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Multistage Median Ranked Set Sampling for Estimating the Population Median

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Abstract: We modify RSS to come up with new sampling method, namely, Multistage Median Ranked Set Sampling (MMRSS). The MMRSS was suggested for estimating the population median and to increase the efficiency of the estimator for specific value of the sample size. The MMRSS was compared to the Simple Random Sampling (SRS), Ranked Set Sampling (RSS) and Median Ranked Set Sampling (MRSS) methods. It is found that MMRSS gives an unbiased estimate of the population median of symmetric distributions and it is more efficient than SRS, RSS and MRSS based on the same number of measured units. Also, it was found that the efficiency of MMRSS increases in r (r is the number of stage) for specific value of the sample size. For asymmetric distributions considered in this study, MMRSS has a small bias, close to zero as r increases, especially with odd sample size. A set of real data was used to illustrate the method.

Keywords: Simple random sampling; ranked set sampling; median ranked set sampling; multistage ranked set sampling; multistage median ranked set sampling

INTRODUCTION

Many sampling methods are suggested in the literature for estimating the population parameters. In some situations where the experimental or sampling units in a study can be easily ranked than quantified, McIntyre[1] proposed the sample mean based on RSS as an estimator of the population mean. He found that the estimator based on RSS is more efficient than SRS. Takahasi and Wakimoto[2] provided the necessary mathematical theory of RSS. Muttlak[3] suggested using median ranked set sampling (MRSS) to estimate the population mean. Al-Saleh and Al-Omari[4] suggested that the multistage ranked set sampling (MSRSS) method to increase the efficiency when estimating the population mean for specific value of the sample size. Jemain and Al-Omari[5,6] proposed double percentile ranked set sampling (DPRSS) and multistage median ranked set sampling (MMRSS) methods respectively for estimating the population mean. They found that DPRSS and MMRSS are more efficient than the commonly used SRS for the same sample size. Jemain, *et al.*[7] suggested multistage extreme ranked set sampling (MERSS) method for estimating the population mean.

In this study, our objectives is to suggest MMRSS for estimating the population median and to compare the efficiency of this method with SRS, RSS and MRSS.

Sampling methods

Ranked set sampling: To obtain a sample of size m by the usual RSS as suggested by McIntyre[1], select m random samples each of size m from the target population and rank the units within each sample with respect to a variable of interest by visual inspection or any cost free method. For measurement, from the first sample the smallest rank unit is selected, and from the second sample the second smallest rank unit is selected. The process is continued until from the mth sample the mth rank unit is selected. The method is repeated n times if needed to get a RSS of mn.

Multistage median ranked set sampling: The MMRSS procedure is described as follows:

Step 1: Randomly selected m^{r+1} units from the target population, where r is the number of stages and m is the sample size.

Step 2: Allocate the m^{r+1} selected units as randomly as possible into m^r sets, each of size m.

Step 3: For each m^r sets in Step (2), if the sample size m is odd, select from each m^r sets the $\left((m+1)/2\right)$th smallest rank unit, i.e., the

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median of each set. If the sample size m is even, select from the first $m^r/2$ sets the $\left(m/2\right)$th smallest rank unit, and from the second $m^r/2$ sets the $\left((m+2)/2\right)$th smallest rank unit. This step yields m^{r-1} median ranked sets each of size m.

Step.4: Without doing any actual quantification, if the sample size m is odd, select from each m^{r-1} sets the $\left((m+1)/2\right)$th smallest rank unit, i.e., the median of each set. If the sample size m is even, select from the first $m^{r-1}/2$ sets the $\left(m/2\right)$th smallest rank unit, and from the second $m^{r-1}/2$ sets select the $\left((m+2)/2\right)$th smallest rank unit. This step yields m^{r-2} median ranked sets each of size m, i.e., it is the second stage median ranked sets.

Step 5: The process is continued using Steps (3) and (4) until we end up with one rth stage median ranked set sample of size m from MMRSS.

