

Peak to Average Power Ratio Reduction of Orthogonal Frequency Division Multiplexing System with a Significant Low Complexity

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ABSTRACT

The Orthogonal Frequency Division Multiplexing (OFDM) has been widely used in many applications in the last decade. It provides good benefits compared to the single carrier systems. However, it has a bad property represented by the high-output Peak to Average Power Ratio (PAPR) which is considered as its major problem. There are many techniques to reduce the PAPR values such as the best one called Selected Mapping (SLM) method. In this study, a novel technique has been used to reduce the PAPR and it depends on a cascade of Inverse Fast Fourier Transform (IFFT). The analysis proved that the computational complexity has been reduced significantly and the simulation results showed that the PAPR reduced also without affecting the Bit Error Rate (BER).

Keywords: CCDF, OFDM Modulation, Peak to Average Power Ratio (PAPR), Selected Mapping (SLM)

1. INTRODUCTION

In the last ten years, the communications devices have been improved very much and their implementation complexity has been enhanced also when they started to use in their cores the OFDM systems. The OFDM is a very important technique which reduces the wireless system's complexity, increases the bit rate, reduces the Intersymbol Interference (ISI) and even it doesn't need an equalizer as compared to the single carrier modulation techniques.

However, the major problem of the OFDM system can be addressed to be the PAPR-problem. In this regard, there are lot of methods to enhance the problem such as; the SLM-method (Bgoml *et al.*, 1996), the Partial Transmitted Sequence (PTS) (Tsai and Huang, 2008), Amplitude Clipping and filtering (AC) (Al-Kebsi, 2008), the All Pass Filter (APF) (Hong and Har, 2010), Tone Reservation (TR) and

Tone Injection (TI) (Wattanasuwakull and Benjapolakul, 2005), Block Coding (BC) (Jones *et al.*, 1994). These mentioned techniques are divided into two categories; blind and non-blind. The blind techniques are those did not need to send side information (attached with the OFDM-symbol) to the receiver to be able to re-produce the transmitted data such as; AC, TR, TI and the BC which will reduce the bit rate significantly. AC-method is the simplest one among the blind category, but it brings out a high BER for the system. TR and TI techniques are complex techniques which they need to find the best tones to be used to reduce the PAPR. The second category is the non-blind methods like; SLM, PTS, APF methods. The PTS is the most complicated technique between them, but it gives good reduction in the PAPR. The SLM needs less computational complexity than the PTS with the same side information and the same reduction in the PAPR. The APF-technique is the modern technique, but

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it also needs more complexity beside the side information.

However, the side information which must be sent to the receiver will reduce the bit rate also. Anyhow, in this study, cascade blocks of IFFT processes will be used with attaching the mean of the first block to the second block which will give an accepted reduction (around 6 dB) without affecting the BER of the system. This method is an expansion to our previous work in which only cascade of IFFT blocks had been used to reduce the PAPR of the Multicarrier Code Division Multiple Access (MC-CDMA) system (Taher *et al.*, 2011). A complete explanation for this method will be shown in section 3 with the computational complexity calculations and comparison.

1.1. Basics OF OFDM System

OFDM is a multicarrier modulation technique. It increases the bit rate very much because it will divide the carrier to subcarriers and put in each subcarrier a bit of data. So, it is a multicarrier system and it is a direct implementation to the Inverse Discrete Fourier Transform (IDFT) or using the Inverse Fast Fourier Transform (IFFT) because of its ability to do the computations. In the mathematical form, it can be expressed as Equation 1:

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j2\pi \frac{nk}{N}} \tag{1}$$

where the time and frequency index $n, k = 0, 1, \dots, N-1$ and $X(k)$ is being up-sampled (Taher *et al.*, 2011). The PAPR will appear because of the elements of $x(n)$ will be added to each other, which will results in a high PAPR which can be defined as Equation 2:

$$PAPR = \frac{\max \left\{ |x(n)|^2 \right\}}{E \left[|x(n)|^2 \right]} \tag{2}$$

Due to the large number of the elements in $x(n)$, the central limit theorem tells us that the distribution will be approaches the Gaussian distribution, so that, the Cumulative Complementary Distribution Function (CCDF) of the PAPR will be the best technique to present it. This function can be defined as Equation 3:

$$CCDF(PAPR) = \Pr(PAPR > PAPR_o) \tag{3}$$

where, $PAPR_o$ is the clipping level. The most appropriate technique to reduce the PAPR levels is the SLM. The SLM-technique can be best described as; the $X(n)$ packets symbol will be copied U -times. Each copy will be multiplied (bitwise) by its assigned phase rotation vector; the resulted OFDM symbol with the lowest PAPR will be transmitted with its corresponding phase rotation vector as side information. In the previous study (Taher *et al.*, 2011), cascaded IFFT blocks have been used. An expansion to (Taher *et al.*, 2011) is: The mean of the first block of IFFT will be attached to the end of the second IFFT block as will be shown in the next section.

