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# Rarefied Gas Flow Modeling inside Rotating Circular Cylinder

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**Abstract:** In this study a rarefied gas flow inside a rotating circular cylinder by means of direct simulation Monte Carlo (DSMC) method is investigated. A rarefied gas is supposed to be contained in a cylindrical domain, bonded by a rotational cylinder with smooth and diffuse-reflection surface. The top and bottom ends of domain is assumed to be speculary-reflecting. Special attention is focused on the effect of parameter L/D, where L and D are the cylinder length and diameter, respectively. The investigation on the formation of various kinds of flow patterns in the range of various values of the L/D are important aims of the present work. Finally computed results such as density fields and velocity profiles are shown and discussed.

**Key words:** Rarefied gas, DSMC method, Rotating cylinder

## INTRODUCTION

This research is carried out as part of numerical simulation of centrifugation process using the principle of a cylinder rotation at high speed. However, the study of this problem with different emphases[1,2,3] have been carried out by using direct simulation Monte Carlo (DSMC) method[4]. It was concerned generation of rotating flow and contraction of the rotating flow. Schematically, in the flow field we can distinguish two zones. The first one, dense and close to the rotational wall and second one, rarefied and close to the center of the cylinder. In the fluid calculations, in the both domain, the Boltzmann's equation[5] is governed and solved by the DSMC method. We have investigated the transport phenomena in rotating rarefied gas on the basis of the kinetic theory of gases. Hence the field of density, velocity and viscous flow are calculated. From a technological viewpoint, it is essential to understanding the effects of geometry of rotating cylinder (L/D ratio) on the flow patterns. Existence of various vortex flow was demonstrated and was shown in the next sections.

## MATERIALS AND METHODS

**Problem And Basic Formulation:** We consider a rarefied gas inside a rotating cylinder with radius R. The cylinder rotate at angular velocity Ω. The bottom and top ends of cylinders are covered with plates located at z = 0 and z = L, respectively. Thus, we consider a cylindrical domain $0\le r\le R$, $0\le\theta\le 2\pi$ and $0\le z\le L$. The cylinder is rotating around z-axes at surface velocities $V\theta = \Omega R$ in the θ direction. Cylinder surface is kept at a temperature $T_0 = 273°K$. We will investigate the behavior of the gas numerically on the basis of kinetic theory under the following assumption: 1) the flow field is axisymmetric, 2) the gas molecules are Hard-Sphere undergo diffuse reflection on the surface of the cylinder and specular-reflection on the bottom and top boundaries; 3) the speed of rotation of cylinder is high; 4) the system has Knudsen number $Kn_0 = 10$. Here, $Kn_0 = \lambda_0/R$ is the Knudsen number with $\lambda_0$ being the mean free path of the gas molecules in the equilibrium state at rest with temperature $T_0$ and density $\rho_0$.

The cylinder radius R, is assumed the reference length in the Knudsen number evaluation. As the solution method, we use the direct simulation Monte Carlo (DSMC) method by Bird[4]. As in the usual DSMC computation, we obtain the steady flow field as the long-time limit, pursuing the long-time behavior of the solution of the time-dependent boundary-value problem with an appropriately chosen initial condition. We consider the Boltzmann equation[6,7]:

$$\frac{\partial f}{\partial t} + \upsilon.\nabla_x f = \frac{1}{\varepsilon} Q(f,f) \qquad (1)$$

Supplemented with the initial condition

$$f(x,\upsilon,t=0) = f_0(x,\upsilon) \qquad (2)$$

where, $f = f(x,\upsilon,t)$ is a non negative function describing the time evolution of the distribution of particles witch

moves with velocity $\upsilon$ in the position x at time $t\rangle 0$. The parameter $\varepsilon\rangle 0$ is the Knudsen number and is proportional to the mean free path between collisions. The bilinear collision operator Q (f, f) describes the binary collisions of the particles and is given by:

$$Q(f,f)(\upsilon) = \int_{R^3}\int_{S^2} \delta(|\upsilon - \upsilon_1|,\omega) [-f(\upsilon)f(\upsilon_*)]d\omega d\upsilon_* f(\upsilon')f(\upsilon'_*) \quad (3)$$

In the above expression, $\omega$ is a unit vector of the sphere $S^2$, so that $\omega$ is an element of area of the surface of the unit sphere $S^2$ in $R^3$. Moreover $(\upsilon',\upsilon'_*)$ represent the post-collisional velocities $(\upsilon,\upsilon_*)$ and the collision parameter $\omega$ i.e.,

$$\upsilon' = \frac{1}{2}(\upsilon + \upsilon_* + |\upsilon - \upsilon_*|\omega)$$
$$\upsilon'_* = \frac{1}{2}(\upsilon + \upsilon_* + |\upsilon - \upsilon_*|\omega) \quad (4)$$

The kernel $\delta$ is a nonnegative function which characterize the details of the binary interactions.