The whole process can be repeated n times to obtain a sample of size nm from MMRSS. It is necessary to note here that the ranking at all stages are done by visual inspection or by any other cheap method, and the actual quantification is exactly done on the last sample of size m that is obtained at the last stage. To estimate a population median by a sample of size m using SRS method, we only randomly select m units and find the median. And if we use the RSS method for the same estimation, we have to identify m^2 units and measure only m of them. But when we use MMRSS, we randomly select m^{r+1} units and measure only m of them. In each method, SRS, RSS or MMRSS, we randomly select different number of units but measure only the same number of units for comparison. Therefore, the measured units of MMRSS, which are based on m^{r+1} unit, have more information and more representative of the target population when compared to SRS or RSS.

Let us consider the following example to illustrate the MMRSS method.

Example 1: Consider the case of $m=3$ and $r=3$. Therefore, we have a random sample of size $m^{r+1}=81$ units as follows: $X_1, X_2, ..., X_{81}$.

Let $X_{i(j:m)}^{(r)}$ be the jth minimum $(j=1,2,3)$ of the ith set $(i=1,2,...,27)$ at stage r $(r=1,2,3)$. Allocate the 81 selected units into 27 sets each of size 3 at zero stage (SRS).

For $r=1$, rank the units within each sample visually according to the variable of interest. After ranking, the sets appear as shown below:

$$A_i^{(1)}=\left\{X_{i(1:3)}^{(1)}, X_{i(2:3)}^{(1)}, X_{i(3:3)}^{(1)}\right\},\ (i=1,2,...,27).$$

Now, select the median from the 27 sets, for $m=3$, the median is the second smallest rank unit, so that let

$$X_{i(2:3)}^{(1)}=\text{med}\left(A_i^{(1)}\right),\ (i=1,2,...,27).$$

This step yields 27 medians which are $X_{1(2:3)}^{(1)}$, $X_{2(2:3)}^{(1)},...,X_{27(2:3)}^{(1)}$. Allocate them into 9 sets each of size 3 as:

$$A_i^{(1)}=\left\{X_{i(2:3)}^{(1)}, X_{i(2:3)}^{(1)}, X_{i(2:3)}^{(1)}\right\},\ (i=1,2,...,9).$$

For $r=2$, rank the units within the 9 sets yields from the first stage to get

$$A_i^{(2)}=\left\{X_{i(1:3)}^{(2)}, X_{i(2:3)}^{(2)}, X_{i(3:3)}^{(2)}\right\},\ (i=1,2,...,9),$$

and then select the median from each set as:

$$X_{i(2:3)}^{(2)}=\text{med}\left(A_i^{(2)}\right),\ (\text{i}=1,2,...,9).$$

This step yields 9 medians, $X_{1(2:3)}^{(2)}, X_{2(2:3)}^{(2)},..., X_{9(2:3)}^{(2)}$, which are allocated into 3 sets of medians each of size 3 as:

$$A_i^{(2)}=\left\{X_{i(2:3)}^{(2)}, X_{i(2:3)}^{(2)}, X_{i(2:3)}^{(2)}\right\},\ (i=1,2,3).$$

For $r=3$, rank the units within each set yields at stage 2 to obtain

$$A_i^{(3)}=\left\{X_{i(1:3)}^{(3)}, X_{i(2:3)}^{(3)}, X_{i(3:3)}^{(3)}\right\},\ (i=1,2,3).$$

Now, select the median of the three sets as:

$$X_{i(2:3)}^{(3)}=\text{med}\left(A_i^{(3)}\right),\ (i=1,2,3).$$

This step yields $\left\{X_{1(2:3)}^{(3)}, X_{2(2:3)}^{(3)}, X_{3(2:3)}^{(3)}\right\}$ to be third stage median ranked set sample. The actual quantification for estimating the population median of the variable of interest can be achieved using only these three units. It is clear that the number of quantified units, which is 3, is a small portion of 27 sampled units.

Example 2: Consider the case of $m=4$ and $r=3$, so that we have a random sample of size $m^{r+1}=256$ units which are: $X_1, X_2,..., X_{256}$. Allocate the 256 units into 64 sets each of size 4.

