Original Research Paper

Modelling the Error Term of Australia Gross Domestic Product

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Abstract: The main aim of this study is to model the Gross Domestic Product (GDP) with the new Combine White Noise (CWN) Model and compare the results with the Vector Autoregressive (VAR) Model and Exponential Generalized Autoregressive Conditional Heteroscedasticity (EGARCH) Model which are the existing models. The CWN model estimation yields best results with least information criteria and high log likelihood values. While the EGARCH model estimated yields better results with least information criteria and high log likelihood values when compared with VAR model. CWN has the least forecast errors which are indications of best results when compare with the EGARCH and VAR models, dynamic evaluation forecast errors. The minimum forecast error values indicate forecast accuracy. The determinant of the residual of the covariance matrix value indicates that CWN is efficient, while the determinant of the residual of the covariance matrix value indicates that VAR is not efficient. The total results testify that CWN is the most right model. To model the data that exhibit conditional heteroscedasticity with leverage effect in Australia and other societies in the world efficiently, CWN is recommended.

Keywords: Combine White Noise, Determinant of the Residual of Covariance, Efficient, Forecast Accuracy, Log Likelihood

Introduction

The main aim of this study is to model the Gross Domestic Product (GDP) with the new Combine White Noise (CWN) Model which is considered as the most suitable for data that exhibits stochastic time series errors (heteroscedastic errors). GDP is the total value of all the goods and services produced within a country's borders in a given time. Thus, GDP is an indicator of the economic health of a country and is a gauge of a country's standard of living (Hubbard and O'Brien, 2012). When the standard of living is high, it determines the economic health of the nation and reflects the well-being of the citizens. Economic growth is the increase in the market value of the goods and services produced by an economy over time. Economic growth, measured as a change in the GDP as defined in Hubbard and O'Brien (2012).

In order to have a good measure of the standard of living in a society, there is a need for a suitable model that will yield better results for suitable forecast and policy making. Error term described the errors exhibited by the empirical model (Qin and Gilbert, 2001). The actions of the error term in the stochastic time series rely mostly on the data size and high data frequencies. The error term named white noise or heteroscedasticity which rely on the type of data. The error term in VAR model reveals white noise errors, while GARCH family models show heteroscedastic errors.

Sims (1980) introduces Vector Autoregressive (VAR) model for modeling the white noise errors appropriately and overcome the weaknesses of simultaneous system of equations model for unsuitable evaluation of the error term (white noise errors). VARs have implements that are easy for estimation, structure inference and forecasting that serves suitably in policy

making. The error term, white noise is effectively modeled by VAR model. The data analysis exhibition of heteroscedastic error term cannot be modelled by the VAR white noise error term, VAR can only model error term that show equal variances (White, 1980; Harvey, 1993; Kennedy, 2008; Lazim, 2013).

Engle (1982) recommends Autoregressive Conditional Heteroscedasticity (ARCH) model to overcome the heteroscedastic error term and the time varying volatility. The equations are not normally distributed, with regards to changes in stock market distribution and fat tail measuring effect and this effect is the ARCH. The group errors are effectively handled by the ARCH models and it can also accommodate the changes made by economic forecaster. The anomalies like crashes, mergers, news effect or threshold effects in the financial and economic sector data analysis cannot be modeled properly by ARCH model (Agboluaje *et al.*, 2015). Bollerslev (1986) introduces generalized ARCH to overcome the volatility persistent that is flexible to uplift the weakness of ARCH model.

Excess kurtosis and volatility persistence are GARCH models weaknesses (Vivian and Wohar, 2012; Ewing and Malik, 2013; Agboluaje *et al.*, 2015).

Threshold GARCH and exponential GARCH suppress the asymmetric effects of positive and negative shocks of the same dimension on conditional volatility in a variety of ways (Nelson, 1991; Hentschel, 1995; McAleer, 2014; McAleer and Hafner, 2014; Kamaruzzaman and Isa, 2015; Al-Hagyan *et al.*, 2015; Farnoosh *et al.*, 2015; Mutunga *et al.*, 2015). Leverage is a particular case of asymmetry. Leverage effects cannot be modeled by GARCH family model, because any restriction imposed on it is positivity restriction which has no leverage effect. The coefficient of variance equation will show negative value for leverage effect to exist (McAleer, 2014; McAleer and Hafner, 2014; Chang *et al.*, 2015; Agboluaje *et al.*, 2015).

