows the effect of densification with respect to flow parameters at the near end in addition to the dilation effect. It is seen from the Fig. 13 that the dilation effect reduces the negative effect of reduced permeability.

## DISCUSSION

Densification effect on the coefficient of volume change is positive in that the maximum induced pore water pressure ratios get reduced and sensitive to the type of variation considered as pore pressure ratios are lesser for the exponential variation compare to linear variation. Densification effect, on the coefficient of permeability alone or in addition to effect on coefficient of volume change, increases the maximum pore water pressure ratios giving a negative effect. The pore pressures ratios are not sensitive to the type of variation of permeability with distance. Densification effect on both coefficients of permeability and volume change result in a either slightly negative or positive effect depending on the degree of densification.

Further research is essential in the area of columnar granular inclusions as liquefaction countermeasure especially regarding encased granular columns.

## CONCLUSION

Liquefaction is the most disastrous feature during an earthquake that causes huge loss and damage to various structures founded on or in the ground. Ground improvements are extensively used to reinforce the *in situ* ground and also as liquefaction countermeasures. Columnar granular inclusions are the most widely used remedial measures against the liquefaction. Columnar granular inclusions provide drainage to mitigate the liquefaction potential of the ground. Various mechanisms such as reinforcement, densification, dilation along with the drainage mitigate the damages due to liquefaction. The study presents an over-view of the recent findings on the topic.

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