For $r=1$, rank the units within each set with respect to the variable of interest as follows:

$$A_i^{(1)}=\left\{X_{i(1:4)}^{(1)}, X_{i(2:4)}^{(1)}, X_{i(3:4)}^{(1)}, X_{i(4:4)}^{(1)}\right\},\ (i=1,2,...,64).$$

Now, select the second rank unit from the first 32 sets, and the third rank unit from the other 32 sets as:

$$X_{i(2:4)}^{(1)} = \text{2nd min}\left(A_i^{(1)}\right),\ (i = 1,2,...,32) \text{, and}$$

$$X_{i(3:4)}^{(1)} = \text{3rd min}\left(A_i^{(1)}\right),\ (i = 33,34,...,64) .$$

This step yields 64 units which are $X_{1(2:4)}^{(1)}, X_{2(2:4)}^{(1)}$, $,..., X_{32(2:4)}^{(1)}\ X_{33(3:4)}^{(1)}, X_{34(3:4)}^{(1)},..., X_{64(3:4)}^{(1)}$. Allocate them into 16 sets each of size 4, as follows:

$A_i^{(1)} = \left\{X_{i(2:4)}^{(1)}, X_{i(2:4)}^{(1)}, X_{i(2:4)}^{(1)}, X_{i(2:4)}^{(1)}\right\}$, $(i = 1,2,...,8)$, and

$A_i^{(1)} = \left\{X_{i(3:4)}^{(1)}, X_{i(3:4)}^{(1)}, X_{i(3:4)}^{(1)}, X_{i(3:4)}^{(1)}\right\}$, $(i = 9,10,...,16)$.

For $r = 2$, rank the units within each the 16 sets yields from the first stage as:

$$A_i^{(2)} = \left\{X_{i(1:4)}^{(2)}, X_{i(2:4)}^{(2)}, X_{i(3:4)}^{(2)}, X_{i(4:4)}^{(2)}\right\},\ (i = 1,2,...,16) .$$

Now, from the first 8 sets select the second rank unit, and from the second 8 sets the third rank unit as shown below:

$$X_{i(2:4)}^{(2)} = \text{2nd min}\left(A_i^{(2)}\right),\ (i = 1,2,...,8),$$

$$X_{i(3:4)}^{(2)} = \text{3rd min}\left(A_i^{(2)}\right),\ (i = 9,10,...,16) .$$

This step yields $X_{1(2:4)}^{(2)}, X_{2(2:4)}^{(2)},...,\ X_{8(2:4)}^{(2)}, X_{9(3:4)}^{(2)}$, $X_{10(3:4)}^{(2)},..., X_{16(3:4)}^{(2)}$. Allocate these units into 4 sets each of size 4, as follows:

$$A_i^{(2)} = \left\{X_{i(2:4)}^{(2)}, X_{i(2:4)}^{(2)}, X_{i(2:4)}^{(2)}, X_{i(2:4)}^{(2)}\right\},\ (i = 1,2) \text{, and}$$

$$A_i^{(2)} = \left\{X_{i(3:4)}^{(2)}, X_{i(3:4)}^{(2)}, X_{i(3:4)}^{(2)}, X_{i(3:4)}^{(2)}\right\},\ (i = 3,4) .$$

For $r = 3$, rank the units within the last 4 sets yields from the second stage as:

$$A_i^{(3)} = \left\{X_{i(1:4)}^{(3)}, X_{i(2:4)}^{(3)}, X_{i(3:4)}^{(3)}, X_{i(4:4)}^{(3)}\right\},\ (i = 1,2,3,4) .$$

Now, from the first 2 sets select the second rank unit and from the second 2 sets the third rank unit as shown below:

$$X_{i(2:4)}^{(3)} = \text{2nd min}\left(A_i^{(2)}\right),\ (i = 1,2) \text{, and}$$

$$X_{i(3:4)}^{(3)} = \text{3nd min}\left(A_i^{(3)}\right),\ (i = 3,4) .$$

The final set $\left\{X_{1(2:4)}^{(3)}, X_{2(2:4)}^{(3)}, X_{3(3:4)}^{(3)}, X_{4(3:4)}^{(3)}\right\}$ is a third stage median ranked set of size 4.