2. MATERIALS AND METHODS

Before we start introducing the proposed technique, the principle idea behind it will be introduced first, so that, the suggested method will be more understandable. Hence; the relationship between the upper bound of the PAPR and the input samples $X(n)$ has been presented in (Tellambura, 1997). Suppose the aperiodic autocorrelation coefficients are given by Equation 4:

$$\eta(r) = \sum_{n=1}^{N-r} X(n+r) X^*(n) \text{ for } r = 0, 1, \dots, N-1 \tag{4}$$

So, the upper bound of the PAPR will be (Tellambura, 1997) Equation 5:

$$PAPR \leq 1 + \frac{2}{N} \sum_{r=1}^{N-1} |\eta(r)| \tag{5}$$

Hence, it is clear from the last expression that when the second term of the right side benign reduced, the PAPR will be reduced. The suggested method will reduce this second term using the IFFT block. Now, let's introduce our technique; for the first IFFT block and according to Parseval's relation, we have Equation 6:

$$\sigma_1^2 = \frac{1}{N} \sum_{n=0}^{N-1} |X(n)|^2 = \sum_{m=0}^{N-1} |S(m)|^2 \tag{6}$$

where, $S(m)$ is the output samples of the first IFFT processes (time domain). The second IFFT processes will reduce this variance by the factor $1/N$ as follows (applying Parseval's relation also) Equation 7:

$$\sigma_2^2 = \sum_{k=0}^{N-1} |x(k)|^2 = \frac{1}{N} \sum_{m=0}^{N-1} |S(m)|^2 = \frac{1}{N} \sigma_1^2 \tag{7}$$

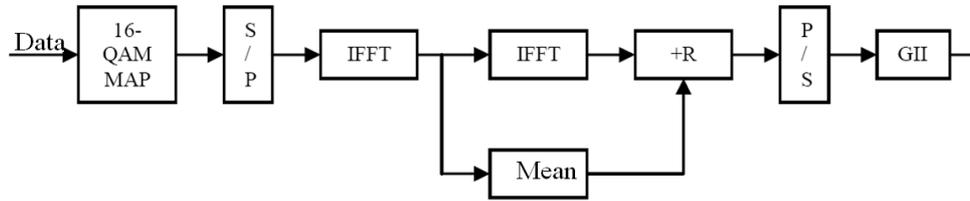


Fig. 1. The suggested system showing where to attach the mean

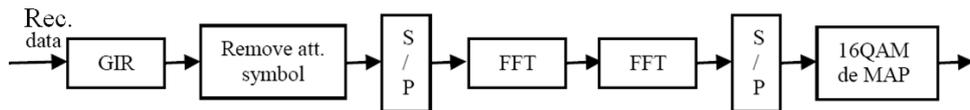


Fig. 2. Reversing the transmitter to give the receiver.

Hence, this proves that the value of $\sum_{r=1}^{N-1} |\eta(r)|$ will be reduced, so that, the PAPR will be reduced accordingly. **Figure 1** shows the suggested system which will be simulated throughout this study. It is showing that the data will be mapped using 16-QAM then serial to parallel them. After this step, the first block of the IFFT blocks was put, the results will be passed to the second IFFT block and the mean of the first block will be attached to the results of the second block.

This can be expressed in the following simple Equation 8:

$$x(n) = \{ \text{IFFT} [\text{IFFT} (X(k))] R \}^T \tag{8}$$

where, R is the sample produced by the mean process to the first IFFT block which can be given as Equation 9:

$$R = \frac{1}{N} \sum_{k'=0}^{N-1} X'(k') \tag{9}$$

where, $X'(k')$ is the same as that in Equation 1 which is re-written here Equation 10:

$$X'(k') = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j2\pi \frac{kk'}{N}} \tag{10}$$

According to this fashion, the computational complexity has been reduced compared to the SLM-method which needs U-times of IFFT blocks. At the receiver, the above steps should be followed reversely as shown in **Fig. 2**.

From **Fig. 2**, the first step is to remove the last sample which has been attached at the receiver then a

serial to the parallel converter is followed. Now the cascade Fast Fourier Transform (FFT) blocks, after that, the parallel to the serial converter to pass it to the 16-QAM de-mapping. The total number of multiplication operations will be lower than the SLM as well as the number of addition operations had been reduced as shown in **Fig. 1** where there are N operations of summations (to compute the mean) and 2-IFFT blocks, hence, $A_{cas} = N + 2N \log_2 N$, so Equation 11:

$$A_{cas} = N(1 + 2 \log_2 N) \tag{11}$$

While the multiplication's operations are only in the cascaded blocks of the IFFT blocks as each block has $N/2[\log_2 N]$, then Equation 12:

$$M_{cas} = N \log_2 N \tag{12}$$

To compare Equation 11 and 12 with the SLM-method, we need to have the following two Equation 13:

$$A_{SLM} = U \cdot N \log_2 N \tag{13}$$

And for the multiplications operations are Equation 14:

$$M_{SLM} = U \cdot N \left(1 + \frac{1}{2} \log_2 N \right) \tag{14}$$

Besides this gain in the computational complexity, the PAPR has been reduced without affecting the BER as will be shown in the next section. Anyhow, the suggested method did not have a limitation to be used with any order of mapping or a mapping family itself

such as the M-PSK. It can be proven it can be used with the M-PSK mapping using the same derivation show above (Equation 4-7). To show the brute force of the method, let's see the simulated results in the following section.