When the data size increases with respect to high frequency data, the existing models cannot have efficient and accurate results because of the behaviors of error terms that are not recognizable in the stochastic volatility time series (McAleer, 2014; McAleer and Hafner, 2014; Agboluaje *et al.*, 2015). The new approach of Combine White Noise uplifts the existing model's weaknesses to model the error terms for suitable estimations and to yield reliable outputs.

In econometric estimation, an essential assumption is that the error term of the series should have equal variance (white noise) (Cuthbertson *et al.*, 1992; Harvey, 1993), it is violated by heteroscedastic variances which are unequal variances of the error series. Then, the error series, which are divided into subseries of equal variances (white noise series) to produce a Combine White Noise to ease the existing model's weaknesses, is introduced (Agboluaje *et al.*, 2015).

Materials and Methods

The data, Australia (AU) Gross Domestic Product (RGDP) quarterly data from 1960Q3 to 2015Q2 is retrieved from the DataStream of Universiti Utara Malaysia library for this study.

The autoregression model below:

$$y_t = \varphi y_{t-1} + \varepsilon_t, \tag{2.1}$$

Let the stochastic procedure of a real-valued time to be ε_t and I is the entire information through t time. The GARCH model is;

$$\varepsilon_t \mid I_{t-1} \sim N(0, h_t), \tag{2.2}$$

$$
\begin{aligned}
h_t &= \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i h_{t-i} \\
&= \omega + A(L)\varepsilon_t^2 + B(L)h_t
\end{aligned} \tag{2.3}
$$

The EGARCH specification is:

$$\log h_t = \alpha + \beta \mid z_{t-1} \mid + \delta z_{t-1} + \gamma \log h_{t-1}, \mid \gamma \mid < 1 \tag{2.4}$$

where, $z_t = \varepsilon_t / \sqrt{h_t}$ is the standardized shocks, $z_t \sim iid(0, \alpha)$. $|\gamma| < 1$ is the stability. The impact is asymmetric if $\delta \neq 0$ although, there is existence of leverage if $\delta < 0$ and $\delta < \beta < -\delta$ While both β and δ must be positive which the variances of two stochastic processes are, then, modeling leverage effect is not possible (McAleer, 2014; McAleer and Hafner, 2014).

The unequal variances (heteroscedastic errors) behaviors of the GARCH processes can be transformed into Combine White Noise models. In order to get rid of heteroscedasticity the standardized residuals of GARCH errors which are unequal variances: Decompose unequal variances into equal variances (white noise) in series. The linear regression model transformed every equal variances series of models (Agboluaje *et al.*, 2015).

Using moving average process of estimation, these white noise series called Combine White Noise has the procedure as follows:

$$
\begin{aligned}
Y_1 &= \varepsilon_{1t} + \theta_{11}\varepsilon_{1,t-1} + \theta_{12}\varepsilon_{1,t-2} + \ldots \theta_{jq}\varepsilon_{j,t-q} \\
Y_2 &= \varepsilon_{2t} + \Phi_{21}\varepsilon_{2,t-1} + \Phi_{22}\varepsilon_{2,t-2} + \ldots \Phi_{jq}\varepsilon_{j,t-q} \\
&\vdots \\
Yj &= \varepsilon_{jt} + \varphi_{j1}\varepsilon_{j,t-1} + \varphi_{j2}\varepsilon_{j,t-2} + \ldots \varphi_{jq}\varepsilon_{j,t-q} \\
Y_{jt} &= \sum_{j=1}^{q} \theta_j \varepsilon_{j,t-q} + \sum_{j=1}^{q} \Phi_j \varepsilon_{j,t-q} + \ldots \sum_{j=1}^{q} \varphi_j \varepsilon_{j,t-q}
\end{aligned} \tag{2.5}
$$

$$
\begin{aligned}
&= A(L)\varepsilon_t + B(L)\varepsilon_t + \ldots \\
&= \varepsilon_t [A(L) + B(L) + \ldots]
\end{aligned} \tag{2.6}
$$

$$= Q\varepsilon_t$$
$$= U_t \tag{2.7}$$

It can be written as:

$$Y_t = U_t, \left(U_t \sim \mathrm{N}(0, \sigma_c^2)\right) \tag{2.8}$$

where, $A(L) + B(L) + \ldots = Q$ which are the matrix polynomial, U_t is the error term of combine white noise model and σ_c^2 is the combination of equal variances.