In the case of inverse Kth power forces between particulars the kernel has the form:

$$\delta(|\upsilon - \upsilon_*|,\theta) = b_\alpha(\theta)|\upsilon - \upsilon_*|^\alpha \quad (5)$$

Where $\alpha = (K-5)/(K-1)$ for numerical purpose, a widely used model is the Variable Hard Sphere (VHS), corresponding to take $b_\alpha(\theta) = C_\alpha$, where $C_\alpha$ is a positive constant. The case $\alpha = 0$ is referred to as the Maxwellian gas, where the case $\alpha = 1$ yields the Hard Sphere gas.

During the evolution process, the collision operator preserves mass, momentum and energy, i.e.,

$$\int_{R^3} Q(f,f)\phi(\upsilon)d\upsilon = 0$$
$$\phi(\nu) = 1,\upsilon,\upsilon^2 \quad (6)$$

And in addition it satisfies Boltzmann's well-known H-theorem

$$\int_{R^3} Q(f,f)\log(f)d\upsilon \le 0 \quad (7)$$

From a physical points of view, Boltzmann's H-theorem implies that any equilibrium distribution function, i.e., any function f which Q (f, f) = 0, has the form of a locally Maxwellian distribution

$$M(\rho,u,T)(\upsilon) = \frac{\rho}{(2\pi T)^{3/2}} \exp\left(-\frac{|u-\upsilon|^2}{2T}\right) \quad (8)$$

Where $\rho$, u, T are the density, mean velocity and temperature of gas defined by

$$\rho = \int_{R^3} f d\upsilon u = \frac{1}{\rho}\int_{R^3} \upsilon f d\upsilon, T = \frac{1}{3\rho}\int_{R^3}[\upsilon - u]^2 f d\upsilon \quad (9)$$

As $\varepsilon \to 0$ the distribution function approaches the Maxwellian. The higher order moments of the distribution f can be computed as function of $\rho$, u and T, by using (8) and we obtain the closed system of governing equation[7].

**Flow Conditions:** More specifically, we restrict ourselves to the cases where L/R = 1, 2, 3 The peripheral speed $V_\theta$ is $V_\theta$ = 600 m $s^{-1}$ and we investigate the flow patterns in each of cases.

Different grids were used to check the sensitivity of the results to the grid parameters and to the extent of the computational domain. Thus, the square cross section of computational domain ($0 \le r \le R$, $0 \le z \le L$) is divided into various numbers of square cells of a uniform size and time step $\delta t$ is $\delta t = 0.5\times10^{-8}$.

It was supposed (rather arbitrary) that the working gas was Argon, characterized by a specific heat ratio $\gamma$ = 5/3 (mono atomic). Considering as a Hard-Sphere gas, with a molecular diameter equal to $d = 4.11\times10^{-10}$ and a molecular mass $m = 6.634\times10^{-26}$ kg $m^{-3}$. It's viscosity is given by Bird[4] as:

$$\mu = \frac{5}{16d^2}\sqrt{\frac{mkT}{\pi}} \quad (10)$$

where, k is the Boltzmann constant.

**DSMC Calculations:** Direct Simulation Monte Carlo (DSMC) method calculations were carried out using the code DSRF (Direct Simulation of Rarefied Flows) developed at the our laboratories in Urmia university, based on the idea of the Bird[4].

The problem definition scheme is shown in Fig.1. The schemes of the grids is shown in Fig. 2 and the interaction of the grid with the cylinders surface was used to divide faces into surface elements.

It is generally admitted that cell size must be small compared with both the gradient length scale and the local mean free path. The first condition is required to ensure a correct space resolution. The second condition

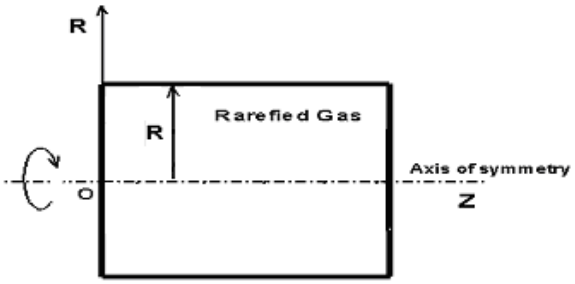


Fig. 1: Definition of the problem

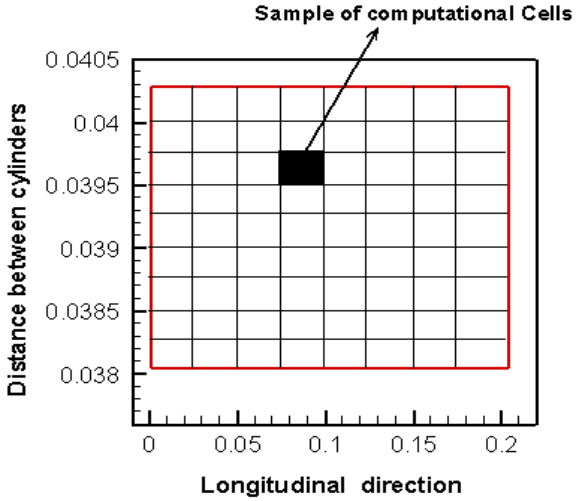


Fig. 2: Schemes of computational domain

is based on the fact that collision partners are chosen within the same cell, irrespective of their actual position[8,9]. Thus the cell should have dimensions smaller than a mean free path to prevent unrealistic collisions to take place.