RESULTS AND DISCUSSION

Let $X_1, X_2,..., X_m$ be a random sample with pdf $f(x)$, cdf $F(x)$, a finite mean μ and variance σ^2. Let $X_{11}, X_{12},\ldots, X_{1m}; X_{21}, X_{22},\ldots, X_{2m};\ldots; X_{m1}, X_{m2},...,$ X_{mm} be independent random variables all with the same distribution function $F(x)$. Let $X_{(i:m)}$ denotes the ith order statistic of a sample of size m $(i = 1,2,....m)$. The SRS estimator of the population median from a sample of size m is defined as:

$$\hat{\eta}_{SRS} = \begin{cases} X_{\left(\frac{m+1}{2}\right)} & \text{,if } m \text{ is odd} \\ \dfrac{X_{\left(\frac{m}{2}\right)} + X_{\left(\frac{m+2}{2}\right)}}{2} & \text{,if } m \text{ is even} \end{cases}, \tag{1}$$

to be the middle or the average of the two middle units after sorting.

The estimator of the population median η for a RSS of size m is given by:

$$\hat{\eta}_{RSS} = \text{median}\left\{X_{i(i:i)}, i = 1,2,...,m\right\}. \tag{2}$$

If the sample size m is odd, let $X_{i\left(\frac{m+1}{2}:m\right)}^{(r)}$ be the median of the ith sample $(i = 1,2,...,m)$ at stage r. The measured units, $X_{1\left(\frac{m+1}{2}:m\right)}^{(r)}, X_{2\left(\frac{m+1}{2}:m\right)}^{(r)},\ldots, X_{m\left(\frac{m+1}{2}:m\right)}^{(r)}$ are iid, and denote the measured MMRSSO. If the sample size m is even, let $X_{i\left(\frac{m}{2}:m\right)}^{(r)}$ be the $(m/2)$th smallest rank unit of the ith sample $(i = 1,2,...,m/2)$, and $X_{i\left(\frac{m+2}{2}:m\right)}^{(r)}$ be the $((m+2)/2)$th smallest rank unit of the ith sample $(i = (m+2)/2, (m+4)/2,...,m)$ at stage r. Note that the first $m/2$ units which are $X_{1\left(\frac{m}{2}:m\right)}^{(r)}$, $X_{2\left(\frac{m}{2}:m\right)}^{(r)},\ldots, X_{\frac{m}{2}\left(\frac{m}{2}:m\right)}^{(r)}$ are iid, and the second $m/2$ units which are $X_{\frac{m+2}{2}\left(\frac{m+2}{2}:m\right)}^{(r)},\ldots, X_{m\left(\frac{m+2}{2}:m\right)}^{(r)}$ are iid. However,

$$X_{1\left(\frac{m}{2}:m\right)}^{(r)}, X_{2\left(\frac{m}{2}:m\right)}^{(r)},..., X_{\frac{m}{2}\left(\frac{m}{2}:m\right)}^{(r)}, X_{\frac{m+2}{2}\left(\frac{m+2}{2}:m\right)}^{(r)},\ldots, X_{m\left(\frac{m+2}{2}:m\right)}^{(r)}$$

which denote the measured MMRSSE, are independent but not identically distributed.

The estimator of the population median using MMRSSO in the case of an odd sample size can be defined as:

$$\hat{\eta}_{MMRSSO}^{(r)} = \text{median}\left\{\begin{array}{l} X_{1\left(\frac{m+1}{2}:m\right)}^{(r)}, X_{2\left(\frac{m+1}{2}:m\right)}^{(r)} \\ ,..., X_{m\left(\frac{m+1}{2}:m\right)}^{(r)} \end{array}\right\}, \tag{3}$$

and for an even sample size, the MMRSSE estimator is defined as:

Table 1: The efficiency of RSS and MMRSS relative to SRS for estimating the population median of some symmetric distributions with $m = 3,4,5$ for $r = 1,2,3,4$

Distributions	η	m	RSS	MMRSS			
				r=1	r=2	r=3	r=4
Uniform (0,1)	0.500	3	1.443	1.873	3.826	8.061	18.131
		4	2.010	2.252	5.594	17.323	51.624
		5	1.860	2.962	9.785	33.972	118.146
Normal (0,1)	0.000	3	1.621	2.235	5.047	11.255	25.376
		4	2.164	2.714	7.136	19.381	52.814
		5	2.115	3.448	12.146	43.792	153.376
Normal (1,2)	1.000	3	1.630	2.258	4.953	11.206	25.581
		4	2.227	2.747	7.201	19.558	53.097
		5	2.131	3.504	12.188	43.495	151.832
Logistic (-1,1)	-1.000	3	1.693	2.373	5.566	12.461	27.964
		4	2.265	2.945	7.459	20.459	55.005
		5	2.181	3.701	13.143	45.869	164.774