The combine variances of the combine white noise is:

$$\sigma_c^2 = \sigma_1^2 + \sigma_2^2 + \ldots \tag{2.9}$$

Considering the best two variances in the best two models produced by the Bayesian model averaging output. The combine variance follows:

$$\sigma_c^2 = \sigma_1^2 + \sigma_2^2 \tag{3.0}$$

The variance of errors, σ_c^2 in the combine white noise can be written:

$$\sigma_c^2 = W^2\sigma_1^2 + (1-W)^2\sigma_2^2 + 2\rho W\sigma_1(1-W)\sigma_2 \tag{3.1}$$

where the balanced weight specified for the model is W. The least of σ_c^2 appearing, when the equation is differentiated with respect to W and equate to zero, obtaining:

$$W = \frac{\sigma_c^2 - \rho\sigma_1\sigma_2}{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2} \tag{3.2}$$

where, ρ is the correlation; intra-class correlation coefficient is used for a reliable measurement.

Results

The data time plot reveals a behavior of non-stationary trending. The data is transformed in returns series to examine the volatility clustering, long tail skewness and excess kurtosis which are the features of heteroscedasticity. The graph exhibits unequal variances that suggest volatility.

Table 1 reveals that there is right tail skewness, excess kurtosis and Jarque-beta test is significant with the signification of non-normality. The standard deviation is slightly greater than one.

In Table 1 ARCH LM tests show that F-Statistic and Obs*R-squared is significant; there is no ARCH effect in the data.

Table 2 shows that EGARCH model is choosing among the GARCH family because it has the least values of AIC, BIC and HQ with high log-likelihood values, this connotes that EGARCH is the best model for further computation.

To overcome the challenges of heteroscedasticity with leverage effect, the standardized residuals graph of the EGARCH model (EGARCH errors) with unequal variances and zero mean is computed into equal variances series (white noise series). Graphs of equal variances (white noise series) with mean zero are obtained from the graph of EGARCH errors. Then, white noise series are fitted into the regression model to get white noise models (Agboluaje *et al.*, 2015).

The performance of Bayesian model averaging reveals two best models from the first grouped best models (Asatryan and Feld, 2014). For authentications, fit linear regression with autoregressive errors and the number of observation, with zero mean and variance one (Higgins and Bera, 1992). The outcome reveals that the best two models are the white noise models.

CWN has the least information criteria with high log likelihood values to get the best results when compare with EGARCH and VAR model estimation. The estimation of EGARCH model and CWN model with their forecasting values is stated in Table 4.

Table 3 testifies that an independent samples test is experimented to test whether a data set of the two white noise models have equal variances or not. The test reveals that the inconsistency in the distribution of the two data sets is no significantly different value which is greater than the p-value 0.05. As a result the two models have equal variances (Lim and Loh, 1996; Boos and Brownie, 2004; Bast *et al.*, 2015).

In Table 4: Stability test reveals that CWN and EGARCH are stable, but VAR is not stable. The three models are stationary. CWN and EGARCH have no autocorrelation, but autocorrelation exists in VAR. In Histogram-Normality tests, CWN and VAR are not normal, while EGARCH appear normal. There is no ARCH effect on the models. In Dynamic Forecast Evaluation: CWN has least forecast error value in Root Mean Standard Error (RMSE) when compare with EGARCH and VAR. Mean Absolute Error (MAE) and Mean Absolute Percentage Error (MAPE) values have the least forecast error in CWN when compare with EGARCH and VAR forecast error values. Therefore, CWN has the best forecast ability among the models under discussion. Ramey RESET tests show that CWN and EGARCH are stable but VAR is not stable. The determinant of the residual of the covariance matrix value indicates that CWN estimation is efficient, since the value is approximately zero. VAR estimation is not efficient. The CWN is efficient and has suitable forecast when compare with the existing models.

