A Novel Ensemble Method for Regression via Classification Problems

1Sami M. Halawani, 2Ibrahim A. Albidewi and 1Amir Ahmad
1Department of Information Technology,
Faculty of Computing and Information Technology,
King Abdul Aziz University, Rabigh, Saudi Arabia
2Department of Information Systems,
Faculty of Computing and Information Technology,
King Abdul Aziz University, Jeddah, Saudi Arabia

Abstract: Problem statement: Regression via Classification (RvC) is a method in which a regression problem is converted into a classification problem. A discretization process is used to covert continuous target value to classes. The discretized data can be used with classifiers as a classification problem. Approach: In this study, we use a discretization method, Extreme Randomized Discretization (ERD), in which bin boundaries are created randomly to create ensembles. Results: We show that the proposed ensemble method is useful for RvC problems. We show theoretically that the proposed ensembles for RvC perform better than RvC with the equal-width discretization method. We also show the superiority of the proposed ensemble method experimentally. Experimental results suggest that the proposed ensembles perform competitively to the method developed specifically for regression problems. Conclusion: As the proposed method is independent of the choice of the classifier, various classifiers can be used with the proposed method to solve the regression method.

Key words: Regression via Classification (RvC), ERD ensembles, classification problem, decision trees, Extreme Randomized Discretization (ERD), Monothetic Contrast Criteria (MCC), RvC perform, Mean Square Error (MSE), neural network

INTRODUCTION

In machine learning and data mining fields, supervised learning plays an important role (Bishop 2006; Mitchell, 1997; Halawani and Albidewi, 2010; Ahmad and Dey, 2005; Ahmad and Brown, 2009; Chandra et al., 2010; Alfred, 2010; Ishrat et al., 2010). In a regression problem, the target values are continuous, whereas in the classification problem we have discrete set of classes. The other difference is that regression values have a natural ordering, whereas for the classification the class values are unordered (Bishop, 2006; Mitchell, 1997). Regression models are not easily understood by domain experts and thus provide little help in understanding the problem, whereas classification models are more comprehensible, but not very useful, when the target values are continuous. There are some learning schemes, like naive Bayes, which are very successful as classification techniques, however, they are difficult to use as regression schemes. Decision trees (Breiman et al., 1984; Quinlan, 1993), neural networks (Bishop, 2006; Mitchell, 1997), naive Bayes (Bishop, 2006; Mitchell, 1997), support vector machines (Vapnik, 1998; Burges, 1998) are quite popular for classification problems, whereas regression trees (Breiman et al., 1984), neural networks (Bishop, 2006; Mitchell, 1997), support vector machines (Vapnik, 1998; Burges, 1998) are used for regression problems.

Discretization (Dougherty et al., 1995) is a process that divides continuous numeric values into a set of intervals (bins) that can be considered as categorical values. Dougherty et al. (1995) define three axes upon which discretization methods can be classified; global vs. local, supervised vs. unsupervised and static vs. dynamic. Supervised methods use the information of class labels whereas, unsupervised methods do not. Local methods as the one used in C4.5, produce partitions that are applied to localized regions of the instance space. Global methods are applied to the entire dataset. In static methods, attributes are discretized independently of each other; whereas, dynamic methods take into account the interdependencies between them. Equal-width intervals, equal-frequency intervals and unsupervised Monothetic Contrast Criteria
(MCC) (Merck, 1993) are unsupervised methods. Discretization methods based on entropy (supervised MCC (Merck, 1993) and Vector Quantization (Kohonen, 1989) are supervised methods. Equal-width intervals and equal-frequency intervals are global methods. The discretization used in the C4.5 decision tree growing phase and Vector Quantization are local methods. All these methods are static methods. Dynamic methods are a promising area of research. As these methods are able to capture interdependencies between attributes, it may improve the accuracy of decision rules.

Researchers (Torgo and Gama, 1996; 1997a; 1997b; Indurkhya and Weiss, 2001) suggest that the discretization process can be used to convert continuous target values into a discrete set of classes and then classification models are used to solve the classification problems. In other words, in a RvC problem, a regression problem is solved by converting into a classification problem. This method employs any classier on a copy of the data that has the target attribute discretized. The whole process of RvC comprises two important stages:

- The discretization of the numeric target variable in order to learn a classification model. There are different discretization methods e.g., equal-width, equal-frequency (Dougherty et al., 1995)
- The reverse process of transforming the class output of the classification model into a numeric prediction. We may use the mean value of the target variable for each interval as the final prediction