Table 2: The efficiency of RSS and MMRSSO relative to SRS for estimating the population median of some asymmetric distributions with $m = 3$ for $r = 1,2,3,4$

Distributions	η		RSS	MMRSS			
				r=1	r=2	r=3	r=4
Exponential (1)	0.693	*Eff*	1.802	2.582	6.175	14.042	32.954
		Bias	0.083	0.063	0.029	0.013	0.005
Log Normal (0,1)	1.000	*Eff*	2.120	3.535	9.428	22.025	51.554
		Bias	0.148	0.104	0.044	0.020	0.006
Weibull (1,3)	2.079	*Eff*	1.807	2.660	6.251	14.579	33.712
		Bias	0.262	0.186	0.086	0.036	0.017
Beta (7,4)	0.645	*Eff*	1.612	2.217	4.894	11.026	24.713
		Bias	0.002	0.002	0.001	0.000	0.000
Gamma (3,1)	2.674	*Eff*	1.690	2.307	5.348	12.002	27.321
		Bias	0.092	0.069	0.029	0.015	0.005

Table 3: The efficiency of RSS and MMRSSE relative to SRS for estimating the population median of some asymmetric distributions with $m = 4$ for $r = 1,2,3,4$

Distributions	η		RSS	MMRSS			
				r=1	r=2	r=3	r=4
Exponential (1)	0.693	*Eff*	2.300	2.811	7.865	15.000	17.123
		Bias	0.095	0.063	0.050	0.074	0.103
Log Normal (0,1)	1.000	*Eff*	2.626	4.038	10.706	14.967	15.155
		Bias	0.163	0.099	0.080	0.123	0.168
Weibull (1,3)	2.079	*Eff*	2.252	3.177	7.870	14.807	15.155
		Bias	0.286	0.174	0.149	0.225	0.311
Beta (7,4)	0.645	*Eff*	2.180	2.643	6.793	18.735	48.108
		Bias	0.003	0.002	0.002	0.002	0.004
Gamma (3,1)	2.674	*Eff*	2.246	2.861	7.343	17.297	30.788
		Bias	0.099	0.059	0.052	0.077	0.107

$$\hat{\eta}^{(r)}_{MMRSSE} = \text{median}\left\{ \begin{array}{l} X^{(r)}_{1\left(\frac{m}{2}:m\right)}, \ldots, X^{(r)}_{\frac{m}{2}\left(\frac{m}{2}:m\right)} \\ , X^{(r)}_{\frac{m+2}{2}\left(\frac{m+2}{2}:m\right)}, \ldots, \\ X^{(r)}_{m\left(\frac{m+2}{2}:m\right)} \end{array} \right\}. \quad (4)$$

The MMRSS estimator of the population median has the following properties:

1. If the distribution is symmetric about the population mean μ then for any stage r we have:
   a. $\hat{\eta}^{(r)}_{MMRSS}$ is an unbiased estimator of a population median η.
   b. $\text{Var}\left(\hat{\eta}^{(r)}_{MMRSS}\right) < \text{Var}(\hat{\eta}_{SRS})$.
   c. $\text{Var}\left(\hat{\eta}^{(r)}_{MMRSS}\right) < \text{Var}(\hat{\eta}_{RSS})$.
2. If the distribution is asymmetric, then

Table 4: The efficiency of RSS and MMRSSO relative to SRS for estimating the population median of some asymmetric distributions with $m=5$ for $r=1,2,3,4$

Distributions	η		RSS	MMRSS			
				r=1	r=2	r=3	r=4
Exponential (1)	0.693	*Eff*	2.282	3.962	14.418	52.488	182.290
		Bias	0.044	0.026	0.008	0.002	0.001
Log Normal (0,1)	1.000	*Eff*	2.650	5.111	19.069	69.858	250.184
		Bias	0.070	0.040	0.012	0.002	0.001
Weibull (1,3)	2.079	*Eff*	2.314	3.998	14.305	51.129	186.73
		Bias	0.131	0.079	0.027	0.007	0.006
Beta (7,4)	0.645	*Eff*	2.023	3.345	11.533	41.004	145.361
		Bias	0.001	0.001	0.000	0.000	0.000
Gamma (3,1)	2.674	*Eff*	2.173	3.600	12.919	45.368	160.266
		Bias	0.047	0.253	0.007	0.003	0.001