In fact when the first condition is satisfied, collision parameters are sampled with the correct distribution function, even when they are more than one mean free path apart. An additional condition is due to the fact that computational collisions take place at the end of the molecular moving rather that any time during time interval δt.

## RESULTS AND DISCUSSION

Fig. 3 shows the generation of a double-vortex flow in the flow field. In this case L/D=1 and two vortices are close to each other. The density contour also shown in the Fig. 4.

At the L/D = 2, the distance (along z-axes) of generated vortices increases (Fig. 5) and internal flow is

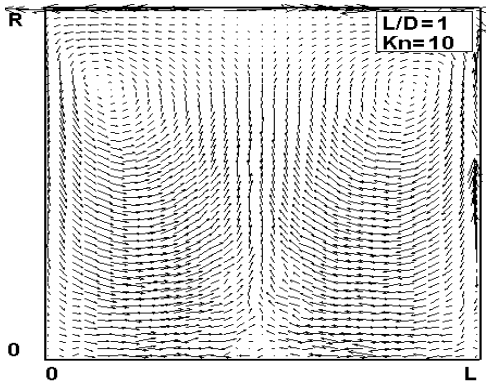


Fig. 3: Flow field of double-vortex Flow at L/D = 1

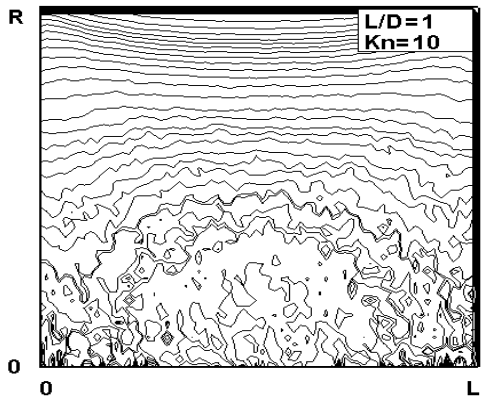


Fig. 4: Density contour at L/D = 1

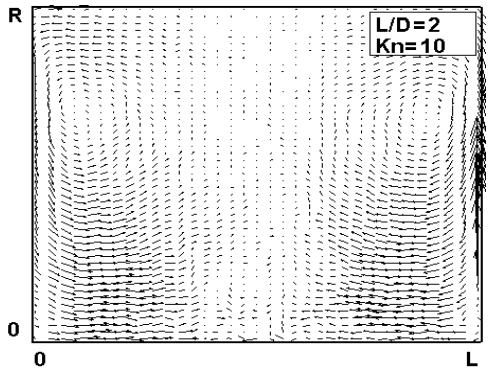


Fig. 5: Flow field of double-vortex Flow at L/D = 2

affected by the strength and rotation directions of two vortices. In this case the density field is different from the last case and shown in Fig. 6.

When the L/D = 3 the effects of generated vortices on each other is less than latest case. In this case higher vortex flow is nearly confined to the top and bottom ends of cylindrical domain (Fig. 7).

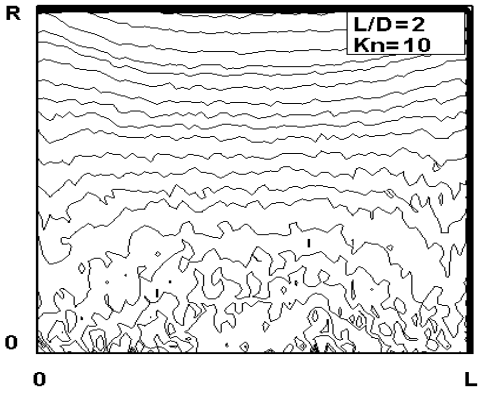


Fig. 6: Density contour at L/D = 2

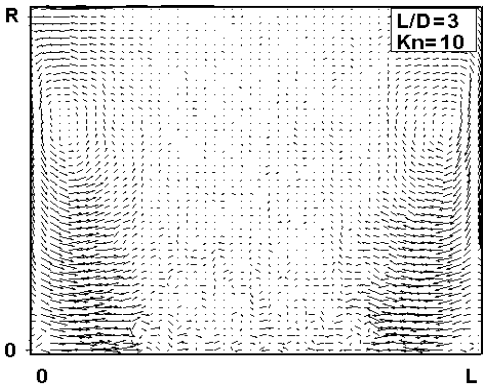


Fig. 7: Flow field of double-vortex Flow at L/D = 3

## CONCLUSIONS

Numerical simulation of rarefied gas flow in a cylindrical domain was carried out using DSMC method. We found that flow patterns change with various values of L/D of cylinder. As the L/D increases, the generated vortices tend to move toward the top and bottom ends of cylindrical domain. Thus, it is the cause of creating a complex vortex flow and this flow field impose a valuable external force on the cylinder surface.

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