Ensembles are a combination of multiple base models (Hansen and Salamon, 1990; Tumer and Ghosh, 1996); the final classification or regression results depends on the combined outputs of individual models. Ensembles have shown to produce better results than single models, provided the models are accurate and diverse (Hansen and Salamon, 1990; Minku and Yao, 2011; Abdulsalam et al., 2011; Zhang and Lu, 2010). Neural networks and decision tree ensembles are quite popular. Bagging (Breiman, 1996) and Boosting methods (Freund and Schapire, 1997) are general and can be used with any classifiers. Bagging and Boosting have been used with regression trees for the regression problem. Several different methods have been proposed to build decision tree ensembles. Breiman (2001) proposes Random Forests. To build a tree, it uses a bootstrap replica of the training sample, then during the tree growing phase, at each node the optimal split is selected from a random subset of size K of candidate features. Geurts et al. (2006) propose Extremely Randomized Trees. Extremely Randomized Trees combines the feature randomization of Random Subspaces with a totally random selection of the cut-point. Random decision trees proposed by Fan et al. (2006; 2003) use completely random splits points. These decision tree ensemble methods have shown excellent performance for the regression problems. In spite of the excellent performance of pure randomization-based ensemble methods, there is little theoretical explanation about their performance (Rodriguez et al., 2006).

The success of an ensemble method depends on its ability to create uncorrelated individual models (Kuncheva, 2004). However, it is very difficult to predict exactly the performance of these ensembles.

Our main contributions in this study are:

- We propose a novel ensemble method for RvC problems
- We show theoretically that for a set of problem, it is possible to predict exactly the performance of the proposed ensembles. Our theoretical predictions match experimental results

**MATERIALS AND METHODS**

We propose an ensemble method for RvC. We also show that the proposed ensembles for RvC performs better than single model with equal-width discretization for RvC, if the number of bins is 3.

**Extreme Randomized Discretization (ERD):** Ahmad (Ahmad, 2010) presents a discretization method, Extreme Randomized Discretization (ERD), for creating ensembles of decision trees. In this method bin boundaries are created randomly. This method was used to discretize attributes. We will use the same method to create ensembles for RvC. Though the same method is used, the theoretical explanation and applications are different. Ahmad (2010) ERD was used to discretize attributes, whereas in this study, ERD is used to discretize the target variable.

We propose that ERD is useful in creating ensembles for RvC. As discussed above, In ERD, bin boundaries for the discretization are created randomly. This may be used in stage (1) of RvC. As it creates diverse datasets, different classifiers can be created. Uncorrelated models are the keys to the success of any ensemble method (Kuncheva, 2004). Now, we will show our theoretical results.

**Theoretical results:** All the results are proved under following conditions:
• The target value is uniformly distributed between 0 and 4L
• Each value is predicted once
• The classification error is 0
• The mean value of the target variable for each interval is the predicted value. As the target value is uniformly distributed, the center of the bin is the predicted value
• y is the target variable
• yp is the target value of the point p
• The number of models in an ensemble is infinity and each model has different bin boundaries
• The final result is the mean of all the predictions

RvC with the equal-width discretization method with two bins: In this case, two equal sized bins are created, the bin boundary is at 2L, all the points at the left side of the bin boundary will be predicted as L (the midpoint of the left bin) and all the points at the right side of the bin boundary will be predicted as 3L (the midpoint of the right bin). Hence, the points with target values around L and 3L will be predicted relatively more accurately, whereas points at the 0, 2L and 4L will have relatively more error.

The Mean Square Error (MSE) in this case is:

\[
\frac{1}{4L} \left[ \frac{1}{4} \int_{0}^{2L} (y - L)^2 dy + \frac{1}{4} \int_{2L}^{4L} (y - 3L)^2 dy \right] = 0.33L^2
\]

(1)

For 4L = 100, the MSE is 208.33.

RvC with ERD with two bins: ERD create different bin boundaries, in different runs (we have assumed that no two bin boundaries are same in different runs, that can be achieved by selecting a new boundary from the boundaries that is not selected before). Hence, the predictions are different in different runs. As given in Fig. 1, the bin boundary (B1) can be anywhere between the minimum value (0) and the maximum value (4L) of the continuous target variable. If the target value we want to predict is yp, if the bin boundary is at the left side of the yp, the predicted value is (4L + B1)/2. If the bin boundary is at the right side of the yp, the predicted value is (0 + B1)/2. As the final result is the mean value of all the predictions. If the number of runs is infinity.