Table 5: The efficiency of RSS and MMRSSE relative to SRS for estimating the population median of asymmetric distributions with $m=6$ for $r=1,2,3,4$

Distributions	η		RSS	MMRSS			
				r=1	r=2	r=3	r=4
Exponential (1)	0.693	*Eff*	2.754	4.286	15.245	50.370	80.861
		Bias	0.050	0.024	0.013	0.021	0.036
Log Normal (0,1)	1.000	*Eff*	3.026	5.062	19.074	56.596	82.160
		Bias	0.080	0.038	0.020	0.032	0.053
Weibull (1,3)	2.079	*Eff*	2.749	4.224	15.059	50.120	77.384
		Bias	0.149	0.076	0.039	0.064	0.107
Beta (7,4)	0.645	*Eff*	2.663	3.839	13.332	50.740	206.501
		Bias	0.002	0.000	0.000	0.000	0.000
Gamma (3,1)	2.674	*Eff*	2.715	4.001	13.745	53.059	136.393
		Bias	0.050	0.025	0.012	0.021	0.040

Table 6: The efficiency of MMRSSO with respect to SRS for estimating median olive yields per tree for $m=3$, and $r=1,2,3,4$

r	SRS			MMRSSO			Efficiency
	Median	Bias	MSE	Median	Bias	MSE	
1	9.339	1.239	15.702	9.041	0.941	8.191	1.917
2	9.382	1.282	15.942	8.758	0.658	3.594	4.435
3	9.371	1.271	15.854	8.535	0.435	1.335	11.873
4	9.385	1.285	15.978	8.362	0.262	0.383	41.665

a. $\hat{\eta}^{(r)}_{MMRSS}$ is a biased estimator of the population median and the bias is very small. In the case of odd sample size this bias is close to zero as *r* increases.

b. $\text{MSE}\left(\hat{\eta}^{(r)}_{MMRSS}\right) < \text{MSE}\left(\hat{\eta}_{SRS}\right)$.

c. $\text{MSE}\left(\hat{\eta}^{(r)}_{MMRSS}\right) < \text{MSE}\left(\hat{\eta}_{RSS}\right)$.

3. The efficiency of $\hat{\eta}^{(r)}_{MMRSS}$ is increasing in *r* for both distributions, either symmetric or asymmetric about the population mean μ.

Simulation study based on MMRSS for median estimation: In this section, we shall compare the proposed estimators for the population median using MMRSS with SRS, RSS and MRSS methods. Several probability distribution functions are considered: uniform, normal, logistic, exponential, lognormal, weibull, beta and gamma. The efficiency of $\hat{\eta}_{RSS}$ and $\hat{\eta}^{(r)}_{MMRSS}$ relative to $\hat{\eta}_{SRS}$ if the distribution is symmetric is defined as:

$$eff\left(\hat{\eta}_{SRS},\hat{\eta}_{RSS}\right)=\frac{\text{Var}\left(\hat{\eta}_{SRS}\right)}{\text{Var}\left(\hat{\eta}_{RSS}\right)}, \tag{5}$$

$$eff^{(r)}\left(\hat{\eta}_{SRS},\hat{\eta}^{(r)}_{MMRSS}\right)=\frac{\text{Var}\left(\hat{\eta}_{SRS}\right)}{\text{Var}\left(\hat{\eta}^{(r)}_{MMRSS}\right)}, \tag{6}$$

respectively, and if the distribution is asymmetric the efficiency, respectively, is given by

$$eff\left(\hat{\eta}_{SRS},\hat{\eta}_{RSS}\right)=\frac{\text{MSE}\left(\hat{\eta}_{SRS}\right)}{\text{MSE}\left(\hat{\eta}_{RSS}\right)}, \tag{7}$$

$$eff^{(r)}\left(\hat{\eta}_{SRS}, \hat{\eta}^{(r)}_{MMRSS}\right) = \frac{\mathrm{MSE}\left(\hat{\eta}_{SRS}\right)}{\mathrm{MSE}\left(\hat{\eta}^{(r)}_{MMRSS}\right)}. \quad (8)$$

The efficiency values for symmetric distributions, which are uniform, normal and logistic for estimating the population median with $m = 3,4,5$ are presented in Table 1. While for asymmetric distributions, which are exponential, lognormal, weibull, beta and gamma, the efficiency and the bias values for estimating the population median using MMRSS with $m = 3,4,5,6$, and $r = 1,2,3,4$ are presented in Tables 2-5 respectively.