3. RESULTS AND DISCUSSION

In this study, the simulation will be run for 16-QAM mapping, subcarriers $N = 256$. The number of simulated OFDM symbols was more than 36000 symbols. **Figure 3** depicts the PAPR results. It is seen in **Fig. 3** that the PAPR has been reduced significantly to 6 dB while the original system gives more than 12 dB. The

computational complexity has been reduced also without affecting the BER behavior as drawn in **Fig. 4**.

It is shown that both the original and the proposed method match each other as that if the SLM-method has been applied. However, in the current situation, the proposed method did not need to send any side information as needed by the SLM-technique, where the number of bits needed as side information will be $\log_2 U$. This side information will reduce the bit rate of the system. This can be considered as another achievement for the suggested method.

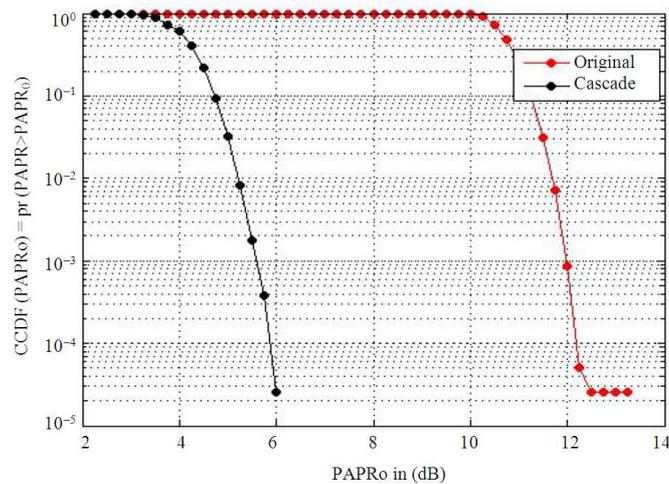


Fig. 3. PAPR results for the original-and the proposed-OFDM systems

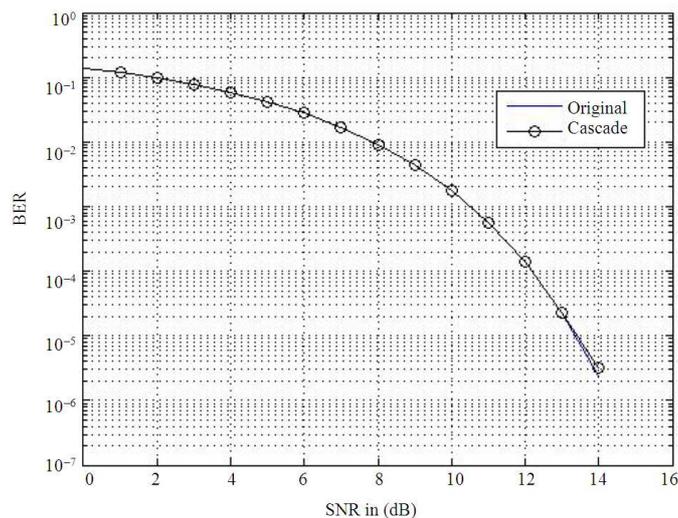


Fig. 4. BER for the proposed and the original systems

On the other hand, the computational complexity can be calculated for $N = 256$, the number of phase rotation vectors of the SLM-method $U = 8$, then, from Equation 11, the number of addition operations are 4352 and using Equation 13, there will be 16384 operations of additions for the proposed-and the SLM-methods respectively. From which, around 73.4% reduction in the number of addition operations has been achieved. The number of multiplications operations is 2048 and 10240 using Equation 12 and 14 for the novel and the SLM methods respectively. Hence, an 80% reduction in the number of multiplications has been obtained. The number of bits of the side information using the suggested method was zero while for the SLM-technique it was 3 bits.

4. CONCLUSION

A novel method to reduce the PAPR of the multicarrier system represented by the OFDM system has been proposed. The proposed method is a cascade of two blocks of the IFFT processes with adding the mean of the first IFFT block to the end of the symbol which produced by the second block. The PAPR has been reduced more than 6 dB and the BER did not change as known in the SLM-method but, the SLM will produce some side information to be sent to the receiver. The computational complexity has been enhanced to about 73.4 and 80% for the addition and multiplication operations respectively.

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