The predicted value is:

\[
\frac{1}{4L} \left[ \frac{1}{4} \int_{0}^{2L} ((4L + B_1)/2) dB_1 + \frac{1}{4} \int_{2L}^{4L} (0 + B_1)/2 dB_1 \right]
\]

(2)

The predicted value = yp/2 + L.

(The general formula is, The predicted value = yp/2 + (y_min + y_max)/4, where y_min is the minimum value of the target and y_max is the maximum value of the target.)

We discuss some of the properties of this result.

For yp = 0 the predicted value is L.
For yp = 2L the predicted value is 2L.
For yp = 4L the predicted value is 3L.

This behavior is different from the RvC with the equal-width method with two bins as in this case target points near the midpoint of the range are predicted more accurately. One of the important point about the predicted value function is that it is a continuous function with respect to the target value. In other words, the predicted values change smoothly with respect to the target value. This is similar to the Guerts et al. (2006) study about the ERT, “extremely and totally randomized tree ensembles hence provide an interpolation of any output variable which for M goes to infinity is continuous”, where M is the size of the ensemble.

The MSE in this case is:

\[
\frac{1}{4L} \left[ \int_{0}^{4L/3} (y - (y/2 + L))^2 dy + \int_{4L/3}^{8L/3} (y - 2L)^2 dy + \int_{8L/3}^{4L} (y - 20L/6)^2 dy \right] = 0.33L^2
\]

(3)

For 4L = 100, the MSE is 208.3.

The MSE in this case is equal to the RvC with a single classifier with the equal-width discretization method. Hence, there is no advantage of the proposed ensembles over single model with equal-width discretization if the number of bins is 2.

RvC with the equal-width discretization method with three bins: In this case the target variable is divided into equal width bins. The size of these bins is 4L/3 and mid points of these bins will be 4L/6, 2L and 20L/6. Hence, the predicted values will be 4L/6, 2L and 20L/6 depending upon in which bin the point lies. The MSE for this case is:

\[
\frac{1}{4L} \left[ \int_{0}^{4L/3} (y - 4L/6)^2 dy + \int_{4L/3}^{8L/3} (y - 2L)^2 dy + \int_{8L/3}^{4L} (y - 20L/6)^2 dy \right] = 0.14L^2
\]

(4)

For 4L = 100, the MSE is 87.5.
Fig. 1: In the subfigure 1 (top figure) the bin boundary $B_1$ is at the left side of the point to be predicted, $y_p$, whereas in the subfigure 2 (bottom figure), the bin boundary, $B_1$ is at the right side of $y_p$.

**RvC with ERD with three bins:** In this case, there are two bin boundaries; $B_1$ and $B_2$. To calculate the predicted value, we will calculate the mean value of all the predicted values by different models. There are two cases (Fig. 2):

- The bin boundary $B_1$ is left of the given point $y_p$. The two conditions are possible:
  - The bin boundary $B_2$ is at the right of the $B_1$. In this case, for different runs $B_2$ is placed at different points between points $B_1$ and 4L. This case is similar to the two bin case with the boundaries; $B_1$ and 4L. Hence, for a given $B_1$, the mean value is $y_p/2 + (4L + B_1)/4$ (by using Eq. 2)
  - The bin boundary $B_2$ is at the left of the $B_1$. In this case, the predicted values is the center of the rightmost bin, which is $(B_1 + 4L)/2$, this value is independent of $B_2$. Hence, the mean value for a given $B_1$ is $(B_1 + 4L)/2$

The probability of the first condition = $(4L - B_1)/4L$. The probability of the second condition = $B_1/4L$. As $B_1$ can take value from 0 to $y_p$. The mean value of this case (the bin boundary $B_1$ is left of the given point $y_p$) is:

$$\frac{1}{y_p} \int_{0}^{y_p} \left[ \frac{y_p/2 + ((4L + B_1)/4)}{(4L - B_1)/4L + ((B_1 + 4L)/2)(B_1/4L)} \right] dB_1$$

(5)

$$= \frac{y_p^2}{24L} + 3y_p/4 + L$$

(6)

Fig. 2: (1) The first bin boundary $B_1$ is at the left side of the $y_p$. The second bin boundary $B_2$ is at the right side $B_1$. (2) The first bin boundary $B_1$ is at left side of the $y_p$. The second bin boundary $B_2$ is at the left side $B_1$. (3) The first bin boundary $B_1$ is at the right side of the $y_p$. The second bin boundary $B_2$ is at the right side $B_1$. (4) The first bin boundary $B_1$ is at the right left side of the $y_p$. The second bin boundary $B_2$ is at the left side $B_1$