Table 1. Histogram-Normality and ARCH Tests of transformed data

	Coefficient/value	probability
Normal test		
Standard deviation	1.055567	
Skewness	0.364743	
Kurtosis	3.949680	
Jarque-Bera	13.085650	0.001440
ARCH Tests		
F-Statistic	4.908379	0.000300
Obs*R-squared	22.576580	0.000400

Table 2. ARCH, EGARCH, VAR, CWN models

	α	β	δ	γ	AIC	BIC	HQ	LL
ARCH	0.13645 (0.000)	0.31623(0.006)			2.90733	2.96942	2.93241	-312.89
EGARCH	-0.0462 (0.448)	-0.0157 (0.811)	0.02031(0.422)	1.0106 (0.000)	2.65324	2.76191	2.69713	-282.20
VAR					11.2244	11.27090	11.24320	-1226.10
CWN					-6.3362	-6.2433		699.813

Note: α is the coefficient of the mean equation, β and δ are the coefficients of the variance equations, while γ is the coefficient of the log of variance equation. In the parentheses are the probability values (PV).

Table 3. Levene's test for equal variances

Independent Samples Test

	Levene's test for equality of variances		t-test for Equality of Means					95% Confidence Interval of the Difference	
	F	Sig.	t	df	Sig.(2-tailed)	Mean Difference	Std.Error Difference	Lower	Upper
B Equal variances assumed	0.045	0.833	-2.993	438	0.003	-0.01409	0.00471	0.02334	-0.0048
Equal variances not assumed			-2.993	424.759	0.003	-0.01409	0.00471	-0.02335	-0.0048

Table 4. The summary of CWN, EGARCH and VAR models estimation and forecasting evaluation

	CWN	EGARCH	VAR
Estimation Residual Diagnostic			
Stability Test(Lag structure)	Stable	Stable	Not Stable
Correlogram (square) residual	covariance stationary	Stationary	covariance stationary
Portmanteau Tests	No autocorrelation	No autocorrelation	Autocorrelation
Histogram-Normality Tests	Not Normal	Appear Normal	Not Normal
ARCH Test	No ARCH effect	No ARCH effect	No ARCH effect
Dynamic Forecast Evaluation			
RMSE	0.0333325	0.489917	53253.79
MAE	0.007404	0.366493	46226.78
MAPE	1.233974	107.6098	15.61704
Residual Diagnostic			
Correlogram (square) residual	Stationary	Stationary	Stationary
Histogram-Normality Tests	Not Normal	Appear Normal	Not Normal
Serial Correlation LM Tests	No Serial Correlation	No Serial Correlation	Serial Correlation
Heteroscedasticity Test	No ARCH effect	No ARCH effect	ARCH effect
Stability Diagnostic			
Ramsey Reset Tests	Stable	Stable	Not Stable
Determinant residual covariance	0.001923		12.86549

Discussion

Chuffart (2015) reveals that wrong specifications can be the use of only Bayesian Information Criteria (BIC). The Logistic Smooth Transition GARCH and Markov-Switching GARCH models were employed to confirm the weakness of BIC. Obtaining the right model specification, CWN model employs Akaike Information Criteria (AIC), Bayesian Information Criteria (BIC) and log likelihood for specification of the model.

Mutunga *et al.* (2015) emphases that the EGARCH model has the minimum mean square error and mean

absolute error when compare with Glosten-Jagannathan-Runkle GARCH model; this reveals that EGARCH forecast is more precise. Conversely, CWN model has least information criteria and minimum forecast errors that indicate the performance of CWN to be better than the EGARCH model.

Chang *et al.* (2015) extend reasonable conditions of strict stationary and ergodicity in favor of three nonlinear models of Self-Exciting Threshold Autoregressive (SETAR) -GARCH process, multiple-regime logistic Transition AutoRegressive (STAR) model by GARCH errors and Exponential STAR-GARCH model. The STAR-GARCH model estimation results are regarded as essential in financial Econometrics. The GARCH family errors are disintegrated into CWN model. CWN is examined by employing different countries' data set, having better performance when compare with family GARCH model (EGARCH) which Mutunga *et al.* (2015) show as appropriate.

McAleer (2014) argues that the asymmetry and leverage are identical. The challenge is that leverage effect has no statistical properties to check the EGARCH estimation. The positivity restriction of the parameters cannot estimate the leverage effect. CWN model estimation has the available statistical properties of maximum likelihood estimation to get efficient estimation, which provides a better estimation than the existing models.

McAleer and Hafner (2014) show one line derivation of EGARCH to model the asymmetric leverage effect, but no resolution for stationarity and invertibility conditions. This makes it impossible to model the leverage effect. But as for CWN model stationarity and invertibility are possible. CWN estimation is more efficient.