Based on Tables 1-5, we can conclude the followings:

1. If the underlying distribution is symmetric, we have:
   a. A gain in efficiency is obtained by using MMRSS for different values of *m*.
   b. $\hat{\eta}^{(r)}_{MMRSS}$ is an unbiased of the population median, η.
   c. The efficiency of $\hat{\eta}^{(r)}_{MMRSS}$ relative to $\hat{\eta}_{SRS}$ is increasing in *r*. For example, with $m = 5$, the efficiency of MMRSS for estimating the median of the logistic distribution for $r = 1,2,3$ and 4, respectively are 3.701, 13.143, 45.869 and 164.774.
   d. $\hat{\eta}^{(r)}_{MMRSS}$ is more efficient than $\hat{\eta}_{RSS}$ for all sample sizes considered in this study.
   e. For $r = 1$, the MMRSS is same as MRSS. It is found that MMRSS is more efficient than MRSS for $r \geq 2$.
2. If the underlying distribution is asymmetric, then we have the followings:
   a. A gain in efficiency is obtained using MMRSS for estimating the population median.
   b. The efficiency of $\hat{\eta}^{(r)}_{MMRSS}$ is increasing in *r* for specific value of the sample size.
   c. In the case of odd sample size, $\hat{\eta}^{(r)}_{MMRSS}$ has a small bias which approaches to zero as *r* increases. For example, for $m = 5$, the efficiency of MMRSS for estimating the population median of an exponential distribution when $r = 1$ is 3.962 with bias 0.026, while when $r = 4$, the efficiency is 182.290, with bias 0.001.
   d. In the case of even sample size, although there is no clear pattern of the bias value for $\hat{\eta}^{(r)}_{MMRSS}$ but this value is small and generally close to zero for all asymmetric distributions considered.
   e. It is found that, MMRSS is more efficient than MRSS. As an example, for estimating the median of the standard exponential distribution, with $m = 5$, the efficiency of MRSS 3.962 with bias 0.026, while the efficiency MMRSS is 182.290 with bias 0.001 for $r = 4$.

Application to real data set: We illustrate the performance of MMRSS method for median estimation using a set of real data consisting of the olive yields of 64 trees. All sampling was done without replacement. We obtain the median and the MSE of each sample using SRS and MMRSS method with sample size $m = 3$. We compared the averages of the 70,000 sample estimates.

Let u_i be the olive yield of the *i*th tree $i = 1, 2, \ldots,$ 64. The mean μ and the variance σ^2 of the population, respectively, are

$$\mu = \frac{1}{64}\sum_{i=1}^{64} u_i = 9.777 \text{ kg/tree}$$

and

$$\sigma^2 = \frac{1}{64}\sum_{i=1}^{64}(u_i - \mu)^2 = 26.112 \text{ kg}^2/\text{tree}$$

The coefficient of skewness and median of the population are 0.484 and 8.250 respectively. It is known that the coefficient of skewness is zero for symmetric distribution, but for our data the coefficient is 0.484, indicating that these data are asymmetrically distributed. For illustration, we consider $m = 3$ for $r = 1,2,3,4$. The efficiency of MMRSSO relative to SRS are computed using Equation (8) and are presented in Table 6 along with the associated bias.

It can be seen from Table 6 that the medians based on MMRSSO are much closer to the population median when compared to those obtained using SRS. It is also found that the efficiency of MMRSSO increases in *r* but the bias decreases in *r*.

CONCLUSION

It can be concluded that MMRSS is more efficient than SRS, RSS and MRSS methods in estimating the population median based on the same sample size. Also, estimator of the population median obtained by MMRSS method is an unbiased when the underlying distribution is symmetric about its mean. If the underlying distribution is asymmetric the estimator is found to have a small bias. The MMRSS is recommended to be used for estimating the population median for symmetric distributions. For asymmetric distributions, this method is suggested for odd sample size as the bias decreases in *r*.

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