- The bin boundary $B_1$ is at right of the given point $y_p$. The two conditions are possible:
  - The bin boundary $B_2$ is at the right of the $B_1$. In this case, the predicted values is the center of the leftmost bin, which is $(B_1)/2$. Hence, the mean value, for a given $B_1$, is $(B_1)/2$
  - The bin boundary $B_2$ is at the left of the $B_1$. In this case, for different runs $B_2$ is placed at different points between points 0 and $B_1$. This case is similar to the two bin case with the range of the target variable between 0 and $B_1$. Hence, the mean value, for a given $B_1$ is, $y_p/2 + (0 + B_1)/4$

The probability of the first case = $(4L - B_1)/4L$. The probability of the second case = $B_1/4L$. As $B_1$ can take value from $y_p$ to 4L. The mean value of this case (the bin boundary $B_1$ is at 4L. The mean value of this case (the bin boundary $B_1$ is at the given point $y_p$) is:

$$\frac{1}{4L - y_p} \int_{y_p}^{4L} \left[ \frac{(B_1/2)(4L - B_1)/4L + ((B_1/2)(B_1/4L))}{y_p/2 + (B_1/4)(B_1/4L)} \right] dB_1$$

(7)
The mean value of all the cases: (The mean value of case 1) + (The mean value of case 2):

\[
= \frac{- (y_p)^2}{24L} + \frac{5y_p}{12} + \frac{3L}{4} \quad (8)
\]

For \( y_p = 0 \) the predicted value is 2L/3.
For \( y_p = 2L \) the predicted value is 2L.
For \( y_p = 4L \) the predicted value is 14L/3.

The MSE for this case is:

\[
MSE = \frac{0.12L^2}{4L} = \frac{1}{4} \left( y - \left( y/2 + \frac{2L/3 + (y_p)^2/8L - (y_p)^2/48L^2}{8L - (y_p)^2/48L^2} \right)^2 \right) \quad (11)
\]

**RESULTS**

We carried out experiments with \( y = x \) function. This is a uniformly distributed function. We generated 10000 points between 0\( \leq x \leq 100 \), 5000 points were used for training and 5000 points were used for testing. We used unpruned C4.5 decision tree (J48 decision tree of WEKA software (Witten and Frank, 2005)) as the classifier. The final result from a classifier was the mean value of the target variable (y in this case) of all the points in the predicted bin. In the results, we found that the classification error was almost 0. As in these experiments all the conditions of our theoretical results were fulfilled, we expected that experimental results should follow the theoretical results. We carried out experiments with two bins and three bins. The size of the ensemble was set to 100. The experiments were conducted following 5×2 cross-validation. The average results are presented in the Table 1. Results suggest that there is an excellent match in experimental results with theoretical results for two bins and three bins cases. We also carried out experiments with 5, 10 and 20 bins. Results suggest that the ratio of the average MSE of RvC with equal-width discretization to the average MSE of RvC with ERD is increasing with the number of bins. This suggests that there is more performance advantage with ERD when we have large number of bins. This verifies our intuition that as we increase the number of bins the performance advantage increases for ERD ensembles. We also carried out experiments with other popular datasets used for regression studies. We also did experiments with REP regression trees (available in WEKA software) with the Bagging procedure. The size of the ensembles was set to 100 for all the experiments. The number of bins was set to 10 for RvC methods.

**DISCUSSION**

Results (MSE) presented in Table 2 suggest the proposed ensemble method (RvC with ERD) perform consistently better than a single model (RvC with equal width discretization method). This shows the effectiveness of the our approach. The comparative study, with REP tree regression trees with Bagging, suggests that our method perform similar to this method. This shows that our method is comparable to the method that is developed specifically for the regression method. In the proposed ensemble method, we used decision trees as the classifier; however, we may use any other classifier. The number of bins is an important variable, as a small number of bins lead to the better classification, however, the value represented by bins will be less representative of the values. If the number of bins is large, the number of points in each bin will be small; this leads to the poor classification accuracy. However, the values represented by bins will more representatives of the points in the bins. One may use cross validation to find out the best number of bins for the best regression results. However, in the present setup, we used the default value of bins as 10 and results suggest that even with the default number of bins, we got the results similar to the models specifically designed for regression. This shows the effectiveness of our proposed ensemble method. Hence, we may use classifier models with the proposed ensemble method to solve regression problems.
Table 1: MSE in different cases. For experimental results, the average results are given. s.d. is given in the bracket.