Riposo and Bianaca (2015) show the distinctiveness of ARCH (1)-M and GARCH(1;1)-M with the continuous limit of a time-discrete processes. They show that there is more volatility variation in the time-discrete GARCH(1;1) when compared with the time-discrete ARCH(1). Whereas, the EGARCH error is decomposed for the formulation of CWN processes to dealt with different types of heteroscedastic error behaviors in the data including the leverage effect that none of GARCH family model can tackle effectively.

Agboluaje *et al.* (2015) employs US GDP data to show that CWN is efficient and gives suitable results when compare with existing models. Equally, tests have been conducted using different data from different countries to show the suitability of the CWN. This CWN can handle other financial/economic data that possesses asymmetry and the leverage effect (Agboluaje *et al.*, 2015). When the standardized residuals of EGARCH are disintegrated into equal variances, few points are zeros on the graph which can cause some errors. There is no package to take care of the zero points when re-graphing the points.

Future research: To disintegrate the conditional standard deviation graph into equal graph series and model it.

Conclusion

The standardized residual GARCH errors are decomposed into Combine White Noise (CWN). CWN has proved to be more efficient and it takes care of GARCH weaknesses. The estimation of Combine White Noise model passes the stability condition, stationary, serial correlation, the ARCH effect tests and it also passes the Levene's test of equal variances.

The CWN model estimation yields best results with minimum information criteria and high log likelihood values. While the EGARCH model estimated yields better results with minimum information criteria and high log likelihood values when compared with VAR model.

CWN has the minimum forecast errors which are indications of best results when compare with the EGARCH and VAR models dynamic evaluation forecast errors (Ismail and Muda, 2006; Fildes *et al.*, 2011; Lazim, 2013). The minimum forecast error values reveal the forecast accuracy.

The determinant of the residual of the covariance matrix value indicates that CWN is efficient. But, the determinant of the residual of the covariance matrix value indicates that VAR is not efficient.

Based on the every result of the empirical analysis, CWN is the most appropriate model. For this reason, CWN is recommended for the modeling of data that exhibits conditional heteroscedasticity and the leverage effect in Australia and other societies in the world.

The contribution of this study to the scientific community is that the CWN gives suitable results that improve the weaknesses of the existing models. The CWN forecast output is more reasonable for effective policy making. Implementation of this CWN will boost the economy of the society.

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Author's Contributions

Ayodele Abraham Agboluaje: Analyzing, producing the results and writing the paper.

Suzilah Ismail: Supervising the contents and flow of the paper.

Chee Yin Yip: Offering ideas, encouragement, proof read text and equations.

Ethics

This manuscript is original; there will be no expectation of any ethical issues after the publication. The three authors have read and approved the manuscript.

References

Agboluaje, A.A., S. Ismail and C.Y. Yip, 2015. Modeling the error term by moving average and generalized autoregressive conditional heteroscedasticity processes. Am. J. Applied Sci., 12: 896-901. DOI: 10.3844/ajassp.2015.896.901

Al-Hagyan, M., M. Misiran and Z. Omar, 2015. Content analysis of stochastic volatility model in discrete and continuous time setting. Res. J. Applied Sci. Eng. Technol., 10: 1185-1191. DOI: 10.19026/rjaset.10.1886

Asatryan, Z. and L.P. Feld, 2014. Revisiting the link between growth and federalism: A Bayesian model averaging approach. J. Comparative Econom., 43: 772-781. DOI: 10.1016/j.jce.2014.04.005

Bast, A., W. Wilcke, F. Graf, P. Lüscher and H. Gärtner, 2015. A simplified and rapid technique to determine an aggregate stability coefficient in coarse grained soils. CATENA, 127: 170-176. DOI: 10.1016/j.catena.2014.11.017

Bollerslev, T., 1986. Generalized autoregressive conditional heteroskedasticity. J. Econometr., 31: 307-327. DOI: 10.1016/0304-4076(86)90063-1

Boos, D.D. and C. Brownie, 2004. Comparing variances and other measures of dispersion. Stat. Sci., 19: 571-578.

Chang, C.L., Y. Li and M. McAleer, 2015. Volatility spillovers between energy and agricultural markets: A critical appraisal of theory and practice.

Chuffart, T., 2015. Selection criteria in regime switching conditional volatility models. Econom. 3: 289-316. DOI: 10.3390/econometrics3020289

Cuthbertson, K., S.G. Hall and M.P. Taylor, 1992. Applied Econometric Techniques. 1st Edn., Harvester University of Michigan Press, Ann Arbor, pp: 274.

Engle, R.F., 1982. Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. Econometrica, 50: 987-1007. DOI: 10.2307/1912773

Ewing, B.T. and F. Malik, 2013. Volatility transmission between gold and oil futures under structural breaks. Int. Rev. Econom. Finance, 25: 113-121. DOI: 10.1016/j.iref.2012.06.008

Farnoosh, R., M. Ebrahimi and S. Dalirian, 2015. Testing homogeneity of mixture of skew-normal distributions via Markov chain Monte Carlo simulation. Res. J. Applied Sci. Eng. Technol., 10: 112-117.

Fildes, R., Y. Wei and S. Ismail, 2011. Evaluating the forecasting performance of econometric models of air passenger traffic flows using multiple error measures. Int. J. Forecast., 27: 902-922. DOI: 10.1016/j.ijforecast.2009.06.002

Harvey, A.C., 1993. Time Series Models. 2nd Edn., The MIT Press, Cambridge, ISBN-10: 0262082241, pp: 308.

Hentschel, L., 1995. All in the family nesting symmetric and asymmetric GARCH models. J. Financial Econom., 39: 71-104. DOI: 10.1016/0304-405X(94)00821-H

Higgins, M.L. and A.K. Bera, 1992. A class of nonlinear ARCH models. Int. Economic Rev., 33: 137-158. DOI: 10.2307/2526988

Hubbard, R.G. and A.P. O'Brien, 2012. Economics. 4th Edn., ISBN-10: 013281725X.

Ismail, S. and T.Z. Muda, 2006. Comparing forecasting effectiveness through air travel data. Proceedings of Knowledge Management International Conference and Exhibition, Jun. 6-8, Sintok, pp: 594-602. http://repo.uum.edu.my/2389/

Kamaruzzaman, Z.A. and Z. Isa, 2015. Modelling stock market return via normal mixture distribution. Res. J. Applied Sci., 10: 324-333. DOI: 10.3923/rjasci.2015.324.333

Kennedy, P., 2008. A Guide to Econometrics. 6th Edn., Blackwell Publishing, Malden, ISBN 1405182571, pp: 585.

Lazim, M.A., 2013. Introductory Business Forecasting: A Practical Approach. 3rd Edn., Penerbit Press, University Technology Mara, ISBN-13: 9789833643103, pp: 311.

Lim, T.S. and W.Y. Loh, 1996. A comparison of tests of equality of variances. Comput. Statist. Data Analysis, 22: 287-301. DOI: 10.1016/0167-9473(95)00054-2

McAleer, M., 2014. Asymmetry and leverage in conditional volatility models. Econometrics, 2: 145-150. DOI: 10.3390/econometrics2030145

McAleer, M. and C.M. Hafner, 2014. A one line derivation of EGARCH. Econometrics, 2: 92-97. DOI: 10.3390/econometrics2020092

Mutunga, T.N., A.S. Islam and L.A.O. Orawo, 2015. Implementation of the estimating functions approach in asset returns volatility forecasting using first order asymmetric GARCH models. Open J. Stat., 5: 455. DOI: 10.4236/ojs.2015.55047

Nelson, D.B., 1991. Conditional heteroskedasticity in asset returns: A new approach. Econometrica, 59: 347-370. DOI: 10.2307/2938260

Qin, D. and C.L. Gilbert, 2001. The error term in the history of time series econometrics. Econometric Theory, 17: 424-450.

Riposo, J. and C. Bianca, 2015. On volatility variation in ARCH (l) and GARCH (1, 1) continuous limits. Nonlinear Studies, 22: 359-371.

Sims, C.A., 1980. Macroeconomics and Reality, Econom. 48: 1-48. DOI: 10.2307/1912017

Vivian, A. and M.E. Wohar, 2012. Commodity volatility breaks. J. Int. Financial Markets Institut. Money, 22: 395-422. DOI: 10.1016/j.intfin.2011.12.003

White, H., 1980. A heteroskedasticity-consistent covariance matrix estimator and a direct test for heteroskedasticity. Econometrica, 48: 817-838. DOI: 10.2307/1912934