<table>
<thead>
<tr>
<th>The number of bins</th>
<th>Error with equal sized bins (Theoretical)</th>
<th>Error with equal sized bins (Experimental)</th>
<th>Error with ERD (Theoretical)</th>
<th>Error with ERD (Experimental)</th>
<th>(1)/(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>208.3</td>
<td>209.1(2.2)</td>
<td>208.3</td>
<td>(2)210.3(3.1)</td>
<td>0.99</td>
</tr>
<tr>
<td>3</td>
<td>87.5</td>
<td>90.3(1.7)</td>
<td>75</td>
<td>77.3(1.5)</td>
<td>1.17</td>
</tr>
<tr>
<td>5</td>
<td>-</td>
<td>33.1(0.8)</td>
<td>-</td>
<td>18.6(0.4)</td>
<td>1.78</td>
</tr>
<tr>
<td>10</td>
<td>-</td>
<td>8.3(0.2)</td>
<td>-</td>
<td>2.6(0.1)</td>
<td>3.19</td>
</tr>
<tr>
<td>20</td>
<td>-</td>
<td>2.9(0.1)</td>
<td>-</td>
<td>0.4(0.1)</td>
<td>7.25</td>
</tr>
</tbody>
</table>

Table 2: Experimental results for different methods for different datasets. The average results for RMSE are presented. s.d. is given in the bracket.

<table>
<thead>
<tr>
<th>Name of dataset</th>
<th>RvC with ERD</th>
<th>RvC with equal-width bins</th>
<th>Bagging with REP Regression trees</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abalone</td>
<td>2.24(0.05)</td>
<td>2.89(0.08)</td>
<td>2.17(0.05)</td>
</tr>
<tr>
<td>Bank8FM</td>
<td>3.61(11)×10⁻²</td>
<td>5.31(0.17)×10⁻²</td>
<td>3.52(12)×10⁻²</td>
</tr>
<tr>
<td>Cart</td>
<td>1.06(0.02)</td>
<td>1.46(0.06)</td>
<td>1.05(0.03)</td>
</tr>
<tr>
<td>Delta(Ailerons)</td>
<td>1.72(03)×10⁻⁴</td>
<td>2.75(0.05)×10⁻⁴</td>
<td>2.03(0.03)×10⁻⁴</td>
</tr>
<tr>
<td>Delta(Elevator)</td>
<td>1.52(02)×10⁻³</td>
<td>1.91(0.03)×10⁻³</td>
<td>1.55(0.02)×10⁻³</td>
</tr>
<tr>
<td>House(8L)</td>
<td>3.12(05)×10⁻⁴</td>
<td>4.12(0.08)×10⁻⁴</td>
<td>3.06(0.03)×10⁻⁴</td>
</tr>
<tr>
<td>House(16H)</td>
<td>3.51(07)×10⁻⁴</td>
<td>4.62(10)×10⁻⁴</td>
<td>3.55(0.05)×10⁻⁴</td>
</tr>
<tr>
<td>Housing(Boston)</td>
<td>3.98(09)</td>
<td>5.23(0.12)</td>
<td>4.01(0.10)</td>
</tr>
<tr>
<td>Kin8nm</td>
<td>0.17(0.01)</td>
<td>0.24(0.02)</td>
<td>0.17(0.01)</td>
</tr>
<tr>
<td>Puma8NH</td>
<td>3.28(0.04)</td>
<td>4.50(0.06)</td>
<td>3.25(0.04)</td>
</tr>
<tr>
<td>Puma32H</td>
<td>8.21(0.13)×10⁻³</td>
<td>1.2(0.04)×10⁻²</td>
<td>7.94(0.17)×10⁻³</td>
</tr>
</tbody>
</table>

CONCLUSION

In supervised learning, the target values may be continuous or a discrete set of class. The continuous target values (the regression problem) can be transferred to a discrete set of classes (the classification problem). The discretization process is a popular method to achieve this task. In this study, we proposed a ensemble method for RvC problems. We showed theoretically that the proposed ensemble method performed better than a single model with equal-width discretization method. This is also verified with experiments. Experiments results also suggest that our method performed similar to the method developed for the regression purpose. This suggests that the proposed ensemble method is useful for regression problems. As the proposed method is independent of the choice of the classifier, various classifiers can be used with the proposed method to solve the regression method. In the study, we carried out experiments with the decision trees, however, in future we will do the experiments with other classifiers like naive Bayes and support vector machines to study its effectiveness with other classifiers.

REFERENCES


